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A COMPACT PI-STRUCTURE DUAL BAND TRANSFORMER

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Abstract—A compact Pi-structure transformer operating at arbitrary dual band is proposed in this paper. To achieve the ideal impedance matching, the exact design formulas with no restrictions are obtained. In addition, it is found that there are infinite solutions for this novel transformer considering the fact that three independent variables exist in two equations. Furthermore, to verify the design formulas, the reflection characteristics in different cases are shown by numerical simulations. The horizontal length of this transformer is half of the Monzon's dual band transformer. The proposed dual band transformer can be used in many compact dual band components such as antennas, couplers and power dividers.

1. INTRODUCTION

With the development of mobile communication, the utility ratio of the frequency band has been improved dramatically. In many cases, devices are required to work at two different frequencies (namely dual band)[1-5]. Based on the principle of two-section transformers, Chow et al. proposes a novel transformer of one-third wavelength in two sections for a frequency and its first harmonic [6]. However, the performance of the transformer designed by the inexact method in [6] is deteriorated by an elevating ratio between the input and load impedance. And then, [7] and [8] represent comprehensive analysis and exact solutions of flexible dual band transformer. Recently, this small dual band transformer has been applied in dual band power dividers [9–11] and unequal dual band power dividers [12, 13].

In this paper, we present a novel Pi-structure transformer operating at arbitrary dual band which is more compact than one in [8]. By solving the matching equations, it is found that there are infinite numbers of solutions for this novel transformer considering the fact that three independent variables exist in two equations. Since different matching parameters can be obtained from close-form design formulas in different cases, this compact transformer can be designed conveniently and flexibly in compact dual band components design. For example, in the special case when characteristic impedances of the matching stubs are equal, the proposed transformer has been used in dual band couplers [14] and power dividers [15], which have certified this dual frequency Pi structure design concept simultaneously. In addition, it is necessary to point out that the effects of circuit layouts are not considered in the analysis of transmission lines to obtain final closed-form design equations.

2. DESIGN EQUATIONS

The Pi-structure dual band transformer is illustrated in Fig. 1. Considering that the transmission lines are connected in parallel, the admittances are applied to analyze the design equations.

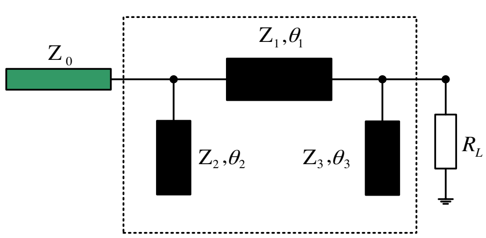

Figure 1. Circuit of Pi-structure dual band transformer.

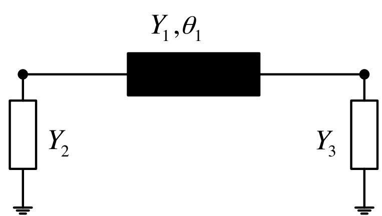

Figure 2. Equivalent circuit of Fig. 1.

In order to be analyzed conveniently, the circuit shown in Fig. 1 can be represented as an equivalent one shown in Fig. 2, whose admittances can be expressed as [16]:

$$\begin{aligned} Y_1 &= \frac{1}{Z_1}, \quad Y_2 = g_2 + jb_2 = \frac{1}{Z_0} + j\frac{\tan(\theta_2)}{Z_2}, \\ Y_3 &= g_3 + jb_3 = \frac{1}{R_L} + j\frac{\tan(\theta_3)}{Z_3}. \end{aligned} \tag{1}$$

The input impedance is desired to equal to Z_0 at dual band. Therefore, this design goal can be given equivalently by

$$Y_2^* = Y_1 \frac{Y_3 + jY_1 \tan(\theta_1)}{Y_1 + jY_3 \tan(\theta_1)}, \tag{2}$$

where the asterisk denotes the complex conjugate symbol. This matching problem is similar to the single transmission line transformation in [17, 18]. From (1) and (2), the following equation can be obtained as

$$g_2 - jb_2 = Y_1 \frac{g_3 + j(b_3 + Y_1 \tan(\theta_1))}{Y_1 - b_3 \tan(\theta_1) + jg_3 \tan(\theta_1)}. \tag{3}$$

