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# APPLYING CRITICAL-POINTS METHOD IN THE PRESENCE OF PHASE SHIFT DUE TO FEED LINE

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**Abstract**—The critical-points method is adopted for measuring unloaded Q-factor of microwave resonators in the presence of phase shift caused by the feed line. The result is calculated from four frequencies of three points in the resonator's impedance trace. In fact, the resonator's impedance trace rotates in Smith Chart by the phase shift. If Q-factor were gotten directly from the measured impedance including feed line rather than the equivalent impedance of the resonator without feed line, the performance of measurement will be impaired. To de-embed the phase shift, objective function was introduced to find the proper rotation angle caused by the feed line instead of calibration using extra measurement. Another advantage of the proposed method lies in the fact that no special attention is needed to distinguish magnetic coupling and electric coupling. The effectiveness of the proposed method was demonstrated by one set of simulation data and two measurement examples, namely, a low Q dielectric resonator and a high Q hollow cylindrical cavity.

## 1. INTRODUCTION

Loaded Q-factor $Q_L$ and unloaded Q-factor $Q_0$ are important parameters of microwave resonator. One resonator may have different $Q_L$ if an external circuit or coupling coefficient is variable. On the contrary, the $Q_0$ is unique for one mode. And $Q_0$ is needed when determining material's loss tangent or upper limits for the overall resonator performance [1].

Q-factor can be measured with one-port reflection method or two-port transmission method. The reflection method is adopted in most of publications, due to large errors caused by the inequality of input and output coupling coefficients and the complex structure in the transmission method [2]. When the reflection method is adopted, measurement using vector data, which have more information, is considered to be more accurate than using scalar data [3, 4].

Among those methods, the critical-points method introduced by Sun and Chao is fast and accurate [5]. And this method was further referenced and extended by lots of people [6, 7]. It is based on simple closed-form expressions and is free from curve fitting to measurement data. In this method, resonator is represented as a parallel or series equivalent circuit, and the coupling circuit has loss and energy storage. $Q_0$ is calculated from four frequencies of three points, that is, one detuned crossover point and two critical points of imaginary part of the impedance.

In fact, the input impedance measured by vector network analyzer (VNA) not only is the equivalent impedance of a resonator and coupling circuit, but also includes phase shift due to feed line, because the VNA is calibrated at the measurement point instead of the coupling point. So the measurement impedance trace in Smith chart has the same shape as the equivalent impedance, but not in the same position. It is rotated from the equivalent impedance trace. The phase shift will cause large errors if the measured impedance were directly used in the critical-points method. Lye solved this problem by two feed lines with difference length to de-embed the phase shift [7].

In this paper, an objective function is introduced to find the proper phase shift without extra complex measurement. The argument of the objective function is the phase shift which is the rotation angle of impedance trace in Smith chart. If the objective function reaches zero, the rotation angle is actually the phase shift which is used to de-embed the feed line. And this method does not need to distinguish magnetic coupling and electric coupling as [5], because these two kinds of coupling can change into each other with an extra one-quarter-wavelength transmission line and this line can be unified into the rotation angle mentioned above. To evaluate the performance of the proposed method, results calculated from simulation data and measurement data of two types of resonators, namely, a low Q dielectric resonator and a high Q hollow cylindrical cavity, are presented.

## 2. EQUIVALENT-CIRCUIT MODEL

The equivalent circuit of a microwave resonator including the feed line can be shown in Fig. 1 [7]. In this circuit, the external source and resistance are denoted by $v_s$ and $R_s$ respectively. The coupling mechanism is characterized by $R_e + \mathrm{j}X_e$, where $R_e$ and $X_e$ represent the loss and energy storage in the coupling, respectively. The value of

$X_e$ is positive ($2\pi f L_e$) or negative ($-1/2\pi f C_e$) depending on whether the coupling is inductive or capacitive. The unloaded resonator is represented by a parallel resonant circuit, which is made up of $R_0$, $C_0$, and $L_0$. The feed line has a phase shift of $\theta/2$ and a characteristic impedance of $Z_0$. Typically, $Z_0 = R_s = 50\,\Omega$.

