QUADRILINEAR DECOMPOSITION-BASED BLIND SIGNAL DETECTION FOR POLARIZATION SENSITIVE UNIFORM SQUARE ARRAY

Y. Shi and X. Zhang

Electronic Engineering Department
Nanjing University of Aeronautics & Astronautics
Nanjing 210016, China

Abstract—This paper links the polarization-sensitive-array signal detection problem to quadrilinear decomposition model. Exploiting this link, it generates a deterministic blind quadrilinear decomposition-based signal detection algorithm, which doesn't require DOA (direction of arrival) and polarization information and has blind and robust characteristics. The proposed algorithm fully utilizes the polarization, spatial and temporal diversity. The simulation results reveal that the performance of blind quadrilinear decomposition-based signal detection algorithm for polarization sensitive uniform square array is close to nonblind MMSE method, and even works better than trilinear decomposition algorithm.

1. INTRODUCTION

Polarization sensitive arrays have some inherent advantages over traditional antenna arrays, such as the capability of separating signals based on their polarization characteristics [1, 2]. Intuitively, polarization sensitive antenna arrays will provide significant improvements for signals which have different polarization characteristics. Polarization sensitive arrays are used widely in the communication, radio and navigation [5, 6]. Classically, beamforming [7–12, 36] requires knowledge of a direction vector of the desired source. Maximum likelihood signal estimation method for polarization sensitive arrays is proposed in [13]. Filtering performance of polarization sensitive arrays in completely polarized case is investigated in [14]. The methods mentioned above are nonblind methods, because they require the knowledge of DOA and

polarization information. Blind quadrilinear decomposition-based signal detection algorithm for polarization sensitive uniform square array is investigated in this paper.

It is well known that most of signal processing methods are based on the theory of matrix or the bilinear model. In general, matrix decomposition is not unique, since inserting a product of an arbitrary invertible matrix and its inverse in between two matrix factors preserves their product. Matrix decomposition can be unique only if one imposes additional problem-specific structural properties including orthogonality, Vandermonde, Topeliz, constant modulus or finite-alphabet constraints. Compared with the case of matrices, trilinear/multilinear decomposition has a distinctive and attractive feature: it is often unique. The uniqueness of trilinear/multilinear decomposition is of great practical significance, which is crucial in many applications such as chemometrics [15], spectrophotometric, chromatographic and flow injection analysis. In signal processing field, trilinear/multilinear decomposition can be thought of as a generalization of ESPRIT and joint approximate diagonalization ideas [16, 17]. Trilinear/multilinear decomposition is thus naturally related to linear algebra for multi-way arrays [18]. Trilinear/multilinear decomposition is used widely in blind receiver detection for Direct-sequence code-division multiple access (CDMA) system [19], array signal processing [20–26], blind estimation of Multi-Input-Multi-Output (MIMO) system [27], blind Speech Separation [28], downlink receiver for space-time block-boded CDMA System [29] and multiuser detection for Single-Input-Multi-Output (SIMO) CDMA System [30].

Quadrilinear decomposition, as trilinear decomposition's extension, will be introduced in this paper. Our work links the polarization-sensitive-array signal detection problem to quadrilinear decomposition and builds a deterministic blind quadrilinear decomposition-based signal detection algorithm whose performance is close to nonblind minimum mean-squared error (MMSE). The proposed algorithm supports small sample sizes, and even works better than trilinear decomposition. Our proposed algorithm does not require knowledge of the DOA and polarization information, but relies on a fundamental result of the uniqueness of low-rank four-way array decomposition [31].

This paper is structured as follows. Section 2 develops data model. Section 3 discusses identifiability issues and deals with algorithmic issues. Section 4 presents simulation results, and Section 5 summarizes our conclusions.

Denote: We denote by $(.)^*$ the complex conjugation, by $(.)^T$ the matrix transpose, and by $(.)^H$ the matrix conjugate transpose. The notation $(.)^+$ refers to the Moore-Penrose inverse (pseudo inverse).

$\| \ \|_F$ stands for Forbenius norm. $\mathbf{I}_P$ is a $P \times P$ identity matrix.

2. THE DATA MODEL

A uniform square array consisted of $M \times N$ pairs of crossed dipoles is shown in Fig. 1. Each dipole in the array is a short dipole, so the output voltage from each dipole is proportional to the electric field component along that dipole. There are orthogonal short dipoles, the x- and y-axis dipoles, parallel to the x, and y axes, respectively. The distance between nth sublinear array with M dipole pairs and x-axis is $(n-1)d$, $n = 1, 2, \ldots, N$. Similarly, the distance between mth sublinear array with N dipole pairs and y-axis is $(m-1)d$, $m = 1, 2, \ldots, M$. As the distance between two adjacent dipole pairs, d is assumed to be a half wavelength to avoid angle ambiguity problems. We consider signals in the far-field, in which case the signal sources are far enough away that the arriving waves are essentially planes over the length of the array. Assume that the noise is independent of the source, and noise is additive i.i.d. Gaussian.
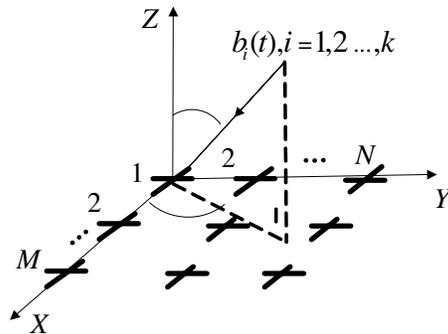
Figure 1. The structure of polarization sensitive uniform square array.