Separating and rearranging the real and imagine parts of (3), the equations can be obtained as

$$\begin{cases} Y_1(g_2 - g_3) = (g_2 b_3 - g_3 b_2) \tan(\theta_1), \\ Y_1(b_2 + b_3) = \left(b_2 b_3 + g_2 g_3 - Y_1^2\right) \tan(\theta_1). \end{cases} \tag{4}$$

Substituting (1) into (4), and after some straightforward manipulation, Equation (4) in terms of characteristic impedances can be expressed as,

$$\begin{cases} Z_2 Z_3 (R_L - Z_0) = Z_1 \tan(\theta_1)[Z_2 R_L \tan(\theta_3) - Z_3 Z_0 \tan(\theta_2)], \\ \qquad Z_0 Z_1 R_L [Z_3 \tan(\theta_2) + Z_2 \tan(\theta_3)] \\ = \left[\left(Z_1^2 - Z_0 R_L\right) Z_2 Z_3 + Z_1^2 Z_0 R_L \tan(\theta_2) \tan(\theta_3)\right] \tan(\theta_1). \end{cases} \tag{5}$$

To express briefly, the normalized coefficients, which are significant when they are positive and real, are defined as,

$$k = \frac{R_L}{Z_0}, \quad z_1 = \frac{Z_1}{Z_0}, \quad z_2 = \frac{Z_2}{Z_0}, \quad z_3 = \frac{Z_3}{Z_0}. \tag{6}$$

Substituting (6) into (5), the Equation (5) can be simplified as,

$$\begin{cases} z_2 z_3 (k - 1) = z_1 \tan(\theta_1) \left[z_2 k \tan(\theta_3) - z_3 \tan(\theta_2)\right], \\ \qquad z_1 k \left[z_3 \tan(\theta_2) + z_2 \tan(\theta_3)\right] \\ = \left[\left(z_1^2 - k\right) z_2 z_3 + z_1^2 k \tan(\theta_2) \tan(\theta_3)\right] \tan(\theta_1). \end{cases} \tag{7}$$

It should be pointed out that θ_n, $n = 1, 2, 3$ are θ_{nf_1} and θ_{nf_2} while operating at dual frequencies f_1, f_2. Solutions of (7) will be discussed in the following sections.

3. SOLUTIONS OF DESIGN EQUATIONS

There are three variables in the design equation (7) for this dual band transformer. So we can determine one variable manually and consider it as an independent variable.

3.1. The Electrical and Physical Length Solutions

To assure (7) can be satisfied in dual band (supposed center frequencies satisfy $f_2 \geq f_1$, namely, $f_2 = pf_1$, $p \geq 1$), the following equation should be applied,

$$\tan\theta_{nf1} = \pm\tan\theta_{nf2}, \quad n = 1, 2, 3. \tag{8}$$

The solution of (8) is given by

$$\theta_{nf1} \pm \theta_{nf2} = m\pi, \quad n = 1, 2, 3. \quad m \in N^+. \tag{9}$$

Since small transformer is helpful for microwave engineers to fulfill miniaturization, the situation that the + sign and $m = 1$ [8] is chosen and from (9) the physical lengths can be obtained as follows,

$$l = l_1 = l_2 = l_3 = \frac{1}{2(1+p)}\lambda_1, \tag{10}$$

where λ_1 is the wavelength of f_1. With the line lengths known, we can obtain the following parameters,

$$\alpha = \tan\theta_1 = \tan\theta_2 = \tan\theta_3 = \tan\left(\frac{\pi}{1+p}\right). \tag{11}$$

It is necessary to note that the l_i, $i = 1, 2, 3$ can also be unequal (For example: $l_1 = ml_2$), as long as the final expression (7) is unchanged with operating at both f_1 and f_2. Considering that the corresponding physical lengths are large, this unequal l_i case will not be discussed in the following sections.

3.2. The Characteristic Impedances Discussion

Using (11), (7) can be expressed as

$$\begin{cases} z_2 z_3(k-1) = z_1\alpha^2(z_2 k - z_3), \\ z_1 k(z_3 + z_2) = (z_1^2 - k)z_2 z_3 + z_1^2 k\alpha^2. \end{cases} \tag{12}$$

Because there are three variables namely z_1, z_2, z_3 in (12), it is necessary to define one of the variables for obtaining the other ones. We suppose that z_1 is known here, the solution of (12) can be discussed in the following different cases of k:

A. One case: when $k = 1$, namely, $Z_0 = R_L$, (12) can be rewritten as

$$\begin{cases} z_2 = z_3, \\ 2z_1 z_2 = \left(z_1^2 - 1\right) z_2^2 + z_1^2\alpha^2. \end{cases} \tag{13}$$

To assure z_2 and z_3 are positive and real, the values can be obtained as,

$$z_2 = z_3 = \begin{cases} \dfrac{z_1\left[1-\sqrt{1+(1-z_1^2)\alpha^2}\right]}{(z_1^2-1)}, & 0 < z_1 \leq 1. \\ \dfrac{z_1\left[1\pm\sqrt{1+(1-z_1^2)\alpha^2}\right]}{(z_1^2-1)}, & 1 < z_1 \leq \sqrt{\dfrac{1+\alpha^2}{\alpha^2}}. \end{cases} \tag{14}$$

B. Another case: when $k \neq 1$, namely, $Z_0 \neq R_L$, (12) can be rewritten as,

$$\begin{cases} Cz_2^2 + Dz_2 + E = 0, \\ z_3 = Az_2 + B. \end{cases} \tag{15}$$

where

$$A = \frac{z_1^2 k\alpha^2 - k^2\alpha^2 - k^2 + k}{z_1^2\alpha^2 - k\alpha^2 + k^2 - k}, \quad B = \frac{z_1 k\alpha^2(k-1)}{z_1^2\alpha^2 - k\alpha^2 + k^2 - k},$$
$$C = A(k-1), \quad D = -2B, \quad E = z_1\alpha^2 B.$$

The solution of (15) can be obtained as,

$$\begin{cases} z_2 = \dfrac{B \pm \sqrt{B^2 - CE}}{A(k-1)}, \\ z_3 = Az_2 + B. \end{cases} \tag{16}$$

To assure z_2, z_3 are positive and real, and after some straightforward manipulation, (16) can be rewritten as,

$$z_2 = \begin{cases} \dfrac{z_1\alpha^2\left(1-\sqrt{(k+\alpha^2 k - z_1^2\alpha^2)}\right)}{z_1^2\alpha^2 - k\alpha^2 - k + 1}, & 0 < z_1 \leq \sqrt{\dfrac{k+\alpha^2 k - 1}{\alpha^2}}. \\ \dfrac{z_1\alpha^2\left(1\pm\sqrt{(k+\alpha^2 k - z_1^2\alpha^2)}\right)}{z_1^2\alpha^2 - k\alpha^2 - k + 1}, & \sqrt{\dfrac{k+\alpha^2 k - 1}{\alpha^2}} < z_1 < \sqrt{\dfrac{k+\alpha^2 k}{\alpha^2}}. \end{cases}$$
$$z_3 = Az_2 + B. \tag{17}$$

Obviously, there are two solutions in (14) and (17) in different cases. Considering the practical microwave implementation, one of them will be discarded, which will be discussed in the following.

3.3. The Final Characteristic Impedances Solutions

From the above discussion, it is interesting that (17) includes (14). Combining (14) and (17), we can choose (17) to analyze finally. And then (17) is separated into two different parts, which follow as

$$z_2 = \frac{z_1\alpha^2\left(1-\sqrt{\left(k+\alpha^2 k - z_1^2\alpha^2\right)}\right)}{z_1^2\alpha^2 - k\alpha^2 - k + 1},\ 0 < z_1 \leq \sqrt{\frac{k+\alpha^2 k}{\alpha^2}}. \tag{18a}$$

$$z_2 = \frac{z_1\alpha^2\left(1+\sqrt{\left(k+\alpha^2 k - z_1^2\alpha^2\right)}\right)}{z_1^2\alpha^2 - k\alpha^2 - k + 1},\ \sqrt{\frac{k+\alpha^2 k - 1}{\alpha^2}} < z_1 < \sqrt{\frac{k+\alpha^2 k}{\alpha^2}}. \tag{18b}$$

If z_1 is in the scope of (18b), the subtraction of (18b) and (18a) is

$$\Delta z_2 = \frac{2z_1\alpha^2\left(\sqrt{\left(k+\alpha^2 k - z_1^2\alpha^2\right)}\right)}{z_1^2\alpha^2 - k\alpha^2 - k + 1} > 0, \tag{19}$$

(19) means that the value of (18b) is larger than one of (18a) when z_1 is the same. Especially, when $z_1 \rightarrow \sqrt{k+\alpha^2 k - 1}/\sqrt{\alpha^2}$ in (18b), $z_2 \rightarrow \infty$ can be obtained. But in (18a), z_2 will be a positive and real number, which is more practical in microwave engineering. Considering practical realization in terms of microstrip line, the values of z_1, z_2, z_3 should be in the adequate range. Apparently, (18b) is not suitable to practical design because the characteristic impedances are very high. So, we only choose (18a) as the final design for different values of z_1 in this paper.