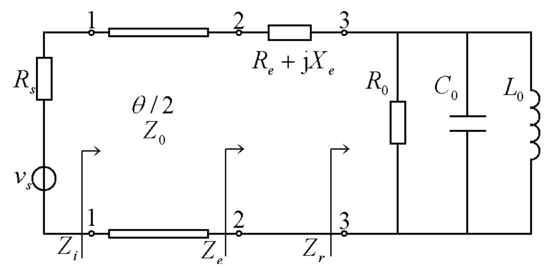


**Figure 1.** General equivalent circuit of microwave resonator including the feed line.

The input impedance at plane 2-2 is shown as following:

$$Z_e = R_e + jX_e + \frac{1}{\frac{1}{R_0} + j\left(\omega C_0 - \frac{1}{\omega L_0}\right)} = R_e + jX_e + \frac{R_0}{1 + jQ_0\left(\frac{f}{f_0} - \frac{f_0}{f}\right)}$$
$$= \left[R_e + \frac{R_0}{1 + (2Q_0\delta D)^2}\right] + j\left[\omega_0(1+\delta)L_e - \frac{2Q_0\delta D R_0}{1 + (2Q_0\delta D)^2}\right] \quad (1)$$

where $Q_0 = R_0\sqrt{C_0/L_0}$, $f_0 = 1/2\pi\sqrt{L_0 C_0}$, $\delta = (f - f_0)/f_0$, and $D = (1 + \delta/2)/(1 + \delta)$.

A typical impedance trace of $Z_e$ in Cartesian coordinate is given in Fig. 2.

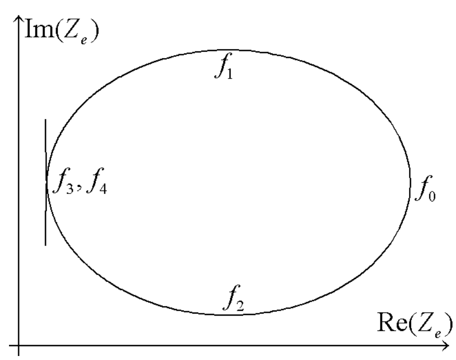


**Figure 2.** Equivalent impedance trace.

In Fig. 2, the frequencies of the maximum and the minimum of the imaginary part of $Z_e$ are marked as $f_1$ and $f_2$. And the frequencies of detuned crossover point are indicated as $f_3$ and $f_4$. The unloaded resonant frequency $f_0$ and unloaded Q-factor $Q_0$ can be calculated from $f_1 \sim f_4$ using the critical-points method [5]:

$$f_0 = (f_1 + f_2)/2 \tag{2}$$

$$Q_0 = |x|\, f_0/|f_1 - f_2| \approx f_0/|f_1 - f_2| \tag{3}$$

where

$$x^2 = \frac{b - 2a - 1 + \sqrt{(b - 2a - 1)^2 - 4(b + a)(a - 1)}}{2(b + a)} \tag{4}$$

$$a = 1 + \frac{f_1^2 + f_2^2 + 2f_1 f_2 - 4f_3 f_4}{2(f_1 f_3 + f_1 f_4 + f_2 f_3 + f_2 f_4 + 4f_3 f_4)} \tag{5}$$

$$b = \left(\frac{\delta_4 D_4 - \delta_3 D_3}{\delta_2 - \delta_1}\right)^2 \tag{6}$$

$$D_k = (1 + \delta_k/2)\,/\,(1 + \delta_k)\,, \quad \delta_k = (f_k - f_0)\,/f_0, \quad k = 1 \sim 4 \tag{7}$$