2.1. The Received Signal Model for Polarization Sensitive Antenna

We begin by considering the polarization of an incoming signal. Suppose that an antenna is at the origin of a spherical coordinate system, and a signal $b(t)$ is arriving from direction θ, φ, where φ is the elevation angle and θ is the azimuth angle. Let this signal be a transverse electromagnetic (TEM) wave, and consider the polarization ellipse produced by the electric field in a fixed transverse plane. Polarization parameters are γ, η. We characterize the antenna in terms of its response to linearly polarized signals in the x, and y

directions. Let v_x be the complex voltage induced at the antenna output terminals by an incoming electromagnetic signal with a unit electric field polarized entirely in the x direction. Similarly, let v_y be the output voltages induced by signals with unit electric fields polarized in the y directions. According to [4], the total output voltage from polarization antenna is

$$y_p(t) = \begin{bmatrix} \cos\theta\cos\varphi & -\sin\varphi \\ \cos\theta\sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} \sin\gamma e^{j\eta} \\ \cos\gamma \end{bmatrix} b(t) = \mathbf{s}b(t) \qquad (1)$$

where $\mathbf{s} = \begin{bmatrix} \cos\theta\cos\varphi & -\sin\varphi \\ \cos\theta\sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} \sin\gamma e^{j\eta} \\ \cos\gamma \end{bmatrix}$ is the polarization vector, and it relates to polarization and DOA information.

2.2. The Received Signal Model for Polarization Sensitive Array

Assume that the kth signal $b_k(t)$, $k = 1, 2, \ldots, K$, arrives at the uniform square array with $M \times N$ pairs of crossed dipoles.

When **K** sources impinge the polarization sensitive antenna on the original point, the received signal at the output of the polarization sensitive antenna is

$$\mathbf{R}_{11} = \mathbf{S}\mathbf{B}^T \qquad (2)$$

where $\mathbf{S} = [s_1, s_2, \ldots, s_K]$ is the polarization matrix, $\mathbf{B} = [b_1^T, b_2^T, \ldots, b_K^T]$ is the source matrix with $L \times K$.

When the sublinear array tends to the x direction, the mth ($m = 1, 2, \ldots, M$) element on the x-axis phase-lag compared with original point is $-2\pi(m-1)d\cos\varphi_i\sin\theta_i/\lambda$ and its spatial shift factor is $p_i = \exp(-j2\pi d\cos\varphi_i\sin\theta_i/\lambda)$. Then analyze the situation on the y-axis, in comparison to the original point the nth($n = 1, 2, \ldots, N$) element phase-lag is $-2\pi(n-1)d\sin\varphi_i\sin\theta_i/\lambda$. $q_i = \exp(-j2\pi d\sin\varphi_i\sin\theta_i/\lambda)$ is the spatial shift factor. The phase difference between the (m,n)th dipole pair and the original point is

$$\Delta\phi_{mn}(i) = -2\pi((m-1)d\cos\varphi_i\sin\theta_i + (n-1)d\sin\varphi_i\sin\theta_i)/\lambda \quad (3)$$

Compared with the received signal of the element on the original point, which's shown as Eq. (2), the received signal of the (m,n)th element

in the uniform square array can be described as:

$$\mathbf{R}_{m,n} = \mathbf{S}\begin{bmatrix} \exp(\Delta\phi_{mn}(1)) & & & \\ & \exp(\Delta\phi_{mn}(2)) & & \\ & & \ddots & \\ & & & \exp(\Delta\phi_{mn}(K)) \end{bmatrix}\mathbf{B}^T$$
$$= \mathbf{S}D_m(\mathbf{G})D_n(\mathbf{H})\mathbf{B}^T, \ m=1,2,\ldots,M; \ n=1,2,\ldots,N; \quad (4)$$

where

$$\mathbf{G} =$$
$$\begin{bmatrix} 1 & \cdots & 1 \\ \exp(-j2\pi d\sin\varphi_1\sin\theta_1/\lambda) & \cdots & \exp(-j2\pi d\sin\varphi_K\sin\theta_K/\lambda) \\ \vdots & \ddots & \vdots \\ \exp(-j2\pi d\sin\varphi_1\sin\theta_1/\lambda) & \cdots & \exp(-j2\pi(N-1)d\sin\varphi_K\sin\theta_K/\lambda) \end{bmatrix} \in \mathbb{C}^{N\times K}(5)$$

$$\mathbf{H} =$$
$$\begin{bmatrix} 1 & \cdots & 1 \\ \exp(-j2\pi d\cos\varphi_1\sin\theta_1/\lambda) & \cdots & \exp(-j2\pi d\sin\cos_K\sin\theta_K/\lambda) \\ \vdots & \ddots & \vdots \\ \exp(-j2\pi(M-1)d\cos\varphi_1\sin\theta_1/\lambda) & \cdots & \exp(-j2\pi(M-1)d\sin\cos_K\sin\theta_K/\lambda) \end{bmatrix}$$
$$\in \mathbb{C}^{M\times K} \quad (6)$$

$D_i(.)$ is understood as an operator that extracts the ith row of its matrix argument and constructs a diagonal matrix out of it.

