Progress In Electromagnetics Research M, Vol. 6, 9–22, 2009

DYADIC GREEN FUNCTIONS FOR A DIELECTRIC LAYER ON A PEMC PLANE

J. Komijani and J. Rashed-Mohassel

School of Electrical and Computer Engineering
University of Tehran
Iran

A. Mirkamali

Electrical Engineering Department
Zanjan University
Iran

Abstract—Perfect electromagnetic conductor (PEMC) is a medium where certain linear combinations of electromagnetic fields are required to vanish. Since PMC has found important applications in antenna design, one may expect that PEMC will also have potential for similar applications; therefore it is important to investigate its radiation properties. In this paper, dyadic Green functions in integral forms have been derived for a structure with a dielectric layer on a PEMC plane. Whereas electric and magnetic dyadic Green functions is required to satisfy the dyadic mixed boundary condition on PEMC surface, a new classification of the electric and magnetic dyadic Green functions has been introduced based on parameter M of PEMC boundary. This classification is general and contains classes of dyadic Green functions which satisfy Dirichlet and Neumann boundary conditions.

1. INTRODUCTION

The concept of perfect electromagnetic conductors (PEMC) was introduced as an extension of both the perfect electric conductor (PEC) and the perfect magnetic conductor (PMC) [1]. It is characterized by a real medium parameter, M, which denotes the admittance of the PEMC boundary. This parameter for two special cases, the PMC and PEC, takes the values zero and infinity, respectively.

Corresponding author: J. Komijani (j.komijani@ece.ut.ac.ir).

In the perfect electromagnetic conductor, certain linear combinations of electromagnetic fields are required to vanish as follows.

$$\vec{H} + M\vec{E} = 0 \quad \text{and} \quad \vec{D} - M\vec{B} = 0 \tag{1}$$

Hence, at the surface of a PEMC the boundary conditions are as follows.

$$\hat{n} \times (\vec{H} + M\vec{E}) = 0 \quad \text{and} \quad \hat{n} \cdot (\vec{D} - M\vec{B}) = 0 \tag{2}$$

The most notable specification of PEMC is the nonreciprocity of its boundary when the parameter M has a finite nonzero value [2]. In fact, it has been demonstrated theoretically that a PEMC material acts as a perfect reflector of electromagnetic waves, but differs from the PEC and the PMC in that the reflected wave has a cross-polarized component [3]. Since PEMC does not allow electromagnetic energy to enter, it can be served as a boundary material [2]. Realizations for the PEMC boundary are studied in terms of a layer of some nonreciprocal medium backed by a PEC plane [2].

Lindell and Sihvola introduced a two-parameter duality transformation which is able to transform problems involving PEMC objects with the parameter M in air to corresponding problems where PEMC is replaced by PEC [3]. Thus, problems involving PEMC objects in an isotropic medium can be transformed to ordinary problems. Using the duality transformation, they solved some typical examples: plane-wave reflection from a PEMC plane, rectangular PEMC waveguide, scattering from a small PEMC object, and image theory for a source above a PEMC ground.

Jancewicz has investigated plane electromagnetic wave propagation in PEMC [4]. Also, Ruppin has presented an analytical theory for the scattering of an electromagnetic plane-wave by a perfect electromagnetic conductor sphere [5, 6].

In this paper, the dyadic Green functions in integral form for a structure with a dielectric layer on a PEMC plane have been derived.

In Section 2, a new classification of the electric and magnetic dyadic Green functions has been introduced based on parameter M of PEMC boundary. This classification is general and contains classes of the dyadic Green functions which satisfy the dyadic Dirichlet and Neumann boundary conditions. In Section 3, the dyadic Green functions in integral form for a structure with a dielectric layer on a PEMC plane have been derived. We will use $\exp(-i\omega t)$ convention in our analysis.

2. CLASSIFICATION OF DYADIC GREEN FUNCTIONS

The boundary conditions of a PEMC surface are represented in (2). The tangential boundary conditions can be written in terms of electric or magnetic fields as follows:

$$\hat{n} \times \left[\nabla \times \vec{E} + iM\eta k\vec{E}\right] = 0 \tag{3}$$

$$\hat{n} \times \left[\nabla \times \vec{H} - \frac{ik}{M\eta}\vec{H} - \vec{J}\right] = 0, \tag{4}$$

where k and η are respectively the wave number and the wave impedance in the electromagnetic medium in contact with the PEMC medium. Also, $\hat{n}$ denotes the unit normal vector pointed from boundary of PEMC medium, and $\vec{J}$ denotes the volume current density infinitesimal beyond the PEMC boundary. By considering three sets of fields due to three orthogonal infinitesimal electric dipoles, we can transform (3) into a dyadic form [7].

The technique of dyadic Green function (DGF) is introduced mainly to formulate various canonical electromagnetic problems in a systematic manner to avoid treatments of many special cases which can be treated as one general problem [7]. In general, the notations $\bar{\bar{G}}_e$ and $\bar{\bar{G}}_m$ are used to denote, respectively, the electric and the magnetic DGFs which are introduced in [7]; they are solutions of the dyadic differential equations

$$\nabla \times \nabla \times \bar{\bar{G}}_e(\bar{R}, \bar{R}') - k^2\bar{\bar{G}}_e(\bar{R}, \bar{R}') = \bar{\bar{I}}\delta(\bar{R}, \bar{R}') \tag{5}$$

$$\nabla \times \nabla \times \bar{\bar{G}}_m(\bar{R}, \bar{R}') - k^2\bar{\bar{G}}_m(\bar{R}, \bar{R}') = \nabla \times [\bar{\bar{I}}\delta(\bar{R}, \bar{R}')]. \tag{6}$$