Here, the aforementioned discussion can be summarized. If z_1 is known and in the range of (20a), the final generalized design equations of z_2 and z_3 can be expressed as:

$$0 < z_1 \leq \sqrt{\frac{k+\alpha^2 k}{\alpha^2}}, \tag{20a}$$

$$z_2 = \frac{z_1\alpha^2\left(1-\sqrt{\left(k+\alpha^2 k - z_1^2\alpha^2\right)}\right)}{z_1^2\alpha^2 - k\alpha^2 - k + 1}, \tag{20b}$$

$$z_3 = \frac{kz_1\alpha^2\left(k-\sqrt{k+\alpha^2 k - z_1^2\alpha^2}\right)}{z_1^2\alpha^2 - k\alpha^2 + k^2 - k}. \tag{20c}$$

So, we can obtain different impedances solutions with different values of z_1 according to (20). When the values of impedances are very large or small, they can be adjusted using different values of z_1. This is the main advantage of this proposed dual band transformer.

3.4. Special Cases of Solutions

Let us consider some special cases of (20).

When $z_1 = \sqrt{k}$, (20) can be simplified as the following equation,

$$z_1 = \sqrt{k}, \quad z_2 = \frac{\sqrt{k}\alpha^2}{1+\sqrt{k}}, \quad z_3 = \frac{k\alpha^2}{1+\sqrt{k}}. \tag{21}$$

When $z_1 = \sqrt{k(1+\alpha^2)-1}/\sqrt{\alpha^2}$, $(k \geq 1)$, (20) can be simplified as the following equation based on the limitation characteristic,

$$z_1 = \sqrt{\frac{k(1+\alpha^2)-1}{\alpha^2}}, \quad z_2 = \frac{z_1\alpha^2}{2}, \quad z_3 = \frac{kz_1\alpha^2}{k+1}. \tag{22}$$

When $z_1 = \sqrt{k(1+\alpha^2)}/\sqrt{\alpha^2}$, (20) can become as follows,

$$z_1 = \sqrt{\frac{k(1+\alpha^2)}{\alpha^2}}, \quad z_2 = z_3 = z_1\alpha^2. \tag{23}$$

This special case including (23) is the same with the results of [14] and [15], and (23) has been applied in dual band couplers and power dividers in compact structure.

4. ANALYSIS OF SOLUTIONS

In this section, using (11), we analyze the characteristics of (20) against different p and k.

4.1. Single Matching Band

The p will be very large when $f_2 \gg f_1$. Considering $\alpha \approx \pi/(1 + p) \to 0$, the electrical length of this transformer will become very small. When $f_2 \to \infty$, the total transformer will be considered as a lumped inductance and capacitance transforming network, namely the resistances are connected directly with lumped components. This characteristic is similar with the dual band transformer in two sections when $f_2 \to \infty$ [8].

In addition, in the case that p is very large, we can increase the length of transmission lines by choosing a larger m to avoid too small physical length,

$$l = \frac{m}{2(1+p)}\lambda_1. \tag{24}$$

And the corresponding parameter becomes,

$$\alpha = \tan\left(\frac{m\pi}{1+p}\right). \tag{25}$$

Based on (24), the value of m can increase along with the value of p, it is suitable that m satisfies $4m < p + 5$ [9] because the physical length should be kept in the adequate range.

4.2. Equal Dual Band and Quarter Wavelength Transformer

If $f_2 = f_1$, then $p = 1$ and $\alpha \to \infty$, the electrical length is $\pi/2$ which stands for one quarter wavelength of f_1. Based on the results of (21)–(23) and conventional transmission line matching concept, $z_1 = \sqrt{k}$ is only chosen in this case and $z_2, z_3 \to \infty$ can be obtained. So, this transformer in this case can be considered as the conventional single band quarter wavelength transformer [16].