If adding noise,

$$\tilde{\mathbf{R}}_{mn} = \mathbf{R}_{mn}+\mathbf{N}_{mn}=\mathbf{S}D_m(\mathbf{G})D_n(\mathbf{H})\mathbf{B}^T+\mathbf{N}_{mn},$$
$$m = 1,2,\ldots,M; n=1,2\ldots,N \quad (7)$$

where $\mathbf{N}_{mn}$ can be regarded as the slice of the received noise.

When accumulating $M \times N$ slices into $M \times N \times L \times 2$ four dimensional data set R, the signal in Eq. (4) is also denoted through rearrangements

$$r_{m,n,l,p}=\sum_{k=1}^{K} g_{m,k}h_{n,k}b_{l,k}s_{p,k},$$
$$m = 1,\ldots,M; n=1,\ldots,N; l=1,\ldots,L; p=1,2; \quad (8)$$

where $g_{m,k}$ and $h_{n,k}$ stand for the elements of matrix G and H, respectively, $s_{p,k}$ represents the (p,k) element of polarization matrix S

and $b_{l,k}$ means the (l,k)th element of transmit signal source matrix B. Eq. (8) is called our quadrilinear decomposition.

The symmetry of the quadrilinear model in Eq. (8) allows three more matrix system rearrangements, which can be interpreted as slicing the 4-dimensional data along different directions.

$$\mathbf{U}_{n,p} = \mathbf{B}D_n(\mathbf{H})D_p(\mathbf{S})\mathbf{G}^T, \ n = 1, 2, \dots, N, \ p = 1, 2; \qquad (9)$$

$$\mathbf{V}_{p,l} = \mathbf{G}D_p(\mathbf{S})D_l(\mathbf{B})\mathbf{H}^T, \ p = 1, 2; \ l = 1, \dots, L; \qquad (10)$$

$$\mathbf{W}_{l,m} = \mathbf{H}D_l(\mathbf{B})D_m(\mathbf{G})\mathbf{S}^T, \ l = 1, \dots, L; \ m = 1, 2, \dots, M; \ (11)$$

3. QUADRILINEAR DECOMPOSITION-BASED BLIND SIGNAL DETECTION FOR POLARIZATION SENSITIVE SQUARE ARRAY

3.1. Quadrilinear Alternating Least Squares

QALS (Quadrilinear Alternating Least Square) algorithm is the common data detection method for quadrilinear model. The basic idea of QALS is as follows: (a) Each time, update a matrix by using least squares conditioned on previously obtained estimates of the remaining matrix; (b) proceed to update another matrix; (c) repeat until convergence. QALS algorithm is discussed in detail as follows.

$$\mathbf{R}_{mn} = \begin{bmatrix} \mathbf{R}_{11} \\ \mathbf{R}_{12} \\ \vdots \\ \mathbf{R}_{MN} \end{bmatrix} = \begin{bmatrix} \mathbf{S}D_1(\mathbf{G})D_1(\mathbf{H}) \\ \mathbf{S}D_1(\mathbf{G})D_2(\mathbf{H}) \\ \vdots \\ \mathbf{S}D_M(\mathbf{G})D_N(\mathbf{H}) \end{bmatrix} \mathbf{B}^T$$

According to Eq. (7), Least squares fitting is

$$\min_{G,H,S,B} \left\| \begin{bmatrix} \tilde{\mathbf{R}}_{11} \\ \tilde{\mathbf{R}}_{12} \\ \vdots \\ \tilde{\mathbf{R}}_{MN} \end{bmatrix} - \begin{bmatrix} \mathbf{S}D_1(\mathbf{G})D_1(\mathbf{H}) \\ \mathbf{S}D_1(\mathbf{G})D_2(\mathbf{H}) \\ \vdots \\ \mathbf{S}D_M(\mathbf{G})D_N(\mathbf{H}) \end{bmatrix} \mathbf{B}^T \right\|_F \qquad (12)$$

where $\| \ \|_F$ stands for the Frobenius norm. $\tilde{\mathbf{R}}_{mn}, m = 1, 2, \dots, M$; $n = 1, 2 \dots, N$ are the noisy slices.

Least squares update for $\mathbf{B}$ is

$$\hat{\mathbf{B}}^T = \begin{bmatrix} \hat{\mathbf{S}}D_1(\hat{\mathbf{G}})D_1(\hat{\mathbf{H}}) \\ \hat{\mathbf{S}}D_1(\hat{\mathbf{G}})D_2(\hat{\mathbf{H}}) \\ \vdots \\ \hat{\mathbf{S}}D_M(\hat{\mathbf{G}})D_N(\hat{\mathbf{H}}) \end{bmatrix}^+ \begin{bmatrix} \tilde{\mathbf{R}}_{11} \\ \tilde{\mathbf{R}}_{12} \\ \vdots \\ \tilde{\mathbf{R}}_{MN} \end{bmatrix} \qquad (13)$$

where $[\ ]^{+}$ stands for pseudo-inverse. $\widehat{\mathbf{G}}$, $\hat{\mathbf{H}}$ and $\hat{\mathbf{S}}$ denote previously obtained estimates of $\mathbf{G}$, $\mathbf{H}$ and $\mathbf{S}$.