Considering a region bounded interiorly by a surface S_0, which is the boundary of a PEMC medium, and exteriorly by a surface S_∞ at infinity, a new classification of the electric and magnetic DGFs can be defined based on parameter M of the PEMC boundary. Consider two groups of dyadic functions which satisfy the dyadic mixed boundary condition with a dimensionless parameter α on S_0,

$$\hat{n} \times [\nabla \times \bar{\bar{G}}_{e(\alpha)}(\bar{R}, \bar{R}') + i\alpha k\bar{\bar{G}}_{e(\alpha)}(\bar{R}, \bar{R}')] = 0 \tag{7}$$

$$\hat{n} \times [\nabla \times \bar{\bar{G}}_{m(\alpha)}(\bar{R}, \bar{R}') + i\alpha k\bar{\bar{G}}_{m(\alpha)}(\bar{R}, \bar{R}')] = 0. \tag{8}$$

Actually, $\bar{\bar{G}}_{e(\alpha)}$ and $\bar{\bar{G}}_{m(\alpha)}$ are mathematical dyadic functions which can be used to designate as the electric and the magnetic DGFs. For example, in the case that S_0 denotes the PEC surface where M approaches infinity, if one substantiates $\alpha = \infty$ and $\alpha = 0$ respectively in Equations (7) and (8), these equations reduce to $\hat{n}\times\bar{\bar{G}}_{e(\infty)}(\bar{R},\bar{R}')=0$ and $\hat{n}\times\nabla\times\bar{\bar{G}}_{m(0)}(\bar{R},\bar{R}')=0$. Obviously, $\bar{\bar{G}}_{e(\infty)}$ and $\bar{\bar{\mathrm{G}}}_{\mathrm{m}(0)}$ can be nominated as electric and magnetic DGFs in the PEC case, as it is mentioned in [7]. This example shows that, in the general case which S_0 denotes the PEMC surface with parameter M, the relationship between the electric and the magnetic DGFs can be obtained from the following equations,

$$\nabla\times\bar{\bar{G}}_{e(\alpha)}(\bar{R},\bar{R}') = \bar{\bar{G}}_{m(\bar{\alpha})}(\bar{R},\bar{R}') \tag{9}$$

$$\nabla\times\bar{\bar{G}}_{m(\bar{\alpha})}(\bar{R},\bar{R}') = k^2\bar{\bar{G}}_{e(\alpha)}(\bar{R},\bar{R}')+\bar{\bar{I}}\delta(\bar{R},\bar{R}'), \tag{10}$$

where $\alpha=M\eta$ and $\bar{\alpha}=-1/\alpha$. These relationships, can be examined if one substitutes these equations in Equations (7), i.e.,

$$\hat{n}\times[\bar{\bar{G}}_{m(\bar{\alpha})}(\bar{R},\bar{R}')+i\alpha k(\nabla\times\bar{\bar{G}}_{m(\bar{\alpha})}(\bar{R},\bar{R}')-\bar{\bar{I}}\delta(\bar{R},\bar{R}'))/k^2]=0.$$

Since source point isn't in boundary, the above equation can be written as

$$\hat{n}\times[(-ik/\alpha)\bar{\bar{G}}_{m(\bar{\alpha})}(\bar{R},\bar{R}')+(\nabla\times\bar{\bar{G}}_{m(\bar{\alpha})}(\bar{R},\bar{R}')]=0,$$

which is the same Equation (8) with a dimensionless parameter $\bar{\alpha}$. It means if $\bar{\bar{G}}_{e(\alpha)}$ is considered as the electric DGF of supposed structure, then $\bar{\bar{G}}_{m(\bar{\alpha})}$ is the magnetic DGF of supposed structure. Now, for instance, one can compare Equation (10) with Equation (4.96) in [7], which can be written as

$$\nabla\times\bar{\bar{G}}_{m(0)}(\bar{R},\bar{R}')=k^2\bar{\bar{G}}_{e(\pm\infty)}(\bar{R},\bar{R}')+\bar{\bar{I}}\delta(\bar{R},\bar{R}').$$

Obviously, one can see that equation is special case of Equation (10) when M approaches infinity.

3. DYADIC GREEN FUNCTIONS FOR A DIELECTRIC LAYER ON A PEMC PLANE

The structure under consideration is shown in Fig. 1. The surface of PEMC plane is denoted by S_0 and the interface is denoted by S. The wave number in region 1 (air) and that of region 2 (dielectric) is denoted, respectively, by k_1 and k_2. As it is typical for more than two media, the Green functions will be denoted by $\bar{\bar{G}}^{(ij)}_{e(\alpha)}$ and $\bar{\bar{G}}^{(ij)}_{m(\alpha)}$, which i and j indicate field point and source point, respectively. Therefore, the electric DGFs which will be derived are $\bar{\bar{G}}^{(11)}_{e(\alpha)}$, $\bar{\bar{G}}^{(21)}_{e(\alpha)}$, $\bar{\bar{G}}^{(22)}_{e(\alpha)}$ and $\bar{\bar{G}}^{(12)}_{e(\alpha)}$ where $\alpha=M\eta_2$.

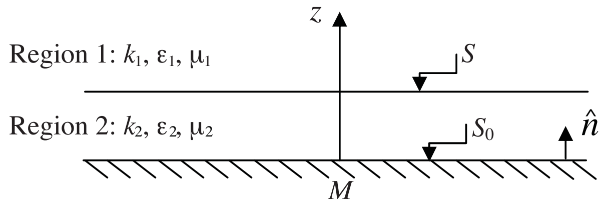

Figure 1. A dielectric layer on a PEMC plane.