4.3. Symmetry Properties

Similar with the transformer of [8], this proposed compact transformer is also symmetry. If the k in (20) is replaced by k_0 and $1/k_0$. The following relationship can be expressed as,

$$z_1|_{k=k_0} = k_0 z_1|_{k=1/k_0}, \tag{26}$$

$$k_0 z_2|_{k=k_0} = z_3|_{k=1/k_0}. \tag{27}$$

In fact, the results of the impedances in the case $k = k_0 < 1$ can be obtained by changing the sequence of ones in the case $k_1 = 1/k_0 > 1$.

5. NUMERICAL SIMULATIONS

In this section, some numerical examples are presented based on (20)–(23). In these examples, the reflections coefficients $|\Gamma|$ are with respect to normalized transmission line characteristic impedance $Z_0 = 1$. Therefore, it is convenient to use the normalized parameters (6).

5.1. Example 1

Considering that the example $f_1 = 1$ GHz, $f_2 = 2$ GHz, $k = 4$, we can obtain that the physical length is $\lambda_1/6$ and α equals $\sqrt{3}$. Four different solutions with different z_1, where case 2 is $z_1 = \sqrt{k}$ and case 4 is $z_1 = \sqrt{k + \alpha^2 k}/\sqrt{\alpha^2}$, are presented in Table 1. The responding reflection coefficients are shown in Fig. 3. We can find that the matching bandwidth of case 4 is wider than the other cases. However, the corresponding characteristic impedances are larger which makes it difficult to be fabricated in practical microstrip lines.

Table 1. Solutions of Example 1.

	Case 1 $z_1 = 1$	**Case 2** $z_1 = 2$	**Case 3** $z_1 = 2.2$	**Case 4** $z_1 = 4/\sqrt{3}$
z_2	0.651	2.000	2.978	6.928
z_3	1.578	4.000	5.061	6.928

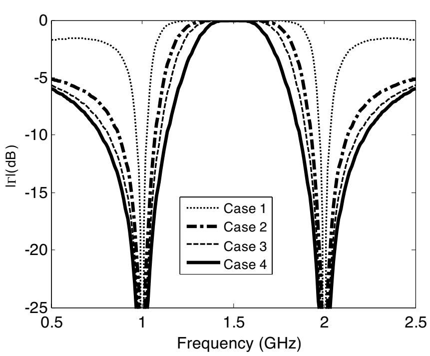

Figure 3. Reflection coefficients of Case 1–4 in Example 1.

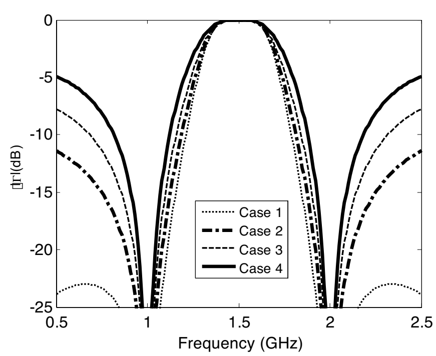

Figure 4. Reflection coefficients of Case 1–4 in Example 2.

5.2. Example 2

Let us consider the example $f_1 = 1$ GHz, $f_2 = 2$ GHz, where $\alpha = \sqrt{3}$ can be obtained. The special solution (23) (namely, the case 4 in example 1) is adopted, and that is $z_1 = 2\sqrt{k}/\sqrt{3}$. But the parameter k is variable.

The corresponding results are listed in Table 2, and the reflection coefficients are shown in Fig. 3. It can be observed from Fig. 3 that the bandwidth will become narrow as the parameter k increases.

Table 2. Solutions of Example 2.

	Case 1 $k = 1$	**Case 2** $k = 2$	**Case 3** $k = 3$	**Case 4** $k = 5$
z_1	1.155	1.633	2.000	2.582
$z_2 = z_3$	3.464	4.899	6.000	7.746

5.3. Example 3

Let us consider the example $f_1 = 1\,\text{GHz}$, $f_2 = pf_1$, $k = 4$, which two kinds of the parameter p (p is close to 1 (A) and p is very large (B)) are used, the special solution (23) is only considered.

Table 3. Solutions of Example 3(A).

	Case 1 $p = 1$	**Case 2** $p = 1.5$	**Case 3** $p = 2.5$	**Case 4** $p = 3$
z_1	2.000	2.103	2.558	2.828
$z_2 = z_3$	∞	19.919	4.022	2.828
l/λ_1	1/4	1/5	1/7	1/8

It is interesting that case 1 is similar with case 4 in Fig. 5. They are all matched at 1 and 3 GHz. It is necessary to point out that case 1 is the same with quarter wavelength transformer, which can match at its odd harmonics. The difference between case 1 and case 4 is that the reflection coefficient bandwidth of case 4 is smaller than case 1. And the stubs are necessary in case 4 while the stubs do not exist in case 1. Table 3 shows the design parameters employed in the example 3(A).

