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# DIFFRACTION FROM A SLIT IN AN IMPEDANCE PLANE PLACED AT THE INTERFACE OF TWO SEMI-INFINITE HALF SPACES OF DIFFERENT MEDIA

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**Abstract**—Diffraction of an electromagnetic plane wave from a slit in an impedance plane placed at the interface of two different media, has been formulated rigorously. Both the principal polarizations are considered. The method of analysis is Kobayashi Potential (KP). To determine the unknown weighting functions, boundary conditions are imposed which resulted into dual integral equations (DIEs). These DIEs are solved by using the discontinuous properties of Weber-Schafheitlin's integrals. The resulting expressions are then expanded in terms of Jacobi's polynomials. The problems are then, reduced to matrix equations with infinite number of unknowns whose elements are expressed in terms of infinite integrals. These integrals are hard to solve analytically. The integrals contain poles for particular values of surface impedance and are solved numerically. Illustrative computations are given for far diffracted fields and other physical quantities of interest. To check the validity of our work, we compared the far field patterns with those of obtained through Physical Optics (PO). The agreement is good.

## 1. INTRODUCTION

In the recent years, an increasing interest has been devoted to the design and fabrication of composite materials [1, 2]. These materials have numerous applications in the field of antenna and microwave devices technology. Therefore, understanding the effects of the properties of non perfectly electrically conducting (non-PEC) materials on diffraction phenomenon is an important and interesting. In the present study, we investigate the diffraction from a slit in an impedance plane placed at the interface of two semi-infinite half spaces of two different media and study how the surface impedance effects the diffracting properties of the slit. We have also included the effects of the media surrounding the slit in our study. Some interesting works on the topic are [3–5].

The method of analysis adopted here is the Kobayashi Potential (KP) method. This method uses the discontinuous properties of Weber-Schafheitlin integral. This integral is an infinite integral and its integrand consists of the product of two Bessel's functions multiplied by the powered algebraic single term [15]. This integral shows a discontinuous property when a particular relation holds among the power of the algebraic term and orders of the Bessel's functions. This method has been successfully applied to potential [6, 7] as well as scattering problems for different geometries [8–13]. Imposition of the boundary conditions result in dual integral equations (DIEs). These DIEs can be solved using the above properties of Weber-Schafheitlin integrals and projection method like the method of moment (MoM), in which Jacobi's polynomials are used as the basis functions. One peculiarity of this method is that it provides the option to incorporate the edge conditions. Finally, the problem reduces to matrix equations whose matrix elements are the infinite integrals. These equations can be solved for the determination of unknown expansion coefficients. Numerical computations are conducted for the physical quantities of interest. We compared our results with those obtained through Physical Optics (PO) and found that these are in good agreement.

## 2. FORMULATION AND SOLUTION OF THE PROBLEM

### 2.1. $E$-polarization

The configuration of the problem is shown in Fig. 1. Let the surface impedances of the upper and lower surfaces of the plane are $Z_+$ and $Z_-$. We take $\epsilon_0$, $\mu_0$ as the constitutive parameters of the upper space $y > 0$ and $\epsilon$, $\mu_0$ as the constitutive parameters of the lower space $y < 0$. The width of the slit is $2a$. If $\phi_0$ is the angle of incidence, then $E_z^i$, the incident field and $E_z^r$, the reflected field from the impedance plane,

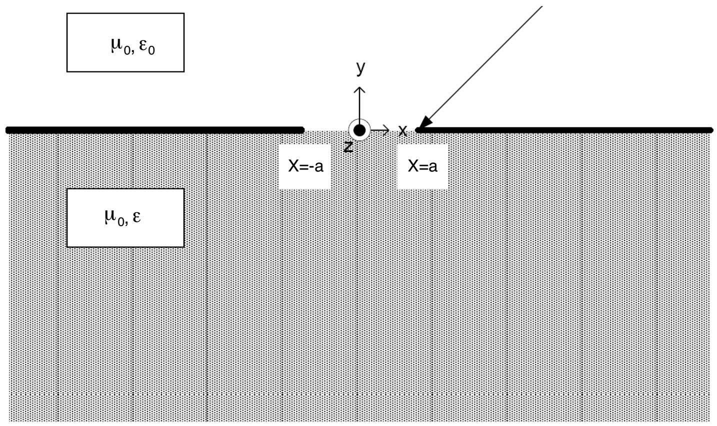


**Figure 1.** Geometry of the problem.

can be written as

$$E_z^i = \exp\left[jk_0(x\cos\phi_0 + y\sin\phi_0)\right] \tag{1a}$$

$$E_z^r = -\frac{Z_0 - Z_+\sin\phi_0}{Z_0 + Z_+\sin\phi_0}\exp\left[jk_0(x\cos\phi_0 - y\sin\phi_0)\right] \tag{1b}$$

We assume the scattered fields $E_z^{d+}$ in the upper space $y > 0$ and $E_z^{d-}$ in the lower space $y < 0$ in the form

$$E_z^{d+} = \int_0^\infty \{g_1(\xi)\cos(x_a\xi) + g_2(\xi)\sin(x_a\xi)\}\exp\left[-\sqrt{\xi^2-\kappa_0^2}y_a\right]d\xi \quad y > 0 \tag{1c}$$

$$E_z^{d-} = \int_0^\infty \{h_1(\xi)\cos(x_a\xi) + h_2(\xi)\sin(x_a\xi)\}\exp\left[\sqrt{\xi^2-\kappa^2}y_a\right]d\xi \quad y < 0 \tag{1d}$$

where $\kappa_0 = k_0 a$, $\kappa = ka$, $x_a = \frac{x}{a}$, $y_a = \frac{y}{a}$ and $k_0$, $k$ are the propagation constant of the upper and lower space respectively. The $g_{1,2}(\xi)$ and $h_{1,2}(\xi)$ are the weighting functions to be determined from the boundary conditions.