The corresponding vector wave functions to represent the DGFs associated with plane stratified media, both electric and magnetic, are denoted by $\bar{M}$ and $\bar{N}$ which can be written as

$$\bar{M}(\bar{k}) = \nabla\times\left(\hat{z}\psi(\bar{k})\right), \quad \bar{M}(\bar{k}) = i(k_y\hat{x} - k_x\hat{y})\psi(\bar{k}) \tag{11}$$

$$\bar{N}(\bar{k}) = \frac{1}{\kappa}\nabla\times\bar{M}(\bar{k}), \qquad \bar{N}(\bar{k}) = \left(-\frac{k_z}{\kappa}(k_x\hat{x}+k_y\hat{y})+\frac{k_x^2+k_y^2}{\kappa}\hat{z}\right)\psi(\bar{k}) \tag{12}$$

where, $\kappa^2 = k_z^2 + k_x^2 + k_y^2$ and $\psi(\bar{k}) = \exp(ik_xx + ik_yy + ik_zz)$.

It can be shown that $\bar{M}$ and $\bar{N}$ are orthogonal, i.e.,

$$\iiint\bar{M}(\bar{k})\cdot\bar{N}(\bar{k}')dV = 0, \quad \iiint\left(\hat{z}\times\bar{M}(\bar{k})\right)\cdot\left(\hat{z}\times\bar{N}(\bar{k}')\right)dV = 0 \tag{13}$$

These orthogonality relations can be used to break up a dyadic equation into several distinct equations.

Considering the vector wave functions, the eigenfunction expansions of the electric dyadic Green functions will be derived in two cases namely; electric current source is in region 1 (3.1) or in region 2 (3.2).

3.1. Source in Region (1)

In this case, one assumes a localized electric current source in region 1. By the method of scattering superposition, the electric DGFs can be written as

$$\bar{\bar{G}}^{(11)}_{e(\alpha)}(\bar{R},\bar{R}') = \bar{\bar{G}}^{(1)}_{e0}(\bar{R},\bar{R}') + \bar{\bar{G}}^{(11)}_{es}(\bar{R},\bar{R}') \tag{14}$$

$$\bar{\bar{G}}^{(21)}_{e(\alpha)}(\bar{R},\bar{R}') = \bar{\bar{G}}^{(21)}_{es}(\bar{R},\bar{R}'), \tag{15}$$

where the dyadic function $\bar{\bar{G}}^{(1)}_{e0}$ denotes the free-space electric DGF defined in a medium of the same constitutive constants as that of region 1 and can be written, by integrating with respect to k_z and using contour integration [7], as

$$\bar{\bar{G}}_{e0}^{(1)}(\bar{R},\bar{R}') = -\frac{1}{(k_1)^2}\hat{z}\hat{z}\delta(\bar{R}-\bar{R}') + \iint\limits_{-\infty}^{+\infty} dk_x dk_y C_{k_1} \left\{\bar{M}(\pm h_1)\bar{M}'(\mp h_1) + \bar{N}(\pm h_1)\bar{N}'(\mp h_1)\right\}, z \gtrless z' \quad (16)$$

where

$$\begin{aligned} \bar{M}(\pm h_j) &= \nabla \times \left(\hat{z}e^{i(k_x x + k_y y \pm h j z)}\right), \\ \bar{M}'(\pm h_j) &= \nabla' \times \left(\hat{z}e^{i(k_x x' + k_y y' \pm h j z')}\right) \end{aligned} \quad (17)$$

$$\begin{aligned} \bar{N}(\pm h_j) &= \frac{1}{k_j}\nabla \times \bar{M}(\pm h_j) \\ \bar{N}'(\pm h_j) &= \frac{1}{k_j}\nabla' \times \bar{M}'(\pm h_j) \end{aligned} \quad (18)$$

$$\begin{aligned} {h_j}^2 &= {k_j}^2 - {k_x}^2 - {k_y}^2 \\ C_{k_j} &= \frac{i}{8\pi^2 h_j (k_x^2 + k_y^2)} \end{aligned}$$

where, indices j, ($j = 1, 2$), represent the regions.

Using some manipulations, the scattered functions must have the form

$$\begin{aligned} \bar{\bar{G}}_{es}^{(11)}(\bar{R},\bar{R}') = \iint\limits_{-\infty}^{+\infty} & dk_x dk_y C_{k_1} \left\{[a_1\bar{M}(h_1) + a_1'\bar{N}(h_1)]\bar{M}'(h_1)\right. \\ & \left. + [b_1\bar{N}(h_1) + b_1'\bar{M}(h_1)]\bar{N}'(h_1)\right\} \end{aligned} \quad (19)$$

$$\begin{aligned} \bar{\bar{G}}_{es}^{(21)}(\bar{R},\bar{R}') = \iint\limits_{-\infty}^{+\infty} & dk_x dk_y C_{k_1} \left\{[a_2^+\bar{M}(+h_2) + a_2^-\bar{M}(-h_2)\right. \\ & + a_2'^+\bar{N}(+h_2) + a_2'^-\bar{N}(-h_2)]\bar{M}'(h_1) \\ & + [b_2^+\bar{N}(+h_2) + b_2^-\bar{N}(-h_2) \\ & \left. + b_2'^+\bar{M}(+h_2) + b_2'^-\bar{M}(-h_2)]\bar{N}'(h_1)\right\}. \end{aligned} \quad (20)$$

Assuming $\mu_2 = \mu_1 = \mu_0$, the boundary condition to be satisfied at S_0 is

$$\hat{z} \times \left(\nabla \times \bar{\bar{G}}_{e(\alpha)}^{(21)} + i\alpha k_2 \bar{\bar{G}}_{e(\alpha)}^{(21)}\right) = 0 \quad (21)$$

and the boundary conditions at S are

$$\begin{cases} \hat{z} \times \left(\bar{\bar{G}}^{(11)}_{e(\alpha)} - \bar{\bar{G}}^{(21)}_{e(\alpha)} \right) = 0 & (22) \\ \hat{z} \times \left(\nabla \times \bar{\bar{G}}^{(11)}_{e(\alpha)} - \nabla \times \bar{\bar{G}}^{(21)}_{e(\alpha)} \right) = 0 & (23) \end{cases}$$