Similarly, from the second way of slices: $\mathbf{U}_{n,p} = \mathbf{B}D_n(\mathbf{H})D_p(\mathbf{S})\mathbf{G}^T$, $n = 1, 2, \ldots, N$, $p = 1, 2$; the LS update for G is

$$\hat{\mathbf{G}}^T = \begin{bmatrix} \hat{\mathbf{B}}D_1(\hat{\mathbf{H}})D_1(\hat{\mathbf{S}}) \\ \hat{\mathbf{B}}D_1(\hat{\mathbf{H}})D_2(\hat{\mathbf{S}}) \\ \vdots \\ \hat{\mathbf{B}}D_N(\hat{\mathbf{H}})D_P(\hat{\mathbf{S}}) \end{bmatrix}^{+} \begin{bmatrix} \tilde{\mathbf{U}}_{11} \\ \tilde{\mathbf{U}}_{12} \\ \vdots \\ \tilde{\mathbf{U}}_{NP} \end{bmatrix} \tag{14}$$

From the third way of slices: $\mathbf{V}_{p,l} = \mathbf{G}D_p(\mathbf{S})D_l(\mathbf{B})\mathbf{H}^T$, $p = 1, 2$; $l = 1, \ldots, L$; and then LS update for H is

$$\hat{\mathbf{H}}^T = \begin{bmatrix} \hat{\mathbf{G}}D_1(\hat{\mathbf{S}})D_1(\hat{\mathbf{B}}) \\ \hat{\mathbf{G}}D_1(\hat{\mathbf{S}})D_2(\hat{\mathbf{B}}) \\ \vdots \\ \hat{\mathbf{G}}D_P(\hat{\mathbf{S}})D_L(\hat{\mathbf{B}}) \end{bmatrix}^{+} \begin{bmatrix} \tilde{\mathbf{V}}_{11} \\ \tilde{\mathbf{V}}_{12} \\ \vdots \\ \tilde{\mathbf{V}}_{PL} \end{bmatrix} \tag{15}$$

Finally, using the fourth mode of slices: $\mathbf{W}_{l,m} = \mathbf{H}D_l(\mathbf{B})D_m(\mathbf{G})\mathbf{S}^T$, $l = 1, \ldots, L$; $m = 1, 2, \ldots, M$; we can also obtain the LS update for S

$$\hat{\mathbf{S}}^T = \begin{bmatrix} \hat{\mathbf{H}}D_1(\hat{\mathbf{B}})D_1(\hat{\mathbf{G}}) \\ \hat{\mathbf{H}}D_1(\hat{\mathbf{B}})D_2(\hat{\mathbf{G}}) \\ \vdots \\ \hat{\mathbf{H}}D_L(\hat{\mathbf{B}})D_M(\hat{\mathbf{G}}) \end{bmatrix}^{+} \begin{bmatrix} \tilde{\mathbf{W}}_{11} \\ \tilde{\mathbf{W}}_{12} \\ \vdots \\ \tilde{\mathbf{W}}_{LM} \end{bmatrix} \tag{16}$$

ALS is optimal when noise is additive i.i.d. Gaussian [32]. ALS algorithm has several advantages: it is easy to implement, guarantee to converge and simple to extend to higher order data. ALS algorithm is known to suffer from degeneracy and slow convergence. Although a unique solution exists, it is not always guaranteed to be found, as the ALS algorithm can be stuck in local minima [33]. ALS can be initialized by eigen-decomposition to speed up convergence. According to Eq. (4), we get the two slices, like $\mathbf{R}_{1,1} = \mathbf{S}D_1(\mathbf{G})D_1(\mathbf{H})\mathbf{B}^T$; $\mathbf{R}_{1,2} = \mathbf{S}D_1(\mathbf{G})D_2(\mathbf{H})\mathbf{B}^T$, which are also denoted as

$$\mathbf{R}_{1,1} = \mathbf{S}\mathbf{B}_E; \quad \mathbf{R}_{1,2} = \mathbf{S}\mathbf{D}\mathbf{B}_E \tag{17}$$

where $\Phi_{11} = D_1(\mathbf{G})D_1(\mathbf{H})$; $\Phi_{12} = D_1(\mathbf{G})D_2(\mathbf{H})$, $\mathbf{B}_E = \Phi_{11}\mathbf{B}^T$; $\mathbf{D} = \Phi_{12}\Phi_{11}^{-1}$.