The required boundary conditions are given by

$$E_z^t\Big|_{y=0_+} = -Z_+H_x^t\Big|_{y=0_+}, \quad E_z^t\Big|_{y=0_-} = Z_-H_x^t\Big|_{y=0_-}; \quad |x_a| \geq 1 \tag{2a}$$

$$E_z^t\Big|_{y=0_+} = E_z^t\Big|_{y=0_-}, \quad H_x^t\Big|_{y=0_+} = H_x^t\Big|_{y=0_-}; \quad |x_a| \leq 1 \tag{2b}$$

where superscript $t$ means total. From the condition (2a) we have

$$\int_0^\infty \left[1 - j\frac{\sqrt{\xi^2-\kappa_0^2}}{\kappa_0}\zeta_+\right]\left[g_1(\xi)\cos(x_a\xi) + g_2(\xi)\sin(x_a\xi)\right] d\xi = 0; \quad |x_a| \geq 1 \tag{3a}$$

$$\int_0^\infty \left[1 - j\frac{\sqrt{\xi^2-\kappa^2}}{\kappa}\zeta_-\right]\left[h_1(\xi)\cos(x_a\xi) + h_2(\xi)\sin(x_a\xi)\right] d\xi = 0; \quad |x_a| \geq 1 \tag{3b}$$

where $\zeta_+$ and $\zeta_-$ are the normalized surface impedances of the plane. And from the boundary conditions (2b), we have

$$\int_0^\infty \{[h_1(\xi) - g_1(\xi)]\cos(x_a\xi) + [h_2(\xi) - g_2(\xi)]\sin(x_a\xi)\}\, d\xi = \frac{2\zeta_+\sin\phi_0}{1+\zeta_+\sin\phi_0}\exp[j\kappa_0 x_a\cos\phi_0] \tag{3c}$$

$$\begin{aligned}&\int_0^\infty \left\{\left[\sqrt{\xi^2-\kappa^2}h_1(\xi) + \sqrt{\xi^2-\kappa_0^2}g_1(\xi)\right]\cos(x_a\xi)\right.\\ &\left.+\left[\sqrt{\xi^2-\kappa^2}h_2(\xi) + \sqrt{\xi^2-\kappa_0^2}g_2(\xi)\right]\sin(x_a\xi)\right\} d\xi\\ &= \frac{j2\kappa_0\sin\phi_0}{1+\zeta_+\sin\phi_0}\exp[j\kappa_0 x_a\cos\phi_0] \qquad |x_a| \geq 1\end{aligned} \tag{3d}$$

The above expressions are the dual integral equations. Making using of the discontinuous properties of Weber-Schafheitlin's integrals and the edge conditions of $E$-field, we can decide the nature of weighting functions $g_{1,2}(\xi)$ and $h_{1,2}(\xi)$ as follow

$$\begin{aligned} g_1(\xi) &= \frac{1}{j\kappa_0\eta_+ + \sqrt{\xi^2-\kappa_0^2}}\sum_{m=0}^\infty A_m J_{2m+\frac{3}{2}}(\xi)\xi^{-\frac{3}{2}},\\ g_2(\xi) &= \frac{1}{j\kappa_0\eta_+ + \sqrt{\xi^2-\kappa_0^2}}\sum_{m=0}^\infty B_m J_{2m+\frac{5}{2}}(\xi)\xi^{-\frac{3}{2}}\end{aligned} \tag{4a}$$

$$\begin{aligned} h_1(\xi) &= \frac{1}{j\kappa\eta_- + \sqrt{\xi^2-\kappa^2}}\sum_{m=0}^\infty C_m J_{2m+\frac{3}{2}}(\xi)\xi^{-\frac{3}{2}},\\ h_2(\xi) &= \frac{1}{j\kappa\eta_- + \sqrt{\xi^2-\kappa^2}}\sum_{m=0}^\infty D_m J_{2m+\frac{5}{2}}(\xi)\xi^{-\frac{3}{2}}\end{aligned} \tag{4b}$$

where $\eta_\pm = \zeta_\pm^{-1}$ and $J_m(.)$ be the Bessel's function of order $m$. The above solutions for the functions $g_{1,2}(\xi)$ and $h_{1,2}(\xi)$ signify that the tangential components of electromagnetic field are finite at the edge. Separating even and odd functions of the expressions (3c) and (3d) and then projecting the resulting equations into the functional space with elements $p_n^{\pm\frac{1}{2}}(x_a^2)$ [14], we obtain the matrix equations for the expansion coefficients

$$\sum_{m=0}^{\infty}\left[-A_m G_{SE}\left(2m+\frac{3}{2},2n+\frac{1}{2};\kappa_0^+\right)+C_m G_{SE}\left(2m+\frac{3}{2},2n+\frac{1}{2};\kappa^-\right)\right]$$
$$=\frac{2\zeta_+\sin\phi_0}{1+\zeta_+\sin\phi_0}\frac{J_{2n+\frac{1}{2}}(\kappa_0\cos\phi_0)}{(\kappa_0\cos\phi_0)^{\frac{1}{2}}} \tag{5a}$$

$$\sum_{m=0}^{\infty}\left[-B_m G_{SE}\left(2m+\frac{5}{2},2n+\frac{3}{2};\kappa_0^+\right)+D_m G_{SE}\left(2m+\frac{5}{2},2n+\frac{3}{2};\kappa^-\right)\right]$$
$$=\frac{j2\zeta_+\sin\phi_0}{1+\zeta_+\sin\phi_0}\frac{J_{2n+\frac{3}{2}}(\kappa_0\cos\phi_0)}{(\kappa_0\cos\phi_0)^{\frac{1}{2}}} \tag{5b}$$

$$\sum_{m=0}^{\infty}\left[A_m K_{SE}\left(2m+\frac{3}{2},2n+\frac{1}{2};\kappa_0^+\right)+C_m K_{SE}\left(2m+\frac{3}{2},2n+\frac{1}{2};\kappa^-\right)\right]$$
$$=\frac{j2\kappa_0\sin\phi_0}{1+\zeta_+\sin\phi_0}\frac{J_{2n+\frac{1}{2}}(\kappa_0\cos\phi_0)}{(\kappa_0\cos\phi_0)^{\frac{1}{2}}} \tag{5c}$$

$$\sum_{m=0}^{\infty}\left[B_m K_{SE}\left(2m+\frac{5}{2},2n+\frac{3}{2};\kappa_0^+\right)+D_m K_{SE}\left(2m+\frac{5}{2},2n+\frac{3}{2};\kappa^-\right)\right]$$
$$=-\frac{2\kappa_0\sin\phi_0}{1+\zeta_+\sin\phi_0}\frac{J_{2n+\frac{3}{2}}(\kappa_0\cos\phi_0)}{(\kappa_0\cos\phi_0)^{\frac{1}{2}}} \qquad n=0,1,2,\dots \tag{5d}$$

where

$$G_{SE}\left(\mu,\nu;\kappa^\pm\right)=\int_0^\infty\frac{1}{j\kappa\eta_\pm+\sqrt{\xi^2-\kappa^2}}\frac{J_\mu(\xi)J_\nu(\xi)}{\xi}d\xi \tag{6a}$$

$$K_{SE}\left(\mu,\nu;\kappa^\pm\right)=\int_0^\infty\frac{\sqrt{\xi^2-\kappa^2}}{j\kappa\eta_\pm+\sqrt{\xi^2-\kappa^2}}\frac{J_\nu(\xi)J_\nu(\xi)}{\xi}d\xi \tag{6b}$$