Using orthogonality relations, these three equations can be broken up into twelve distinct equations (Appendix). These twelve equations lead to

$$\begin{cases} a_2^+ = \varrho \frac{(1+\rho')-\alpha^2(1-\rho')}{(1-\rho)(1+\rho')+\alpha^2(1-\rho')(1+\rho)} \\ a_2'^+ = -\varrho \frac{2i\alpha}{(1-\rho)(1+\rho')+\alpha^2(1-\rho')(1+\rho)} \\ b_2^+ = -\varrho' \frac{(1-\rho)-\alpha^2(1+\rho)}{(1-\rho)(1+\rho')+\alpha^2(1-\rho')(1+\rho)} \\ b_2'^+ = \varrho' \frac{2i\alpha}{(1-\rho)(1+\rho')+\alpha^2(1-\rho')(1+\rho)} \end{cases}$$

and $\begin{cases} a_2^- = \rho a_2^+ + \varrho \\ a_2'^- = \rho' a_2'^+ \\ b_2^- = \rho' b_2^+ + \varrho' \\ b_2'^- = \rho b_2'^+ \end{cases}$ and $\begin{cases} a_1 = (a_2^+ + a_2^- e^{-i2\Delta_2}) e^{i\Delta} - e^{-i2\Delta_1} \\ a_1' = (k_2/k_1)(a_2'^+ + a_2'^- e^{-i2\Delta_2}) e^{i\Delta} \\ b_1 = (k_2/k_1)(b_2^+ + b_2^- e^{-i2\Delta_2}) e^{i\Delta} - e^{-i2\Delta_1} \\ b_1' = (b_2'^+ + b_2'^- e^{-i2\Delta_2}) e^{i\Delta} \end{cases}$

where

$$\begin{cases} \Delta_1 = h_1 d \\ \Delta_2 = h_2 d \\ \Delta = \Delta_2 - \Delta_1 \end{cases} \quad \begin{cases} \rho = \frac{h_2 - h_1}{h_2 + h_1} e^{i2\Delta_2} \\ \rho' = \frac{h_2 (k_1)^2 - h_1 (k_2)^2}{h_2 (k_1)^2 + h_1 (k_2)^2} e^{i2\Delta_2} \end{cases} \quad \begin{cases} \varrho = \frac{2h_1}{h_2 + h_1} e^{i\Delta} \\ \varrho' = \frac{2h_1 k_1 k_2}{h_2 (k_1)^2 + h_1 (k_2)^2} e^{i\Delta} \end{cases}$$

3.2. Source in Region (2)

In this case, one assumes a localized electric current source in region 2. Using scattering superposition, the electric dyadic Green functions can be written as

$$\bar{\bar{G}}^{(22)}_{e(\alpha)}(\bar{R}, \bar{R}') = \bar{\bar{G}}^{(2)}_{e0}(\bar{R}, \bar{R}') + \bar{\bar{G}}^{(22)}_{es}(\bar{R}, \bar{R}') \tag{24}$$

$$\bar{\bar{G}}^{(12)}_{e(\alpha)}(\bar{R}, \bar{R}') = \bar{\bar{G}}^{(12)}_{es}(\bar{R}, \bar{R}'), \tag{25}$$

where the dyadic function $\bar{\bar{G}}^{(2)}_{e0}$ denotes the free-space electric DGF defined in a medium of the same constitutive constants as that of region 2 and can be written as, by integrating with respect to k_z and using contour integration [7], as

$$\bar{\bar{G}}_{e0}^{(2)}(\bar{R},\bar{R}') = -\frac{1}{(k_2)^2}\hat{z}\hat{z}\delta(\bar{R}-\bar{R}') + \iint\limits_{-\infty}^{+\infty} dk_x dk_y C_{k_2} \left\{\bar{M}(\pm h_2)\bar{M}'(\mp h_2)+\bar{N}(\pm h_2)\bar{N}'(\mp h_2)\right\}, z \lessgtr z' \quad (26)$$

where C_{k_2} and h_2 are defined after Equation (18).

Using some manipulations, the scattered functions must have the form

$$\begin{aligned}\bar{\bar{G}}_{es}^{(22)}(\bar{R},\bar{R}') = \iint_{-\infty}^{+\infty} & dk_x dk_y C_{k_2}\big\{\bar{M}(h_2)[A_2^{+}\bar{M}'(+h_2)+A_2^{-}\bar{M}'(-h_2)\\ & +A_2'^{+}\bar{N}'(+h_2)+A_2'^{-}\bar{N}'(-h_2)]+\bar{M}(-h_2)[B_2^{+}\bar{M}'(+h_2)\\ & +B_2^{-}\bar{M}'(-h_2)+B_2'^{+}\bar{N}'(+h_2)+B_2'^{-}\bar{N}'(-h_2)]\\ & +\bar{N}(h_2)[C_2^{+}\bar{N}'(+h_2)+C_2^{-}\bar{N}'(-h_2)+C_2'^{+}\bar{M}'(+h_2)\\ & +C_2'^{-}\bar{M}'(-h_2)]+\bar{N}(-h_2)[D_2^{+}\bar{N}'(+h_2)+D_2^{-}\bar{N}'(-h_2)\\ & +D_2'^{+}\bar{M}'(+h_2)+D_2'^{-}\bar{M}'(-h_2)]\big\} \end{aligned} \quad (27)$$

Similarly;

$$\begin{aligned}\bar{\bar{G}}_{es}^{(12)}(\bar{R},\bar{R}') = \iint\limits_{-\infty}^{+\infty} & dk_x dk_y C_{k_2}\big\{\bar{M}(h_1)[A_1^{+}\bar{M}'(+h_2)+A_1^{-}\bar{M}'(-h_2)\\ & +A_1'^{+}\bar{N}'(+h_2)+A_1'^{-}\bar{N}'(-h_2)]+\bar{N}(h_1)[C_1^{+}\bar{N}'(+h_2)\\ & +C_1^{-}\bar{N}'(-h_2)+C_1'^{+}\bar{M}'(+h_2)+C_1'^{-}\bar{M}'(-h_2)]\big\} \end{aligned} \quad (28)$$