Construct auto- and cross-correlation matrices:

$$\mathbf{R}_1 = \mathbf{R}_{1,1}^H \mathbf{R}_{1,1} = \mathbf{B}_E^H \mathbf{S}^H \mathbf{S} \mathbf{B}_E; \quad \mathbf{R}_2 = \mathbf{R}_{1,1}^H \mathbf{R}_{1,2} = \mathbf{B}_E^H \mathbf{S}^H \mathbf{S} \mathbf{D} \mathbf{B}_E \quad (18)$$

Define $\alpha = \mathbf{B}_E^H \mathbf{S}^H \mathbf{S}$, then

$$\mathbf{R}_1 = \alpha \mathbf{B}_E; \quad \mathbf{R}_2 = \alpha \mathbf{D} \mathbf{B}_E \quad (19)$$

According to (19),

$$\alpha^+ \mathbf{R}_1 = \mathbf{D}^{-1} \alpha^+ \mathbf{R}_2 \quad (20)$$

where $[.]^+$ is the pseudo-inverse. Let u_f^H, the fth row of α^+ and λ_k be the kth element along the diagonal of $\mathbf{D}^{-1}$. The general eigen-decomposition for $(\mathbf{R}_1, \mathbf{R}_2)$ is given as

$$\mathbf{u}_k^H (\mathbf{R}_1 - \lambda_k \mathbf{R}_2) = 0, \quad k = 1, 2, \dots, K \quad (21)$$

The λ_k's and $\mathbf{u}_k^H$'s are the generalized eigenvalues and left generalized eigenvectors of $(\mathbf{R}_1, \mathbf{R}_2)$. Once α^+ is recovered, then $\mathbf{B}_E = \alpha^+ \mathbf{R}_1$; $\mathbf{S} = \mathbf{R}_{1,1}[\mathbf{B}_E]^+$.

From other ways of slices, we estimate other parameters' initialization by using eigen-analysis with two slices.

3.2. Identifiability

The k-rank concept is very important in the trilinear algebra.

Definition **1** [18]: Consider a matrix $A \in C^{M \times N}$. If rank$(A) = r$, then A contains a collection of r linearly independent columns. Moreover, if *every* $l \le N$ columns of A are linearly independent, but this does not hold for every $l + 1$ columns, then A has k-rank $k_A = l$. Note that $k_A \le \text{rank}(A), \forall A$.

Theorem 1 [31]: $\mathbf{R}_{mn} = \mathbf{S} D_m(\mathbf{G}) D_n(\mathbf{H}) \mathbf{B}^T + \mathbf{N}_{mn}$, $m = 1, 2, \dots, M$; $n = 1, 2, \dots, N$, where $\mathbf{S} \in C^{2 \times K}$, $\mathbf{B} \in C^{L \times K}$, $\mathbf{H} \in C^{M \times K}$, $\mathbf{G} \in C^{N \times K}$, if

$$k_{\mathbf{S}} + k_{\mathbf{B}} + k_{\mathbf{G}} + k_{\mathbf{H}} \ge 2K + 3 \quad (22)$$

then $\mathbf{G}$, $\mathbf{H}$, $\mathbf{B}$ and $\mathbf{S}$ are unique up to permutation and scaling of columns, that is to say, any other quadruple $\bar{\mathbf{G}}$, $\bar{\mathbf{H}}$, $\bar{\mathbf{B}}$, $\bar{\mathbf{S}}$ that construct $\mathbf{R}_{mn}$ $(m = 1, 2, \dots, M, n = 1, 2, \dots, N)$ is related to $\mathbf{G}$, $\mathbf{H}$, and $\mathbf{S}$ via

$$\bar{\mathbf{G}} = \mathbf{G} \Pi \Delta_1, \quad \bar{\mathbf{H}} = \mathbf{H} \Pi \Delta_2, \quad \bar{\mathbf{B}} = \mathbf{B} \Pi \Delta_3, \quad \bar{\mathbf{S}} = \mathbf{S} \Pi \Delta_4 \quad (23)$$

where Π is a permutation matrix, and Δ_1, Δ_2, Δ_3, Δ_4 are diagonal scaling matrices satisfying

$$\Delta_1 \Delta_2 \Delta_3 \Delta_4 = \mathbf{I} \tag{24}$$

Scale ambiguity and permutation ambiguity are inherent to the separation problem. This is not a major concern. Permutation ambiguity can be resolved by resorting to a priori or embedded information. The scale ambiguity can be resolved by using automatic gain control and embedded information.

Generically, a matrix is full rank and full k-rank. Therefore, Eq. (22) becomes

$$\min(2, K) + \min(L, K) + \min(N, K) + \min(M, K) \geq 2K + 3 \tag{25}$$

For the received noisy signal, we use quadrilinear decomposition to get the estimated matrices

$$\hat{\mathbf{B}} = \mathbf{B}\Pi\Delta_3 + \mathbf{N}_1 \tag{26}$$

where $\mathbf{N}_1$ is the noise. Permutation ambiguity and scale ambiguity are inherent in quadrilinear decomposition. The scale ambiguity can be resolved.

3.3. The Steps of QALS Algorithm

The blind quadrilinear decomposition-based signal detection algorithm for polarization sensitive uniform square array is proposed in this paper. The QALS algorithm can firstly attain the source matrix, and then the hard decision is received for the source matrix. The detailed steps are displayed as follows:

(1) Initialize the direction matrices G, H, the polarization matrix S and the source matrix B;

(2) LS update for B according to Eq. (13);

(3) LS update for G according to Eq. (14);

(4) LS update for H according to Eq. (15);

(5) LS update for S according to Eq. (16);

(6) Repeat step (2) to step(5) until convergence;

(7) Decide for the source matrix.