In writing the Equation (5), we have used the following relations

$$\cos x = \sqrt{\frac{\pi x}{2}} J_{-\frac{1}{2}}(x) \tag{7a}$$

$$\sin x = \sqrt{\frac{\pi x}{2}} J_{\frac{1}{2}}(x) \tag{7b}$$

$$x^{-m/2} J_m\big(\xi\sqrt{x}\big) = \sum_{n=0}^{\infty} \frac{2(2n+m+1)\Gamma(n+m+1)}{\Gamma(n+1)\Gamma(m+1)} \frac{J_{2n+m+1}(\xi)}{\xi} p_n^m(x) \tag{7c}$$

$$p_n^m(x) = \frac{\Gamma(n+1)\Gamma(m+1)}{\Gamma(n+m+1)} x^{-m/2} \int_0^{\infty} J_m(\sqrt{x}\xi) J_{2n+m+1}(\xi) d\xi \tag{7d}$$

where $p_n^m(x)$ be the Jacobi's polynomials [14].

The Equation (5) are the matrix equations and we write them in matrix notation as under

$$\begin{aligned} -\left[G^{+}_{SE,E}\right][A_m] + \left[G^{-}_{SE,E}\right][C_m] &= \zeta_+ [J_E], \\ \left[K^{+}_{SE,E}\right][A_m] + \left[K^{-}_{SE,E}\right][C_m] &= j\kappa_0 [J_E] \end{aligned} \tag{8a}$$

$$\begin{aligned} -\left[G^{+}_{SE,O}\right][B_m] + \left[G^{-}_{SE,O}\right][D_m] &= j\zeta_+ [J_O], \\ \left[K^{+}_{SE,O}\right][B_m] + \left[K^{-}_{SE,O}\right][D_m] &= -\kappa_0 [J_O] \end{aligned} \tag{8b}$$

where the correspondence between the matrices and their elements are given by

$$\begin{aligned} \left[G^{\pm}_{SE,E}\right] &\Longleftrightarrow G_{SE}\left(2n+\frac{1}{2}, 2m+\frac{3}{2}; \zeta_\pm\right), \\ \left[G^{\pm}_{SE,O}\right] &\Longleftrightarrow G_{SE}\left(2n+\frac{3}{2}, 2m+\frac{5}{2}; \zeta_\pm\right) \\ \left[K^{\pm}_{SE,E}\right] &\Longleftrightarrow K_{SE}\left(2n+\frac{1}{2}, 2m+\frac{3}{2}; \zeta_\pm\right), \\ \left[K^{\pm}_{SE,O}\right] &\Longleftrightarrow K_{SE}\left(2n+\frac{3}{2}, 2m+\frac{5}{2}; \zeta_\pm\right) \\ [J_E] &\Longleftrightarrow \frac{2\sin\phi_0}{1+\zeta_+\sin\phi_0} \frac{J_{2n+\frac{1}{2}}(\kappa\cos\phi_0)}{(\kappa\cos\phi_0)^{\frac{1}{2}}}, \end{aligned} \tag{9}$$

$$[J_O] \iff \frac{2\sin\phi_0}{1+\zeta_+\sin\phi_0}\frac{J_{2n+\frac{3}{2}}(\kappa\cos\phi_0)}{(\kappa\cos\phi_0)^{\frac{1}{2}}}$$

Equation (8) can be solved for the expansion coefficients $A_m$, $B_m$, $C_m$, $D_m$ as follow

$$\begin{aligned}
&\left\{\left[G^+_{SE,E}\right]^{-1}\left[G^-_{SE,E}\right]+\left[K^+_{SE,E}\right]^{-1}\left[K^-_{SE,E}\right]\right\}[C_m]\\
&\quad= \left\{\zeta_+\left[G^+_{SE,E}\right]^{-1}+j\kappa_0\left[K^+_{SE,E}\right]^{-1}\right\}[J_E] \qquad (10\text{a})\\
[A_m] &= \left[G^+_{SE,E}\right]^{-1}\left[G^-_{SE,E}\right][C_m]-\zeta_+\left[G^+_{SE,E}\right]^{-1}[J_E] \qquad (10\text{b})\\
&\left\{\left[G^+_{SE,O}\right]^{-1}\left[G^-_{SE,O}\right]+\left[K^+_{SE,O}\right]^{-1}\left[K^-_{SE,O}\right]\right\}[D_m]\\
&\quad= \left\{j\zeta_+\left[G^+_{SE,O}\right]^{-1}-\kappa_0\left[K^+_{SE,O}\right]^{-1}\right\}[J_O] \qquad (10\text{c})\\
[B_m] &= \left[G^+_{SE,O}\right]^{-1}\left[G^-_{SE,O}\right][D_m]-j\zeta_+\left[G^+_{SE,O}\right]^{-1}[J_O] \qquad (10\text{d})
\end{aligned}$$

The geometry supports the surface wave. When the observation point is far from the surface, these waves can be neglected and diffracted waves dominates. A far diffracted fields in the upper region can be evaluated by applying the saddle point method of integration. The result is given by

$$\begin{aligned}
E^d_z &= \sum_{m=0}^{\infty}\int_0^{\infty}\frac{j\kappa_0}{j\kappa_0+\sqrt{\xi^2-\kappa_0^2}\zeta_+}\left\{A_m\frac{J_{2m+\frac{3}{2}}(\xi)}{\xi^{\frac{3}{2}}}\cos(x_a\xi)\right.\\
&\quad\left.+B_m\frac{J_{2m+\frac{5}{2}}(\xi)}{\xi^{\frac{3}{2}}}\sin(x_a\xi)\right\}\exp\left[-\sqrt{\xi^2-\kappa_0^2}y_a\right]d\xi\\
&= \sqrt{\frac{\pi}{2}}\frac{\tan\phi}{1+\zeta_+\sin\phi}\frac{1}{\sqrt{k_0\rho}}\exp\left[-jk_0\rho+j\frac{\pi}{4}\right]\\
&\quad\sum_{m=0}^{\infty}\left[A_m\frac{J_{2m+\frac{3}{2}}(\kappa_0\cos\phi)}{\sqrt{\kappa_0\cos\phi}}+jB_m\frac{J_{2m+\frac{5}{2}}(\kappa_0\cos\phi)}{\sqrt{\kappa_0\cos\phi}}\right] \qquad (11)
\end{aligned}$$

where $(\rho,\phi)$ are the cylindrical coordinates of the observation point. A far field in the lower region can also be derived similarly.