Actually, the impedance measured by VNA is $Z_i$ at plane 1-1, rather than $Z_e$ at plane 2-2. Due to the presence of feed line, $Z_i$ is rotated from $Z_e$ in Smith Chart by $\theta = 4\pi fl/v$, where $l$ is the length of the feed line, and $v$ is the speed of wave. For a high Q resonator, the measurement frequency range is narrowband. So $f$ is approximately a constant. And $\theta$ is only dependent on $l$. In the critical-points method, the impedance trace, which is used to calculate $f_0$ and $Q_0$, is $Z_e$ rather than $Z_i$. Therefore $\theta$ should firstly be obtained after the measurement of $Z_i$. Then $Z_e$ is deduced from $Z_i$ and $\theta$ by transmission line theory. And finally, $f_0$ and $Q_0$ are calculated by (2)–(7) from $Z_e$. It will be shown in the next section that there will be large errors in $f_0$ and $Q_0$ if they were directly calculated from $Z_i$ instead of $Z_e$. The equation of deducing $Z_e$ from $Z_i$ is:

$$Z_e = Z_0 \frac{Z_i - \mathrm{j} Z_0 \tan(\theta/2)}{Z_0 + \mathrm{j} Z_i \tan(\theta/2)} \tag{8}$$

If $\theta$ is the proper rotation angle in Smith chart, $Z_e$ calculated from (8) is the impedance at plane 2-2. The real parts of $Z_e$ of the detuned crossover point are equal. Therefore we get

$$R_e + \frac{R_0}{1 + (2Q_0\delta_3 D_3)^2} = R_e + \frac{R_0}{1 + (2Q_0\delta_4 D_4)^2} \tag{9}$$

resulting in

$$J(\theta) = \delta_3 D_3 + \delta_4 D_4 = 0 \tag{10}$$

substitute (2) and (7) into (10), we get

$$J(\theta) = \frac{f_3}{f_1 + f_2} - \frac{f_1 + f_2}{4f_3} + \frac{f_4}{f_1 + f_2} - \frac{f_1 + f_2}{4f_4} = 0 \tag{11}$$

There are two reasons that (11) can be used as an objective function to find the rotation angle caused by feed line. Firstly, (9) is derived from (1) which is an expression of $Z_e$ rather than $Z_i$. And the expression of $Z_i$ which includes the feed line is different from $Z_e$. Even though the two impedances at the crossover point of $Z_i$ are equal, the expressions of the real parts of them are not (9). Therefore (11) can not be satisfied. Secondly, the critical points of the imaginary part of the impedance in Smith Chart vary with the rotation angle. At the same time, $f_1$ and $f_2$ change according to the critical points. If the wrong rotation angle were chosen, (11) will not be zero, despite $f_3$ and $f_4$ remain the same. An expression of the imaginary part at the crossover point is not recommended to be used as an objective function because of the complex form and ambiguity. The procedure of finding the rotation angle $\theta$ is:

1. let the rotation angle $\theta$ vary throughout a period, such as from $-180^\circ$ to $180^\circ$;
2. calculate the potential $Z_e$ with $Z_i$ and $\theta$ by (8);
3. find four frequencies $f_1 \sim f_4$ of one detuned crossover point and two critical points in the potential $Z_e$ trace calculated in step 2 according to different $\theta$;
4. calculate objective function $J(\theta)$ with $f_1 \sim f_4$ found in step 3 by (11);
5. determine $\theta$ by $J(\theta)$ when it reaches zero.

## 3. SIMULATION RESULTS

One set of simulation data is used to verify the proposed method. In Fig. 1, let $R_0 = 10\,\Omega$, $C_0 = 1.5915\,\text{nF}$, $L_0 = 0.15915\,\text{pH}$, $R_e = 10\,\Omega$, $L_e = 7.8585\,\text{pH}$, and $\theta = 0^\circ$. Then $Q_0 = 1000$ and $f_0 = 10\,\text{GHz}$ theoretically. $Z_e$ is gotten using this set of parameters near the resonant frequency by (1). In this case, $Z_i = Z_e$, because $\theta = 0^\circ$. The objective function $J(\theta)$ is calculated using the data of rotated $Z_i$ in Smith Chart with the assumption that $\theta$ is unknown. And the rotation angle varies from $-180^\circ$ to $180^\circ$. In Fig. 3, the objective function $J(\theta)$ versus the rotation angle $\theta$ is shown. And $J(\theta)$ equals zero at 0° as expected. It means $J(\theta)$ can be used to determine the proper angle that $Z_e$ rotates from $Z_i$ in Smith Chart. It is also seen that $J(\theta)$ has another zero. Fortunately, this unwanted zero can be easily distinguished from the proper one by the trend of the curve. $f_0$ and $Q_0$ are also calculated versus $\theta$, as shown in Fig. 4. The two curves both reach their theoretical values at 0°, namely 10 GHz and 1000 respectively. The slope of $J(\theta)$ is very small near 0°. Thus it will cause big errors when the rotation angle being determined. However it does not distort the precision of $f_0$ and $Q_0$, because the slopes of them are also very small near 0°.