Assuming $\mu_2=\mu_1=\mu_0$, the boundary condition to be satisfied at S_0 is

$$\hat{z}\times\left(\nabla\times\bar{\bar{G}}_{e(\alpha)}^{(22)}+i\alpha k_2\bar{\bar{G}}_{e(\alpha)}^{(22)}\right)=0 \quad (29)$$

and the boundary conditions at S are

$$\begin{cases}\hat{z}\times\left(\bar{\bar{G}}_{e(\alpha)}^{(22)}-\bar{\bar{G}}_{e(\alpha)}^{(12)}\right)=0 & (30)\\ \hat{z}\times\left(\nabla\times\bar{\bar{G}}_{e(\alpha)}^{(22)}-\nabla\times\bar{\bar{G}}_{e(\alpha)}^{(12)}\right)=0. & (31)\end{cases}$$

Using orthogonality relations, these three equations can be broken up into twelve distinct equations. On the other hand, since $\psi'(h)$ and $\psi'(-h)$ are two independent functions of z', if the derived equations are assorted in terms of $\psi'(h)$ and $\psi'(-h)$, then their coefficients are required separately to vanish. Hence, twenty four equations result. After simplifying these twenty four equations (Appendix), one obtains

$$\begin{cases} A_2^{+} = \frac{(1+\rho')-\alpha^2(1-\rho')}{(1-\rho)(1+\rho')+\alpha^2(1-\rho')(1+\rho)} \\ A_2'^{+} = \frac{2i\alpha}{(1-\rho)(1+\rho')+\alpha^2(1-\rho')(1+\rho)} \\ C_2^{+} = -\frac{(1-\rho)-\alpha^2(1+\rho)}{(1-\rho)(1+\rho')+\alpha^2(1-\rho')(1+\rho)} \\ C_2'^{+} = -\frac{2i\alpha}{(1-\rho)(1+\rho')+\alpha^2(1-\rho')(1+\rho)} \end{cases}$$

$$\text{and}\begin{cases} A_2^{-} = \rho A_2^{+} \\ A_2'^{-} = \rho' A_2'^{+} \\ C_2^{-} = \rho' C_2^{+} \\ C_2'^{-} = \rho C_2'^{+} \end{cases} \quad \text{and}\begin{cases} B_2^{+} = \rho A_2^{+} \\ B_2'^{+} = \rho A_2'^{+} \\ D_2^{+} = \rho' C_2^{+} \\ D_2'^{+} = \rho' C_2'^{+} \end{cases} \quad \text{and}\begin{cases} B_2^{-} = \rho A_2^{-} + \rho \\ B_2'^{-} = \rho A_2'^{-} \\ D_2^{-} = \rho' C_2^{-} + \rho' \\ D_2'^{-} = \rho' C_2'^{-} \end{cases}$$

$$\text{and}\begin{cases} A_1^{+} = (A_2^{+} + B_2^{+} e^{-i2\Delta_2})e^{i\Delta} \\ A_1'^{+} = (A_2'^{+} + B_2'^{+} e^{-i2\Delta_2})e^{i\Delta} \\ C_1^{+} = (k_2/k_1)(C_2^{+} + D_2^{+} e^{-i2\Delta_2})e^{i\Delta} \\ C_1'^{+} = (k_2/k_1)(C_2'^{+} + D_2'^{+} e^{-i2\Delta_2})e^{i\Delta} \end{cases}$$

$$\text{and}\begin{cases} A_1^{-} = (1 + A_2^{-} + B_2^{-} e^{-i2\Delta_2})e^{i\Delta} \\ A_1'^{-} = (A_2'^{-} + B_2'^{-} e^{-i2\Delta_2})e^{i\Delta} \\ C_1^{-} = (k_2/k_1)(1 + C_2^{-} + D_2^{-} e^{-i2\Delta_2})e^{i\Delta} \\ C_1'^{-} = (k_2/k_1)(C_2'^{-} + D_2'^{-} e^{-i2\Delta_2})e^{i\Delta} \end{cases}$$

where ρ and ρ' are defined previously.

4. CONCLUSION

In this paper, a new classification of the electric and magnetic dyadic Green functions has been introduced with a dimensionless parameter α. The relation between the electric and magnetic dyadic Green functions with parameter α has been obtained. Using this classification, the Green theorem can be employed in problems involving a PEMC. The symmetrical properties of dyadic Green functions for our new classification are left for another work. As an important instance, the dyadic Green functions in integral form for a structure with a dielectric layer on a PEMC plane have been derived.

ACKNOWLEDGMENT

This work is supported by a grant from the Iran Telecommunication Research Center.

APPENDIX A.

a) Source in Region (1)

In this case, Equations (16), (19), and (20) can be substitute into (21) to (23). Using orthogonality relations, Equations (21) to (23) can be broken up into twelve distinct equations which can be written as

$$(21) \xrightarrow{\text{yields}} \begin{cases} (a_2^+ - a_2^-) + i\alpha(a_2'^+ - a_2'^-) = 0 & \text{(A1a)} \\ (a_2'^+ + a_2'^-) + i\alpha(a_2^+ + a_2^-) = 0 & \text{(A1b)} \\ (b_2^+ + b_2^-) + i\alpha(b_2'^+ + b_2'^-) = 0 & \text{(A1c)} \\ (b_2'^+ - b_2'^-) + i\alpha(b_2^+ - b_2^-) = 0 & \text{(A1d)} \end{cases}$$