## 2.2. $H$-polarization

The field expressions corresponding to expressions (1) for $H$-polarization may be written as

$$H_z^i = \exp\left[jk_0(x\cos\phi_0 + y\sin\phi_0)\right] \tag{12a}$$

$$H_z^r = \frac{-Z_+ + Z_0\sin\phi_0}{Z_+ + Z_0\sin\phi_0}\exp\left[jk_0(x\cos\phi_0 - y\sin\phi_0)\right] \tag{12b}$$

$$H_z^{d+} = \int_0^\infty \{g_1(\xi)\cos(x_a\xi) + g_2(\xi)\sin(x_a\xi)\} \exp\left[-\sqrt{\xi^2-\kappa_0^2}y_a\right]d\xi \qquad y>0 \tag{12c}$$

$$H_z^{d-} = \int_0^\infty \{h_1(\xi)\cos(x_a\xi) + h_2(\xi)\sin(x_a\xi)\} \exp\left[\sqrt{\xi^2-\kappa^2}y_a\right]d\xi \qquad y<0 \tag{12d}$$

All the notations used in the above expressions have the same meaning as described in last section.

The boundary conditions are

$$E_x^t\Big|_{y=0_+} = Z_+H_z^t\Big|_{y=0_+}, \quad E_x^t\Big|_{y=0_-} = -Z_-H_z^t\Big|_{y=0_-}; \quad |x_a|\geq 1 \tag{13a}$$

$$E_x^t\Big|_{y=0_+} = E_x^t\Big|_{y=0_-}, \quad H_z^t\Big|_{y=0_+} = H_z^t\Big|_{y=0_-}; \quad |x_a|\leq 1 \tag{13b}$$

Using (13a), we get

$$\int_0^\infty [u + j\kappa_0\zeta_+]\left[g_1(\xi)\cos(x_a\xi) + g_2(\xi)\sin(x_a\xi)\right]d\xi = 0;\ |x_a|\geq 1 \tag{14a}$$

$$\int_0^\infty [v + j\kappa\zeta_-]\left[h_1(\xi)\cos(x_a\xi) + h_2(\xi)\sin(x_a\xi)\right]d\xi = 0;\ |x_a|\geq 1 \tag{14b}$$

where $u = \sqrt{\xi^2-\kappa_0^2}$, $v = \sqrt{\xi^2-\kappa^2}$ and $\zeta_+$, $\zeta_-$ are the normalized impedances of upper and lower surface of the impedance plane respectively.

Using the discontinuous properties of Weber-Schafheitlin's integrals and incorporating the edge conditions for $H$-field, we get

$$\begin{aligned} g_1(\xi) &= \frac{1}{j\kappa_0\zeta_+ + u}\sum_{m=0}^\infty A_m J_{2m+\frac{1}{2}}(\xi)\xi^{-\frac{1}{2}}, \\ g_2(\xi) &= \frac{1}{j\kappa_0\zeta_+ + u}\sum_{m=0}^\infty B_m J_{2m+\frac{3}{2}}(\xi)\xi^{-\frac{1}{2}} \end{aligned} \tag{15a}$$

$$h_1(\xi) = \frac{1}{j\kappa\zeta_- + v} \sum_{m=0}^{\infty} C_m J_{2m+\frac{1}{2}}(\xi)\xi^{-\frac{1}{2}},$$
$$h_2(\xi) = \frac{1}{j\kappa\zeta_- + v} \sum_{m=0}^{\infty} D_m J_{2m+\frac{3}{2}}(\xi)\xi^{-\frac{1}{2}} \tag{15b}$$

From (13b)

$$\int_0^{\infty} \{[g_1(\xi) - h_1(\xi)] \cos(x_a\xi) + [g_2(\xi) - h_2(\xi)] \sin(x_a\xi)\} \, d\xi$$
$$= \frac{2\sin\phi_0}{\zeta_+ + \sin\phi_0} \exp[j\kappa_0 x_a \cos\phi_0] \tag{16a}$$
$$\int_0^{\infty} \{[vh_1(\xi) + \epsilon_r u g_1(\xi)] \cos(x_a\xi) + [vh_2(\xi) + \epsilon_r u g_2(\xi)] \sin(x_a\xi)\} \, d\xi$$
$$= \frac{j2\kappa_0\zeta_+ \sin\phi_0}{\zeta_+ + \sin\phi_0} \exp[j\kappa_0 x_a \cos\phi_0] \qquad |x_a| \geq 1 \tag{16b}$$