4. SIMULATION AND ANALYSIS

Let the received noisy signal $\tilde{\mathbf{R}}_{m,n} = \mathbf{S}D_m(\mathbf{G})D_n(\mathbf{H})\mathbf{B}^T + \mathbf{N}_{m,n}$, $m = 1, 2, \ldots, M$; $n = 1, 2 \ldots, N$, where $\mathbf{N}_{m,n}$ is the received noise, and then we define SNR

$$SNR = 10 \log_{10} \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} \left| \mathbf{S}D_m(\mathbf{G})D_n(\mathbf{H})\mathbf{B}^T \right|^2}{\sum_{m=1}^{M} \sum_{n=1}^{N} |\mathbf{N}_{m,n}|^2} \text{dB} \qquad (27)$$

We present Monte Carlo simulations which are to assess the performance of blind signal detection for polarization sensitive uniform square array based on quadrilinear decomposition. The number of Monte Carlo trials is 1200.

A uniform square array with 16 pairs (4×4) of crossed dipoles is used in the experiment. The antennas are considered to be completely polarized in the simulations and each signal source only has one path to polarization sensitive array. We adopt Binary Phase Shift Keying (BPSK) to modulate signal and complex additive gauss white noise is added into this system. We compare our proposed algorithm with the nonblind minimum mean-squared error (MMSE) receiver which requires perfect knowledge of DOA (direction of arrival), SNR and polarization information. MMSE receiver offers a performance bound against which blind algorithms are often measured [34, 35]. For all the simulation, the distance between two adjacent dipole pairs is assumed to be half wavelength and SNR varies from -10 dB to 8 dB. Note that L is the number of snapshots and K is the number of sources.
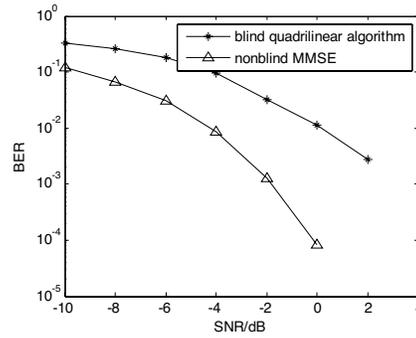
Simulation 1. Quadrilinear decomposition-based signal detection algorithm polarization sensitive square array is compared with trilinear decomposition algorithm.

$$\mathbf{R}_{m,n} = \mathbf{S} \begin{bmatrix} \exp(\Delta\phi_{mn}(1)) & & & \\ & \exp(\Delta\phi_{mn}(2)) & & \\ & & \ddots & \\ & & & \exp(\Delta\phi_{mn}(K)) \end{bmatrix} \mathbf{B}^T$$

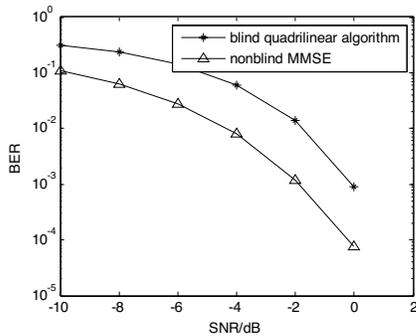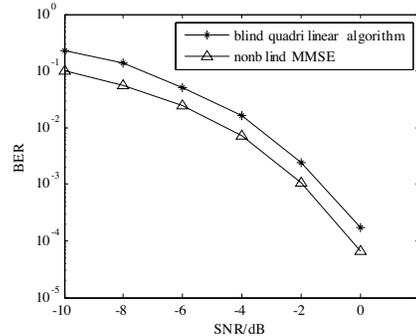$$= \mathbf{S}D_{(m-1)N+n}(\mathbf{A})\mathbf{B}^T$$

where
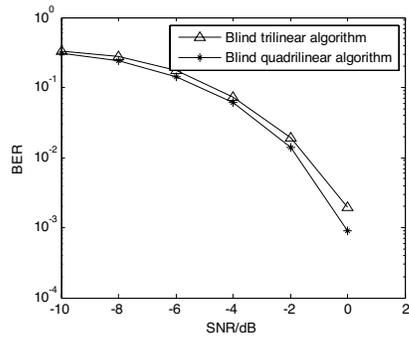
Figure 2. Comparison of algorithm performances.

Figure 3. Algorithm performances comparison with $L = 50$.

Figure 4. Algorithm performances comparison with $L = 100$.

**Figure 5.** Algorithm performances comparison with $L = 300$.

$$\mathbf{A} = \begin{bmatrix} \exp(\Delta\phi_{11}(1)) & \exp(\Delta\phi_{11}(2)) & \cdots & \exp(\Delta\phi_{11}(K)) \\ \exp(\Delta\phi_{12}(1)) & \exp(\Delta\phi_{12}(2)) & \cdots & \exp(\Delta\phi_{12}(K)) \\ \vdots & \vdots & \ddots & \vdots \\ \exp(\Delta\phi_{mn}(1)) & \exp(\Delta\phi_{mn}(2)) & \cdots & \exp(\Delta\phi_{mn}(K)) \\ \vdots & \vdots & \ddots & \vdots \\ \exp(\Delta\phi_{MN}(1)) & \exp(\Delta\phi_{MN}(2)) & \cdots & \exp(\Delta\phi_{MN}(K)) \end{bmatrix}$$

We can use trilinear decomposition [22–26] for blind signal detection for polarization sensitive uniform square array. Trilinear decomposition algorithm is the common data detection method for trilinear model [18], and it has been discussed in detailed in [22]. It can be concluded

from Fig. 2 that our proposed algorithm runs better than trilinear decomposition algorithm. In general, quadrilinear decomposition has better data fitting performance than trilinear decomposition, and it has better bit error rate performance than trilinear decomposition.