When p is very large (3B) and $m = 1$, the design parameters are listed in Table 4 and the corresponding reflections coefficients are shown in Fig. 6.

Figure 6 shows that the matching frequency f_2 is much higher and the matching characteristics at f_1 of case 1–4 change little when $p \gg 1$. The curve of case 4 overlaps the one of case 3 shown in Fig. 6 when $f < 10\,\text{GHz}$ because $p \gg 10$ in case 3 and 4, which means that the reflection coefficients of case 3 and 4 is very similar in the frequency range $f < 10\,\text{GHz}$.

Table 4. Solutions of Example 3(B).

	Case 1 $p = 5$	**Case 2** $p = 9$	**Case 3** $p = 20$	**Case 4** $p = 100$
z_1	4.000	6.472	13.419	64.309
$z_2 = z_3$	1.333	0.683	0.305	0.062
l/λ_1	1/12	1/20	1/42	1/202

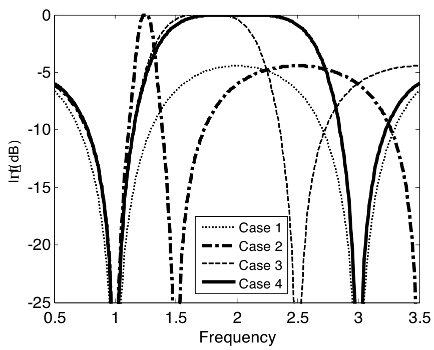

Figure 5. Reflection coefficients of Case 1–4 in Example 3(A).

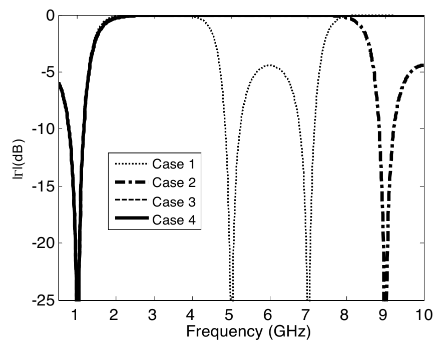

Figure 6. Reflection coefficients of Case 1–4 in Example 3(B).

5.4. Example 4

Let us consider the example $f_1 = 1\,\text{GHz}$, $f_2 = pf_1$, $k = 4$. Different from example 3(A), the solution (21) is chosen and it can be obtained that $z_1 = 2$. The other parameters are listed in Table 5. The corresponding reflection characteristics are shown in Fig. 7. Comparing with the results of the example 3(A), the transformer with lower impedances in this example is easier to be fabricated, but the bandwidth will become narrower slightly.

Table 5. Solutions of Example 4.

	Case 1 $p = 1.5$	**Case 2** $p = 2.5$	**Case 3** $p = 3$
z_2	6.315	1.048	0.667
z_3	12.630	2.097	1.333
l/λ_1	1/5	1/7	1/8

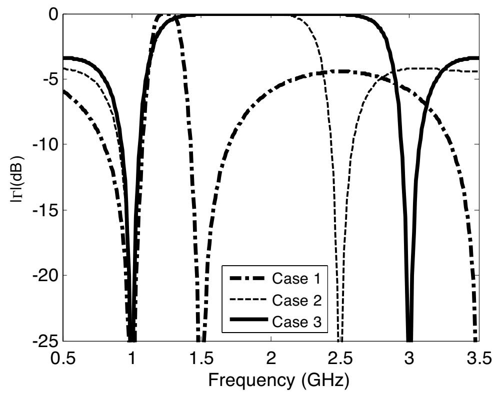

Figure 7. Reflection coefficients of Case 1–3 in Example 4.

6. CONCLUSIONS

A novel compact Pi-structure transformer operating at arbitrary dual band has been presented. This design can shorten the horizontal length of a traditional Monzon's two-section dual band transformer by 50%. In addition, the number of solutions of this transformer can be infinite, which increases the flexibility of applications. This dual band transformer will provide various applicable advances in compact dual band components including antennas' matching, power dividers and couplers.

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