Proceeding in a similar manner as in last section, we get the matrix equations

$$\sum_{m=0}^{\infty} \left[ A_m G_{SH}\left(2m+\frac{1}{2}, 2n+\frac{1}{2}; \kappa_0^+\right) - C_m G_{SH}\left(2m+\frac{1}{2}, 2n+\frac{1}{2}; \kappa^-\right)\right]$$
$$= \frac{2\sin\phi_0}{\zeta_+ + \sin\phi_0} \frac{J_{2n+\frac{1}{2}}(\kappa_0\cos\phi_0)}{(\kappa_0\cos\phi_0)^{\frac{1}{2}}} \tag{17a}$$
$$\sum_{m=0}^{\infty} \left[ B_m G_{SH}\left(2m+\frac{3}{2}, 2n+\frac{3}{2}; \kappa_0^+\right) - D_m G_{SH}\left(2m+\frac{3}{2}, 2n+\frac{3}{2}; \kappa^-\right)\right]$$
$$= \frac{j2\sin\phi_0}{\zeta_+ + \sin\phi_0} \frac{J_{2n+\frac{3}{2}}(\kappa_0\cos\phi_0)}{(\kappa_0\cos\phi_0)^{\frac{1}{2}}} \tag{17b}$$
$$\sum_{m=0}^{\infty} \left[ A_m K_{SH}\left(2m+\frac{1}{2}, 2n+\frac{1}{2}; \kappa_0^+\right) + C_m K_{SH}\left(2m+\frac{1}{2}, 2n+\frac{1}{2}; \kappa^-\right)\right]$$
$$= \frac{j2\kappa_0\zeta_+\sin\phi_0}{\zeta_+ + \sin\phi_0} \frac{J_{2n+\frac{1}{2}}(\kappa_0\cos\phi_0)}{(\kappa_0\cos\phi_0)^{\frac{1}{2}}} \tag{17c}$$
$$\sum_{m=0}^{\infty} \left[ B_m K_{SH}\left(2m+\frac{3}{2}, 2n+\frac{3}{2}; \kappa_0^+\right) + D_m K_{SH}\left(2m+\frac{3}{2}, 2n+\frac{3}{2}; \kappa^-\right)\right]$$
$$= -\frac{2\kappa_0\zeta_+\sin\phi_0}{\zeta_+ + \sin\phi_0} \frac{J_{2n+\frac{3}{2}}(\kappa_0\cos\phi_0)}{(\kappa_0\cos\phi_0)^{\frac{1}{2}}} \quad n = 0, 1, 2, \ldots \tag{17d}$$

where

$$G_{SH}\left(\mu,\nu;\kappa^{\pm}\right) = \int_0^{\infty} \frac{1}{j\kappa\zeta_{\pm}+\sqrt{\xi^2-\kappa^2}} \frac{J_\mu(\xi)J_\nu(\xi)}{\xi}d\xi \quad \text{(18a)}$$

$$K_{SH}\left(\mu,\nu;\kappa^{\pm}\right) = \int_0^{\infty} \frac{\epsilon_r\sqrt{\xi^2-\kappa^2}}{j\kappa\zeta_{\pm}+\sqrt{\xi^2-\kappa^2}} \frac{J_\mu(\xi)J_\nu(\xi)}{\xi}d\xi \quad \text{(18b)}$$

The above expressions are the matrix equations and can be solved for the expansion coefficients $A_m$, $B_m$, $C_m$, $D_m$ by any standard method.

The geometry supports the surface waves but if we use the asymptotic analysis, we can ignore the contribution of these waves. So applying the saddle point method, far diffracted fields in the upper space may be written as

$$\begin{aligned} H_z^{d+} = & \sqrt{\frac{\pi}{2}}\frac{\sin\phi}{\zeta_+ + \sin\phi}\frac{1}{\sqrt{k_0\rho}}\exp\left[-jk_0\rho + j\frac{\pi}{4}\right] \\ & \sum_{m=0}^{\infty}\left[A_m\frac{J_{2m+\frac{1}{2}}(\kappa_0\cos\phi)}{\sqrt{\kappa_0\cos\phi}} + jB_m\frac{J_{2m+\frac{3}{2}}(\kappa_0\cos\phi)}{\sqrt{\kappa_0\cos\phi}}\right] \end{aligned} \quad (19)$$

where $(\rho = \sqrt{x_a^2+y_a^2}, \phi)$ are the coordinates of observation point in cylindrical coordinates.

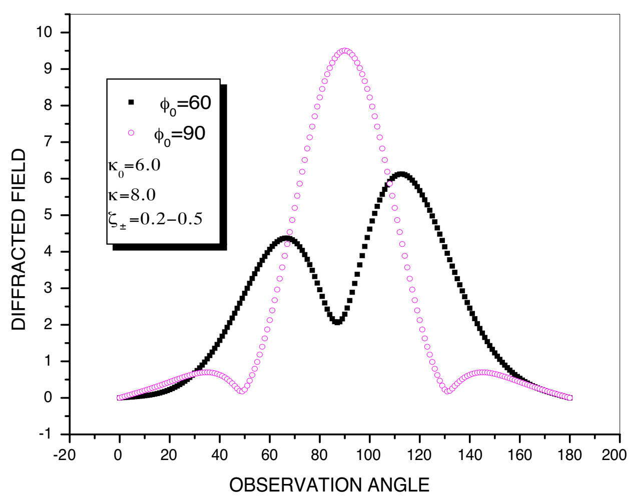


**Figure 2.** Variations of far field patterns with the angle of incidence.

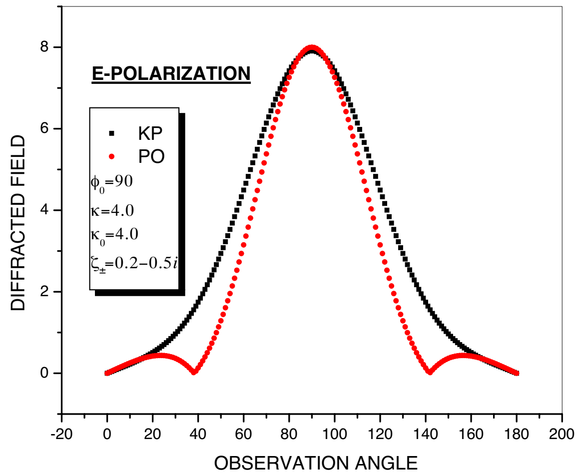


**Figure 3.** Comparison of the patterns obtained through KP and PO.

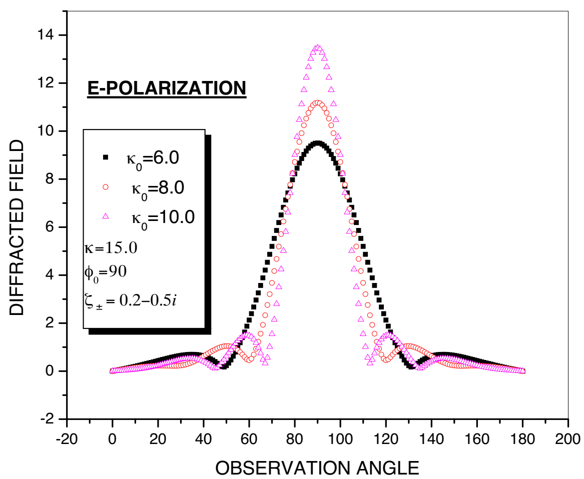


**Figure 4.** Effect of slit width on the field patterns.

## 3. RESULTS AND DISCUSSIONS

The field patterns are computed from Equation (11) for $E$-polarization and Equation (19) for $H$-polarization. But these equations contain the unknowns expansion coefficients $A_m$, $B_m$, $C_m$, $D_m$. The values of these expansion coefficients can be computed from Equations (10a)–(10d) for $E$-polarization and (17a)–(17d) for $H$-polarization. We have taken the matrix size $(2\kappa_0 + 3) \times (2\kappa_0 + 3)$ in our computations. The far field patterns are shown in Fig. 2 and Fig. 6 for $E$- and $H$-polarizations respectively for different angle of incidence. We notice that the peak of the main lobe corresponding to an angle of incidence occurs approximately at $\pi - \phi_0$ and as we increase $\phi_0$, the main lobe shifts towards the lower value of $\phi$. To verify the validity of our