Simulation 2. We examine the difference between MMSE (the nonblind minimum mean-squared error) and our blind Quadrilinear decomposition-based receiver algorithm for the uniform square array. When $K = 3$ and $L = 300$, it's clearly revealed in Fig. 3 that our proposed algorithm is very close to traditional nonblind MMSE method. Then the performance of the blind Quadrilinear decomposition-based receiver algorithm for the uniform square array under different L is also investigated in the simulation. We set $L = 50$, $L = 100$ and $L = 300$. Fig. 3, Fig. 4 and Fig. 5 show that the BER (bit error rate) performance of our new algorithm improves with the increase of the snapshots number. When L gets larger, this algorithm has better suppress noise performance, and then its BER performance is improved.

Simulation 3. In this simulation, we study the different performances of Quadrilinear decomposition-based signal detection algorithm in terms of different source numbers. When setting $K = 2$, $K = 3$ and $K = 4$ respectively, it's evidently shown that the BER performance of our new algorithm degrades with the increase of the signal source number K. When K gets larger, the interference between sources increases, and then its BER performance degrades.
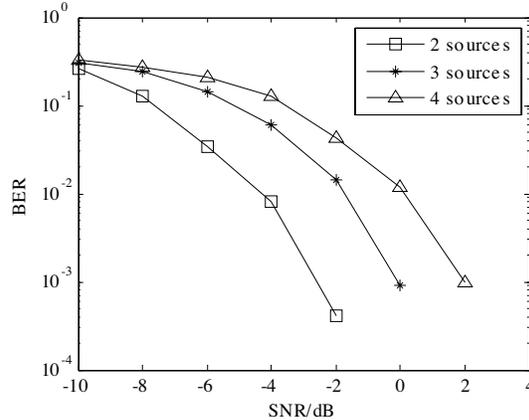
Figure 6. Algorithm performances under different source numbers.

5. CONCLUSIONS

This paper has developed a link between quadrilinear decomposition and blind signal detection for polarization sensitive uniform square array. Relying on the uniqueness of low-rank four-way array decomposition and quadrilinear alternating least squares, we establish a deterministic blind quadrilinear decomposition-based signal detection algorithm. The algorithm, as trilinear decomposition's extention, doesn't require DOA information and polarization information, so it has blind and robust characteristics. The simulation results indicate that the performance of blind quadrilinear decomposition-based signal detection algorithm for polarization sensitive uniform square array is close to nonblind MMSE method, and works better than trilinear decomposition.

ACKNOWLEDGMENT

REFERENCES

1. Costanzo, S. and G. Dimassa, "Direct far-field computation from bi-polar near-field samples," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 9, 1137–1148, 2006.
2. Bao, X. L. and M. J. Ammann, "Comparison of several novel annular-ring microstrip patch antennas for circular polarization," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 11, 1427–1438, 2006.
3. Zhang, M.-T., Y.-B. Chen, Y.-C. Jiao, et al., "Dual circularly polarized antenna of compact structure for radar application," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 14, 1895–1902, 2006.
4. Panagopoulos, A. D. and G. E. Chatzaraki, "Outage performance of single/dual polarized fixed wireless access links in heavy rain climatic regions," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 3, 283–297, 2007.
5. Ng, J. W. P. and A. Manikas, "Polarisation-sensitive array in blind MIMO CDMA system," *Electronics Letters*, Vol. 41, No. 17, 49–50, August 18, 2005.

6. Ioannis, K. and G. B. Keith, "Base station polarization-sensitive adaptive antenna for mobile radio," *Third Annual International Conference on Universal Personal Communications*, 230–235, 1994.

7. Dessouky, M. I., H. A. Sharsha, and Y. A. Albagor, "A novel tapered beamforming window for uniform concentric circular arrays," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 14, 2077–2089, 2006.

8. Guo, B., Y. Wang, J. Li, et al., "Microwave imaging via adaptive beamforming methods for breast cancer detection," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 1, 53–63, 2006.

9. Jeong, Y.-S. and J.-H. Lee, "Estimation of time delay using conventional beamforming-based algorithm for UWB systems," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 15, 2413–2420, 2007.

10. Dessouky, M. I., H. A. Sharshar, and Y. A. Albagory, "Improving the cellular coverage from a high altitude platform by novel tapered beamforming technique," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 13, 1721–1731, 2007.

11. Babayigit, B., A. Akdagli, and K. Guney, "A clonal selection algorithm for null synthesizing of linear antenna arrays by amplitude control," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 8, 1007–1020, 2006.

12. Naghshvarian-Jahromi, M., "Novel Ku band fan beam reflector back array antenna," *Progress In Electromagnetics Research Letters*, Vol. 3, 95–103, 2008.

13. Weiss, A. J. and B. Friedlander, "Maximum likelihood signal estimation for polarization sensitive arrays," *IEEE Transactions on Antennas and Propagation*, Vol. 41, No. 7, 918–925, July 1993.