$$(22) \xrightarrow{\text{yields}} \begin{cases} (a_2^+ + a_2^- e^{-i2\Delta_2}) = (a_1 + e^{-i2\Delta_1})e^{-i\Delta} & \text{(A2a)} \\ h_2 k_1 (a_2'^+ - a_2'^- e^{-i2\Delta_2}) = h_1 k_2 (a_1')e^{-i\Delta} & \text{(A2b)} \\ h_2 k_1 (b_2^+ - b_2^- e^{-i2\Delta_2}) = h_1 k_2 (b_1 - e^{-i2\Delta_1})e^{-i\Delta} & \text{(A2c)} \\ (b_2'^+ + b_2'^- e^{-i2\Delta_2}) = (b_1')e^{-i\Delta} & \text{(A2d)} \end{cases}$$

$$(23) \xrightarrow{\text{yields}} \begin{cases} h_2 (a_2^+ - a_2^- e^{-i2\Delta_2}) = h_1 (a_1 - e^{-i2\Delta_1})e^{-i\Delta} & \text{(A3a)} \\ k_2 (a_2'^+ + a_2'^- e^{-i2\Delta_2}) = k_1 (a_1')e^{-i\Delta} & \text{(A3b)} \\ k_2 (b_2^+ + b_2^- e^{-i2\Delta_2}) = k_1 (b_1 + e^{-i2\Delta_1})e^{-i\Delta} & \text{(A3c)} \\ h_2 (b_2'^+ - b_2'^- e^{-i2\Delta_2}) = h_1 (b_1')e^{-i\Delta} & \text{(A3d)} \end{cases}$$

where, $\Delta_1 = h_1 d$, $\Delta_2 = h_2 d$ and $\Delta = \Delta_2 - \Delta_1$.

Now, there are twelve undefined coefficients and twelve algebraic equations. To solve this algebraic system, one can find simplified equations as

$$\xrightarrow{(A2a),\,(A3a)} h_1 a_1 = h_1(a_2^+ + a_2^- e^{-i2\Delta_2})e^{i\Delta} - h_1 e^{-i2\Delta_1}$$
$$= h_2(a_2^+ - a_2^- e^{-i2\Delta_2})e^{i\Delta} + h_1 e^{-i2\Delta_1} \quad \text{(A4a)}$$

$$\xrightarrow{(A2b),\,(A3b)} h_1 k_1 k_2 a_1' = h_2(k_1)^2(a_2'^+ - a_2'^- e^{-i2\Delta_2})e^{i\Delta}$$
$$= h_1(k_2)^2(a_2'^+ + a_2'^- e^{-i2\Delta_2})e^{i\Delta} \quad \text{(A4b)}$$

$$\xrightarrow{(A2c),\,(A3c)} h_1 k_1 k_2 b_1 = h_2(k_1)^2(b_2^+ - b_2^- e^{-i2\Delta_2})e^{i\Delta} + h_1 k_1 k_2 e^{-i2\Delta_1}$$
$$= h_1(k_2)^2(b_2^+ + b_2^- e^{-i2\Delta_2})e^{i\Delta} - h_1 k_1 k_2 e^{-i2\Delta_1} \quad \text{(A4c)}$$

$$\xrightarrow{(A2d),\,(A3d)} h_1 b_1' = h_1(b_2'^+ + b_2'^- e^{-i2\Delta_2})e^{i\Delta}$$
$$= h_2(b_2'^+ - b_2'^- e^{-i2\Delta_2})e^{i\Delta} \quad \text{(A4d)}$$

Hence,

$$\begin{cases} \xrightarrow{(A4a)} a_2^- = \rho a_2^+ + \varrho & \text{(A5a)} \\ \xrightarrow{(A4b)} a_2'^- = \rho' a_2'^+ & \text{(A5b)} \end{cases} \qquad \begin{cases} \xrightarrow{(A4c)} b_2^- = \rho' b_2^+ + \varrho' & \text{(A5c)} \\ \xrightarrow{(A4d)} b_2'^- = \rho b_2'^+ & \text{(A5d)} \end{cases}$$

where, $\rho = \frac{h_2 - h_1}{h_2 + h_1} e^{i2\Delta_2}$, $\rho' = \frac{h_2(k_1)^2 - h_1(k_2)^2}{h_2(k_1)^2 + h_1(k_2)^2} e^{i2\Delta_2}$, $\varrho = \frac{2h_1}{h_2 + h_1} e^{i\Delta}$, $\varrho' = \frac{2h_1 k_1 k_2}{h_2(k_1)^2 + h_1(k_2)^2} e^{i\Delta}$.

With the aid of the eight equations, (A1a) to (A1d) and (A5a) to (A5d), it is a simple task to show that these equations result in

$$a_2^+ = \varrho \frac{(1+\rho') - \alpha^2(1-\rho')}{(1-\rho)(1+\rho') + \alpha^2(1-\rho')(1+\rho)} \quad \text{(A6a)}$$

$$a_2'^+ = -\varrho \frac{2i\alpha}{(1-\rho)(1+\rho') + \alpha^2(1-\rho')(1+\rho)} \quad \text{(A6b)}$$

$$b_2^+ = -\varrho' \frac{(1-\rho) - \alpha^2(1+\rho)}{(1-\rho)(1+\rho') + \alpha^2(1-\rho')(1+\rho)} \quad \text{(A6c)}$$

$$b_2'^+ = \varrho' \frac{2i\alpha}{(1-\rho)(1+\rho') + \alpha^2(1-\rho')(1+\rho)}. \quad \text{(A6d)}$$

The coefficients $a_2^-, a_2'^-, b_2^-$ and $b_2'^-$ can be obtained by Equations (A5a) to (A5d), and the coefficients a_1, a_1', b_1 and b_1' can be obtained by Equations (A4a) to (A4d).

b) Source in Region (2)

In this case, Equations (26) to (28) can be substitute into (29) to (31). Using orthogonality relations, Equations (29) to (31) can be broken up into twelve distinct equations. On the other hand, since