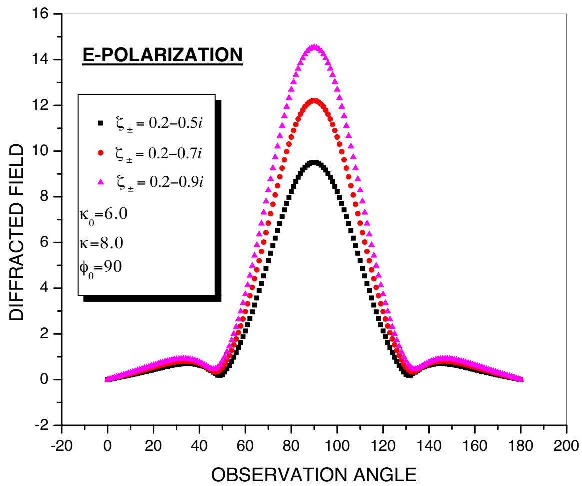


**Figure 5.** Diffracted patterns for different values of impedance of plane.

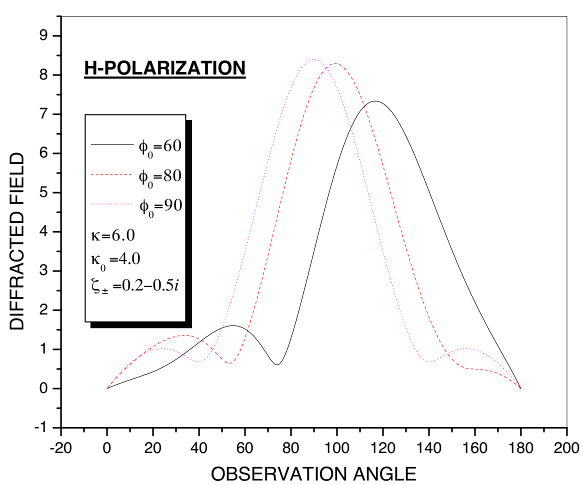


**Figure 6.** Far field patterns for different values of angle of incidence.

computations, we have compared our results with those of obtained through Physical Optics (PO). The comparison for $E$-polarization is given in Fig. 3. The angle of incidence $\phi_0$ is $\frac{\pi}{2}$ and the surface impedances are chosen as $\zeta_{\pm} = 0.2 - 0.5i$. Similarly Fig. 7 shows the comparison for $H$-polarization. All the parameters are same except $\phi_0$, which is $\frac{\pi}{3}$ in this case. The comparison is fairly good for both the cases. Fig. 4 and Fig. 8 give the variations in the field patterns

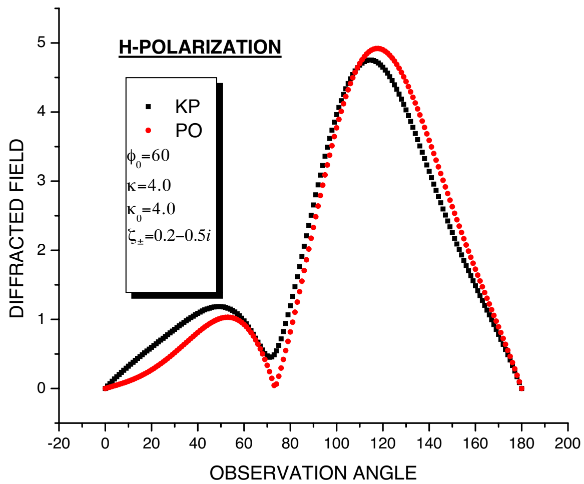


**Figure 7.** Comparison of the two methods for $H$-polarization.

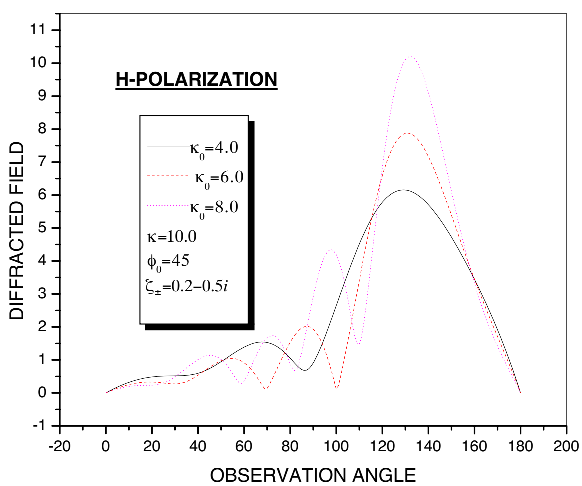


**Figure 8.** Effects of slit width on the patterns for $H$-polarization.

as we increase the slit width. Fig. 5 is intended to show the effects of material properties of the plane on the diffracted fields. It gives, as we decrease the impedances of the plane, the strength of the field patterns intensifies. We also computed the far field patterns to see the effects of medium properties of lower space. Fig. 9 presents the same. The field patterns are given for $\epsilon_r = 1.0$, $\epsilon_r = 2.25$, and $\epsilon_r = 4.0$. We choose the other parameters as $\phi_0 = \frac{\pi}{3}$, $\zeta_\pm = 0.2 - 0.5i$. It shows that as we increase the value of $\epsilon_r$, the strength of the diffracted fields in the upper space $(y > 0)$ also increase.