14. Xu, Z., X. Wang, S. Xiao, et al., "Filtering performance of polarization sensitive array: Completely polarized case," *ACTA ELECTRONICA SINICA*, Vol. 32, No. 8, 1310–1313, 2004.

15. Smilde, A., R. Bro, and P. Geladi, *Multi-way Analysis. Applications in the Chemical Sciences*, John Wiley and Sons, Chichester, U.K., 2004.

16. De Lathauwer, L., B. De Moor, and J. Vandewalle, "Computation of the canonical decomposition by means of a simultaneous generalized schur decompositition," *SIAM J.Matrix Anal. Appl.*, Vol. 26, No. 2, 295–327, 2004.

17. De Lathauwer, L., "A link between the canonical decomposition

in multilinear algebra and simultaneous matrix diagonalization," *SIAM J. Matrix Anal. Appl.*, Vol. 28, No. 3, 642–666, 2006.

18. Kruskal, J. B., "Three-way arrays: Rank and uniqueness of trilinear decompositions, with application to arithmetic complexity and statistics," *Linear Algebra Applicat.*, Vol. 18, 95–138, 1977.

19. Sidiropoulos, N. D., G. B. Giannakis, and R. Bro, "Blind PARAFAC receivers for DS-CDMA systems," *IEEE Transactions on Signal Processing*, Vol. 48, 810–823, March 2000.

20. Sidiropoulos, N. D., R. Bro, and G. B. Giannakis, "Parallel factor analysis in sensor array processing," *IEEE Transactions on Signal Processing*, Vol. 48, No. 8, 2377–2388, August 2000.

21. Yue, R., S. A. Vorobyov, A. B. Gershman, and N. D. Sidiropoulos, "Blind spatial signature estimation via time-varying user power loading and parallel factor analysis," *IEEE Transactions on Signal Processing*, Vol. 53, No. 5, 1697–1710, May 2005.

22. Zhang, X., Y. Shi, and D. Xu, "Novel blind joint direction of arrival and polarization estimation for polarization-sensitive uniform circular array," *Progress In Electromagnetics Research*, PIER 86, 19–37, 2008

23. Zhang, X., D. Wang, and D. Xu, "Novel blind joint direction of arrival and frequency estimation for uniform linear array," *Progress In Electromagnetics Research*, PIER 86, 199–215, 2008.

24. Zhang, X. and D. Xu, "Deterministic blind beamforming for electromagnetic vector sensor array," *Progress In Electromagnetics Research*, PIER 84, 363–377, 2008.

25. Zhang, X., B. Feng, and D. Xu, "Blind joint symbol detection and DOA estimation for OFDM system with antenna array," *Wireless Pers. Commun.*, Vol. 46, 371–383, 2008.

26. Zhang, X. and D. Xu, "Blind PARAFAC signal detection for polarization sensitive array," *EURASIP Journal on Advances in Signal Processing*, Vol. 2007, Article ID 12025, 2007.

27. Yu, Y. and A. P. Petropulu, "Parafac based blind estimation of mimo systemswith possibly more inputs than outputs," *2006 IEEE International Conference on Acoustics, Speech and Signal Processing*, Vol. 3, III-133–III-136, May 14–19, 2006.

28. Mokios, K. N., N. D. Sidiropoulos, and A. Potamianos, "Blind speech separation using parafac analysis and integer least squares," *2006 IEEE International Conference on Acoustics, Speech and Signal Processing*, Vol. 5, V-73–V-76, May 14–19, 2006.

29. Zhang, X. and D. Xu, "Blind PARAFAC receiver for space-

time block-coded cdma system," *2006 International Conference on Communications, Circuits and Systems Proceedings*, 675–678, 2006.

30. Zhang, X. and D. Xu, "PARAFAC multiuser detection for SIMO-CDMA system," *2006 International Conference on Communications, Circuits and Systems Proceedings*, 744–747, 2006.

31. Sidiropoulos, N. D. and R. Bro, "On the uniqueness of multilinear decomposition of N-way arrays," *J. of Chemometrics*, special issue on multi-way analysis, 2000.

32. Vorobyov, S. A., Y. Rong, N. D. Sidiropoulos, and A. B. Gershman, "Robust iterative. Fitting of multilinear models," *IEEE Transactions on Signal Processing*, Vol. 53, No. 8, 2678–2689.

33. Tomasi, G. and R. Bro, "A comparison of algorithms for fitting the PARAFAC model," *Computational Statistics & Data Analysis*, Vol. 50, 1700–1734, 2006.

34. Gesbert, D., J. Sorelius, and A. Paulraj, "Blind multi-user MMSE detection of CDMA signals," *Proc. of Int. Conf. Acoust., Speech, Signal Process.*, 3161–3164, May 1998.

35. Tsatsanis, M. K. and Z. Xu, "Performance analysis of minimum variance CDMA receivers," *IEEE Transactions on Signal Processing*, Vol. 46, 3014–3022, November 1998.

36. Zhang, X., Z. Wang, and D. Xu, "Wavelet packet transform-based least mean square beamformer with low complexity," *Progress In Electromagnetics Research*, PIER 86, 291–304, 2008.