$\psi'(h)$ and $\psi'(-h)$ are two independent functions of z', their coefficients are required to vanish. Hence, twenty four equations can be resulted as follows.

$$(29) \xrightarrow{\text{yields}} \begin{cases} (C_2'^{+} + D_2'^{+}) + i\alpha(A_2^{+} + B_2^{+} + 1) = 0 & \text{(A7a)} \\ (C_2'^{-} + D_2'^{-}) + i\alpha(A_2^{-} + B_2^{-}) = 0 & \text{(A7b)} \\ (A_2^{+} - B_2^{+} - 1) + i\alpha(C_2'^{+} - D_2'^{+}) = 0 & \text{(A7c)} \\ (A_2^{-} - B_2^{-}) + i\alpha(C_2'^{-} - D_2'^{-}) = 0 & \text{(A7d)} \\ (C_2^{+} + D_2^{+} + 1) + i\alpha(A_2'^{+} + B_2'^{+}) = 0 & \text{(A7e)} \\ (C_2^{-} + D_2^{-}) + i\alpha(A_2'^{-} + B_2'^{-}) = 0 & \text{(A7f)} \\ (A_2'^{+} - B_2'^{+}) + i\alpha(C_2^{+} - D_2^{+} - 1) = 0 & \text{(A7g)} \\ (A_2'^{-} - B_2'^{-}) + i\alpha(C_2^{-} - D_2^{-}) = 0 & \text{(A7h)} \end{cases}$$

$$(30) \xrightarrow{\text{yields}} \begin{cases} (A_2^{+} + B_2^{+}e^{-i2\Delta_2}) = A_1^{+}e^{-i\Delta} & \text{(A8a)} \\ (A_2^{-} + B_2^{-}e^{-i2\Delta_2} + 1) = A_1^{-}e^{-i\Delta} & \text{(A8b)} \\ h_2k_1(C_2'^{+} - D_2'^{+}e^{-i2\Delta_2}) = h_1k_2(C_1'^{+})e^{-i\Delta} & \text{(A8c)} \\ h_2k_1(C_2'^{-} - D_2'^{-}e^{-i2\Delta_2}) = h_1k_2(C_1'^{-})e^{-i\Delta} & \text{(A8d)} \\ (A_2'^{+} + B_2'^{+}e^{-i2\Delta_2}) = A_1'^{+}e^{-i\Delta} & \text{(A8e)} \\ (A_2'^{-} + B_2'^{-}e^{-i2\Delta_2}) = A_1'^{-}e^{-i\Delta} & \text{(A8f)} \\ h_2k_1(C_2^{+} - D_2^{+}e^{-i2\Delta_2}) = h_1k_2(C_1^{+})e^{-i\Delta} & \text{(A8g)} \\ h_2k_1(C_2^{-} - D_2^{-}e^{-i2\Delta_2} + 1) = h_1k_2(C_1^{-})e^{-i\Delta} & \text{(A8h)} \end{cases}$$

$$(31) \xrightarrow{\text{yields}} \begin{cases} h_2(A_2^{+} - B_2^{+}e^{-i2\Delta_2}) = h_1A_1^{+}e^{-i\Delta} & \text{(A9a)} \\ h_2(A_2^{-} - B_2^{-}e^{-i2\Delta_2} + 1) = h_1A_1^{-}e^{-i\Delta} & \text{(A9b)} \\ k_2(C_2'^{+} + D_2'^{+}e^{-i2\Delta_2}) = k_1(C_1'^{+})e^{-i\Delta} & \text{(A9c)} \\ k_2(C_2'^{-} + D_2'^{-}e^{-i2\Delta_2}) = k_1(C_1'^{-})e^{-i\Delta} & \text{(A9d)} \\ h_2(A_2'^{+} - B_2'^{+}e^{-i2\Delta_2}) = h_1A_1'^{+}e^{-i\Delta} & \text{(A9e)} \\ h_2(A_2'^{-} - B_2'^{-}e^{-i2\Delta_2}) = h_1A_1'^{-}e^{-i\Delta} & \text{(A9f)} \\ k_2(C_2^{+} + D_2^{+}e^{-i2\Delta_2}) = k_1(C_1^{+})e^{-i\Delta} & \text{(A9g)} \\ k_2(C_2^{-} + D_2^{-}e^{-i2\Delta_2} + 1) = k_1(C_1^{-})e^{-i\Delta} & \text{(A9h)} \end{cases}$$

where, $\Delta_1 = h_1 d, \Delta_2 = h_2 d$ and $\Delta = \Delta_2 - \Delta_1$.

Now, there are twenty four undefined coefficients and twenty four algebraic equations. To solve this algebraic system, one can find simplified equations as

$$\xRightarrow{\text{(A8a), (A9a)}} h_1 A_1^+ e^{-i\Delta} = h_1(A_2^+ + B_2^+ e^{-i2\Delta_2}) = h_2(A_2^+ - B_2^+ e^{-i2\Delta_2}) \quad \text{(A10a)}$$

$$\xRightarrow{\text{(A8b), (A9b)}} h_1 A_1^- e^{-i\Delta} = h_1(1 + A_2^- + B_2^- e^{-i2\Delta_2}) = h_2(1 + A_2^- - B_2^- e^{-i2\Delta_2}) \quad \text{(A10b)}$$

$$\xRightarrow{\text{(A8c), (A9c)}} h_1 k_1 k_2 C_1'^+ e^{-i\Delta} = h_2(k_1)^2(C_2'^+ - D_2'^+ e^{-i2\Delta_2}) = h_1(k_2)^2(C_2'^+ + D_2'^+ e^{-i2\Delta_2}) \quad \text{(A10c)}$$

$$\xRightarrow{\text{(A8d), (A9d)}} h_1 k_1 k_2 C_1'^- e^{-i\Delta} = h_2(k_1)^2(C_2'^- - D_2'^- e^{-i2\Delta_2}) = h_1(k_2)^2(C_2'^- + D_2'^- e^{-i2\Delta_2}) \quad \text{(A10d)}$$