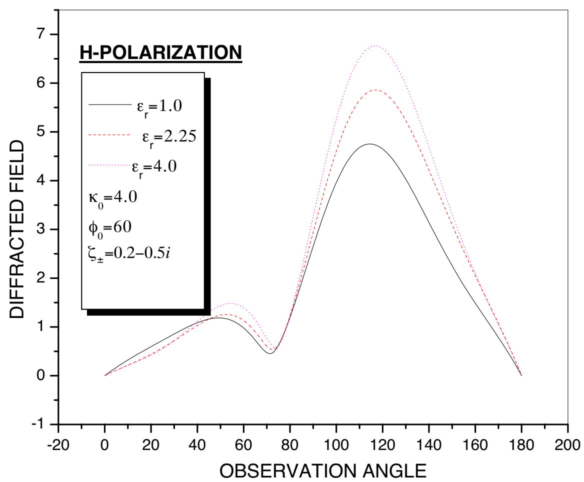


**Figure 9.** Effects of medium materials on the scattered fields.

## APPENDIX A. HOW TO COMPUTE THE INTEGRALS

$G_{SE}(\mu,\nu,\kappa^{\pm})$, $K_{SE}(\mu,\nu,\kappa^{\pm})$, $G_{SH}(\mu,\nu,\kappa^{\pm})$ and $K_{SH}(\mu,\nu,\kappa^{\pm})$

First we take the integral

$$\begin{aligned} G_{SE}(\mu,\nu;\kappa^{\pm}) &= \int_0^\infty \frac{1}{j\kappa\eta_\pm + \sqrt{\xi^2-\kappa^2}} \frac{J_\mu(\xi)J_\nu(\xi)}{\xi} d\xi \\ &= \int_0^x \frac{1}{j\kappa\eta_\pm + \sqrt{\xi^2-\kappa^2}} \frac{J_\mu(\xi)J_\nu(\xi)}{\xi} d\xi \\ &\quad + \int_x^\infty \frac{1}{j\kappa\eta_\pm + \sqrt{\xi^2-\kappa^2}} \frac{J_\mu(\xi)J_\nu(\xi)}{\xi} d\xi \\ &= G_{xSE}(\mu,\nu;\kappa^{\pm}) + G_{\infty SE}(\mu,\nu;\kappa^{\pm}) \qquad \text{(A1a)} \end{aligned}$$

where $x$ is a fairly large constant and we chose $x = 300$ in our computation. The integral $G_{xSE}(\mu, \nu; \kappa^{\pm})$ may be evaluated easily by using the definition of Spherical Bessel Function [17] since $\mu, \nu$ are the half order numbers. And these functions are convergent [17] and so is the integral $G_{xSE}(\mu, \nu; \kappa^{\pm})$. The second integral $G_{\infty SE}(\mu, \nu; \kappa^{\pm})$ can be computed as follow

$$\begin{aligned} G_{\infty SE}(\mu, \nu; \kappa^{\pm}) &= \int_x^{\infty} \frac{1}{j\kappa\eta_{\pm} + \sqrt{\xi^2 - \kappa^2}} \frac{J_\mu(\xi)J_\nu(\xi)}{\xi} d\xi \\ &\simeq \int_x^{\infty} \frac{J_\mu(\xi)J_\nu(\xi)}{\xi^2} - j\kappa\eta_{\pm}\int_x^{\infty} \frac{J_\mu(\xi)J_\nu(\xi)}{\xi^3} d\xi \\ &\quad + \frac{\kappa^2}{2} \int_x^{\infty} \frac{J_\mu(\xi)J_\nu(\xi)}{\xi^4} d\xi \end{aligned} \tag{A1b}$$

To perform the above numerical integrations, we need to have the Hankel approximation of the Bessel function. That is given by

$$\begin{aligned} J_n(\xi) = \sqrt{\frac{2}{\pi\xi}} \Bigg\{ &\Bigg[ 1 - \frac{(4n^2-1)(4n^2-9)}{128\,\xi^2} \\ &+ \frac{(4n^2-1)(4n^2-9)(4n^2-25)(4n^2-49)}{98304\,\xi^4} \Bigg] \cos\left(\xi - \frac{2n+1}{4}\pi\right) \\ &- \left[ \frac{(4n^2-1)}{8\xi} - \frac{(4n^2-1)(4n^2-9)(4n^2-25)}{3072\,\xi^3} \right] \sin\left(\xi - \frac{2n+1}{4}\pi\right) \Bigg\} \end{aligned} \tag{A2}$$

Using this formula we may write

$$\begin{aligned} J_m(\xi)J_s(\xi) &= \frac{1}{\pi}\left[ \frac{1}{\xi} - \frac{a_2 + b_2 - a_1 b_1}{\xi^3} \right. \\ &\quad \left. + \frac{a_4 + b_4 + a_2 b_2 - a_1 b_3 - b_1 a_3}{\xi^5} \right] \cos\frac{(m-s)\pi}{2} \\ &\quad + \frac{1}{\pi}\left[ \frac{a_1 - b_1}{\xi^2} - \frac{a_1 b_2 + a_3 - (a_2 b_1 + b_3)}{\xi^4} \right] \sin\frac{(m-s)\pi}{2} \\ &\quad + \frac{1}{\pi}\left[ \frac{1}{\xi} - \frac{a_2 + b_2 + a_1 b_1}{\xi^3} \right] \cos\left(2\xi - \frac{m+s+1}{2}\pi\right) \\ &\quad - \frac{1}{\pi}\left[ \frac{a_1 + b_1}{\xi^2} - \frac{a_1 b_2 + a_3 + (a_2 b_1 + b_3)}{\xi^4} \right] \sin\left(2\xi - \frac{m+s+1}{2}\pi\right) \\ a_1 &= \frac{4m^2-1}{8}, \quad a_2 = \frac{(4m^2-1)(4m^2-9)}{128}, \\ a_3 &= \frac{(4m^2-1)(4m^2-9)(4m^2-25)}{3072} \end{aligned}$$

$$
\begin{aligned}
a_4 &= \frac{(4m^2-1)(4m^2-9)(4m^2-25)(4m^2-49)}{98304} \\
b_1 &= \frac{4s^2-1}{8}, \quad b_2 = \frac{(4s^2-1)(4s^2-9)}{128},
\end{aligned}
$$

$$
\begin{aligned}
b_3 &= \frac{(4s^2-1)(4s^2-9)(4s^2-25)}{3072} \\
b_4 &= \frac{(4s^2-1)(4s^2-9)(4s^2-25)(4s^2-49)}{98304} \qquad \text{(A3)}
\end{aligned}
$$

Integrating by parts and If we retain the terms up to $\xi^{-5}$ with constant coefficients and $\xi^{-4}$ multiplied by trigonometric functions. Then we can write

$$
\begin{aligned}
\int_x^\infty \frac{J_\mu(\xi)J_\nu(\xi)}{\xi}d\xi &= \frac{1}{\pi}\left[\left(-\frac{1}{x}+\frac{A_1}{3x^3}\right)\cos\frac{\mu-\nu}{2}\pi\right. \\
&\quad \left.+\left(-\frac{A_3}{2x^2}+\frac{A_5}{4x^4}\right)\sin\frac{\mu-\nu}{2}\pi\right] \\
&\quad +\left[\left(\frac{1}{2x^2}-\frac{A_2}{2x^4}+\frac{3A_4}{4x^4}-\frac{3}{4x^4}\right)\sin(2x-\beta)\right. \\
&\quad +\left(-\frac{1}{2x^3}+\frac{3}{2x^5}+\frac{A_2}{x^5}+\frac{A_4}{2x^3}-\frac{3A_4}{2x^5}\right) \\
&\quad \left.\times\cos(2x-\beta)\right]\frac{1}{\pi} \qquad \text{(A4a)} \\
\int_x^\infty \frac{J_\mu(\xi)J_\nu(\xi)}{\xi^2}d\xi &= \frac{1}{\pi}\left[\left(-\frac{1}{2x^2}+\frac{A_1}{4x^4}\right)\cos\frac{\mu-\nu}{2}\pi\right. \\
&\quad +\left(\frac{1}{2x^3}-\frac{3}{2x^5}+\frac{A_4}{x^5}-\frac{A_2}{2x^5}\right)\sin(2x-\beta) \\
&\quad +\left(-\frac{3}{4x^4}+\frac{A_4}{2x^4}\right)\cos(2x-\beta) \\
&\quad \left.+\left(\frac{-A_3}{3x^3}+\frac{A_5}{5x^5}\right)\sin\left(\frac{\mu-\nu}{2}\pi\right)\right] \qquad \text{(A4b)} \\
\int_x^\infty \frac{J_\mu(\xi)J_\nu(\xi)}{\xi^3}d\xi &= \frac{1}{\pi}\left[\left(-\frac{1}{3x^3}+\frac{A_1}{5x^5}\right)\cos\frac{\mu-\nu}{2}\pi\right. \\
&\quad +\frac{1}{2x^4}\sin(2x-\beta)+\left(\frac{1}{x^5}+\frac{A_4}{2x^5}\right)\cos(2x-\beta) \\
&\quad \left.-\frac{A_3}{4x^4}\sin\left(\frac{\mu-\nu}{2}\pi\right)\right] \qquad \text{(A4c)}
\end{aligned}
$$

$$\int_x^{\infty} \frac{J_\mu(\xi)J_\nu(\xi)}{\xi^4} d\xi = \frac{1}{\pi}\left[-\frac{1}{4x^4}\cos\left(\frac{\mu-\nu}{2}\pi\right)+\frac{1}{2x^5}\sin(2x-\beta) -\frac{A_3}{5x^5}\sin\left(\frac{\mu-\nu}{2}\pi\right)\right] \quad \text{(A4d)}$$

where

$$A_1 = \frac{a_2+b_2-a_1b_1}{\xi^3}, \qquad A_2 = \frac{a_2+b_2+a_1b_1}{\xi^3}, \qquad A_3 = \frac{a_1-b_1}{\xi^2}$$

$$A_4 = \frac{a_1+b_1}{\xi^2}, \qquad A_5 = \frac{a_1b_2+a_3-(a_2b_1+b_3)}{\xi^4}$$

Using the expressions (A4a)–(A4d), we can compute the integral $G_{SE}(\mu,\nu;\kappa^{\pm})$ from (A1a). Similarly we can compute the remaining integrals from the expressions

$$K_{SE}\left(\mu,\nu;\kappa^{\pm}\right) = \int_0^{\infty} \frac{\sqrt{\xi^2-\kappa^2}}{j\kappa\eta_{\pm}+\sqrt{\xi^2-\kappa^2}} \frac{J_\mu(\xi)J_\nu(\xi)}{\xi} d\xi = -j\kappa\eta_{\pm} G_{SE}\left(\mu,\nu;\kappa^{\pm}\right) + \int_0^{\infty} \frac{J_\mu(\xi)J_\nu(\xi)}{\xi} d\xi \quad \text{(A5)}$$

$$\begin{aligned} G_{SH}\left(\mu,\nu;\kappa^{\pm}\right) &= \int_0^{\infty} \frac{1}{j\kappa\zeta_{\pm}+\sqrt{\xi^2-\kappa^2}} \frac{J_\mu(\xi)J_\nu(\xi)}{\xi} d\xi \\ &= \int_0^{x} \frac{1}{j\kappa\zeta_{\pm}+\sqrt{\xi^2-\kappa^2}} \frac{J_\mu(\xi)J_\nu(\xi)}{\xi} d\xi \\ &\quad + \int_x^{\infty} \frac{J_\mu(\xi)J_\nu(\xi)}{\xi^2} d\xi + P_1 \int_0^{x} \frac{J_\mu(\xi)J_\nu(\xi)}{\xi^3} d\xi \\ &\quad + P_2 \int_x^{\infty} \frac{J_\mu(\xi)J_\nu(\xi)}{\xi^4} d\xi \end{aligned} \quad \text{(A6)}$$

where $P_1 = -j\kappa\zeta_{\pm}$, $P_2 = (\frac{1}{2}-\zeta_{\pm}^2)\kappa^2$.

$$K_{SH}\left(\mu,\nu;\kappa^{\pm}\right) = \int_0^{\infty} \frac{\sqrt{\xi^2-\kappa^2}}{j\kappa\zeta_{\pm}+\sqrt{\xi^2-\kappa^2}} \frac{J_\mu(\xi)J_\nu(\xi)}{\xi} d\xi = -j\kappa\zeta_{\pm} G_{SH}\left(\mu,\nu;\kappa^{\pm}\right) + \frac{1}{m+n}\delta_{\mu,\nu} \quad \text{(A7)}$$

The same comments on the convergence for the case $G_{xSE}(\mu,\nu;\kappa^{\pm})$ are also valid for these integrals.

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