$$\xRightarrow{\text{(A8e), (A9e)}} h_1 A_1'^+ e^{-i\Delta} = h_1(A_2'^+ + B_2'^+ e^{-i2\Delta_2}) = h_2(A_2'^+ - B_2'^+ e^{-i2\Delta_2}) \quad \text{(A10e)}$$

$$\xRightarrow{\text{(A8f), (A9f)}} h_1 A_1'^- e^{-i\Delta} = h_1(A_2'^- + B_2'^- e^{-i2\Delta_2}) = h_2(A_2'^- - B_2'^- e^{-i2\Delta_2}) \quad \text{(A10f)}$$

$$\xRightarrow{\text{(A8g), (A9g)}} h_1 k_1 k_2 C_1^+ e^{-i\Delta} = h_2(k_1)^2(C_2^+ - D_2^+ e^{-i2\Delta_2}) = h_1(k_2)^2(C_2^+ + D_2^+ e^{-i2\Delta_2}) \quad \text{(A10g)}$$

$$\xRightarrow{\text{(A8h), (A9h)}} h_1 k_1 k_2 C_1^- e^{-i\Delta} = h_2(k_1)^2(1 + C_2^- - D_2^- e^{-i2\Delta_2}) = h_1(k_2)^2(1 + C_2^- + D_2^- e^{-i2\Delta_2}) \quad \text{(A10h)}$$

Hence,

$$\xRightarrow{\text{(A10a)}} B_2^+ = \rho A_2^+ \quad \text{(A11a)}, \qquad \xRightarrow{\text{(A10b)}} B_2^- = \rho A_2^- + \rho \quad \text{(A11b)}$$

$$\xRightarrow{\text{(A10c)}} D_2'^+ = \rho' C_2'^+ \quad \text{(A11c)}, \qquad \xRightarrow{\text{(A10d)}} D_2'^- = \rho' C_2'^- \quad \text{(A11d)}$$

$$\xRightarrow{\text{(A10e)}} B_2'^+ = \rho A_2'^+ \quad \text{(A11e)}, \qquad \xRightarrow{\text{(A10f)}} B_2'^- = \rho A_2'^- \quad \text{(A11f)}$$

$$\xRightarrow{\text{(A10g)}} D_2^+ = \rho' C_2^+ \quad \text{(A11g)}, \qquad \xRightarrow{\text{(A10h)}} D_2^- = \rho' C_2^- + \rho' \quad \text{(A11h)}$$

where, $\rho = \frac{h_2 - h_1}{h_2 + h_1} e^{i2\Delta_2}$, $\rho' = \frac{h_2(k_1)^2 - h_1(k_2)^2}{h_2(k_1)^2 + h_1(k_2)^2} e^{i2\Delta_2}$.

With the aid of the sixteen equations, (A7a) to (A7h) and (A11a) to (A11h), it is a simple task to show that these equations result in

$$A_2^+ = \frac{(1+\rho') - \alpha^2(1-\rho')}{(1-\rho)(1+\rho') + \alpha^2(1-\rho')(1+\rho)} \quad \text{(A12a)}, \qquad A_2^- = \rho A_2^+ \quad \text{(A12b)},$$

$$A_2'^+ = \frac{2i\alpha}{(1-\rho)(1+\rho') + \alpha^2(1-\rho')(1+\rho)} \quad \text{(A12c)}, \qquad A_2'^- = \rho' A_2'^+ \quad \text{(A12d)},$$

$$C_2^+ = -\frac{(1-\rho) - \alpha^2(1+\rho)}{(1-\rho)(1+\rho') + \alpha^2(1-\rho')(1+\rho)} \quad \text{(A12e)}, \qquad C_2^- = \rho' C_2^+ \quad \text{(A12f)},$$

$$C_2'^+ = -\frac{2i\alpha}{(1-\rho)(1+\rho') + \alpha^2(1-\rho')(1+\rho)} \quad \text{(A12g)}, \qquad C_2'^- = \rho C_2'^+ \quad \text{(A12h)},$$

The coefficients $B_2^+, B_2'^+, D_2^+, D_2'^+, B_2^-, B_2'^-, D_2^-$ and $D_2'^-$ can be obtained by Equations (A11a) to (A11h) and the coefficients $A_1^+, A_1^-, C_1'^+, C_1'^-, A_1'^+, A_1'^-, C_1^+$ and C_1^- can be obtained by Equations (A12a) to (A12h).

REFERENCES

1. Lindell, I. V. and A. H. Sihvola, "Perfect electromagnetic conductor," *Journal of Electromagnetic Waves and Applications*, Vol. 19, 861–869, 2005.
2. Lindell, I. V. and A. H. Sihvola, "Realization of the PEMC boundary," *IEEE Trans. Antennas and Propagat.*, Vol. 53, 3012–3018, 2005.
3. Lindell, I. V. and A. H. Sihvola, "Transformation method for problems involving perfect electromagnetic conductor (PEMC) structures," *IEEE Trans. Antennas and Propagat.*, Vol. 53, 3005–3011, 2005.
4. Jancewicz, B., "Plane electromagnetic wave in PEMC," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 5, 647–659, 2006.
5. Ruppin, R., "Scattering of electromagnetic radiation by a perfect electromagnetic conductor sphere," *Journal of Electromagnetic Waves and Applications*, Vol. 20, 1569–1576, 2006.
6. Ruppin, R., "Scattering of electromagnetic radiation by a perfect electromagnetic conductor cylinder," *Journal of Electromagnetic Waves and Applications*, Vol. 20, 1853–1860, 2006.
7. Tai, C. T., *Dyadic Green's Functions in Electromagnetic Theory*, Ed. 2, IEEE Press, 1994.