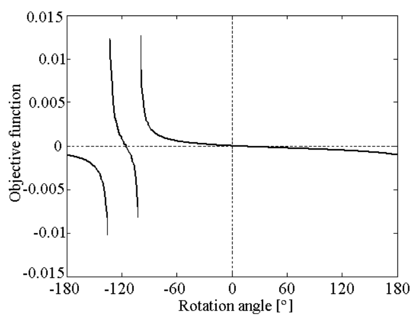


**Figure 3.** Objective function $J(\theta)$ of simulation data.

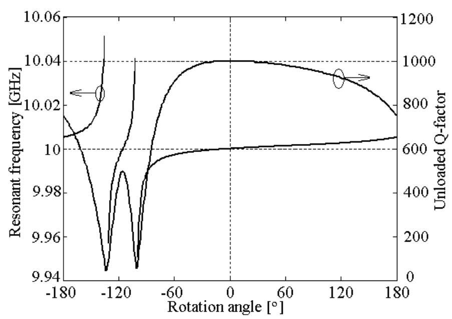


**Figure 4.** Resonant frequency and unloaded Q-factor versus rotation angle of simulation data.

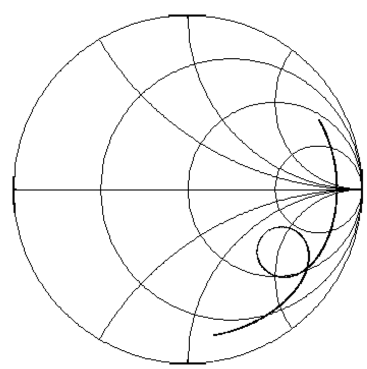

**Figure 5.** Measurement $Z_i$ of the dielectric resonator in Smith Chart.

## 4. EXPERIMENTAL RESULTS

To illustrate the above principle and procedure in Section 2, $f_0$ and $Q_0$ of two resonators, a dielectric resonator and a high Q hollow cylindrical cavity, were measured. The input impedances of the two resonators were measured by Agilent VNA E8363B. In the case of dielectric resonator, the measurement input impedance is shown in Fig. 5. And the objective function versus the rotation angle is calculated using the procedure from step 1 to 4 given in Section 2. In Fig. 6, the objective function reaches zero when rotation angle equals $-55°$. That means the wanted rotation angle caused by the feed line is $-55°$. And the proper $Z_e$ is gotten from $Z_i$ by (8), where $\theta = -55°$. Thus, $f_0 = 6.5021\,\text{GHz}$ and $Q_0 = 1492$ are estimated from (2) ~ (7) with the proper $Z_e$. $f_0$ and

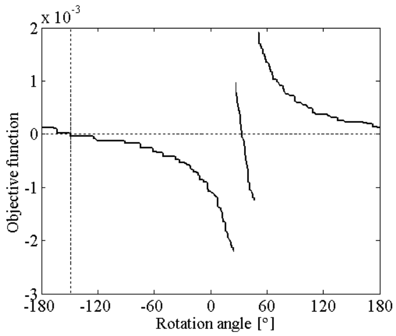


**Figure 6.** Objective function versus rotation angle of the dielectric resonator.

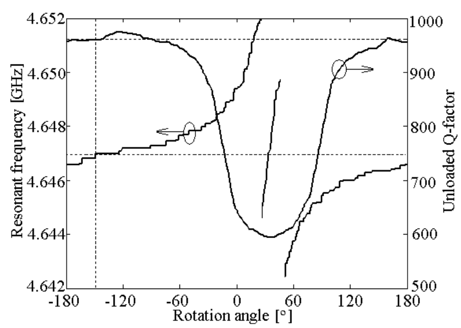


**Figure 7.** Resonant frequency and unloaded Q-factor versus rotation angle of the dielectric resonator.

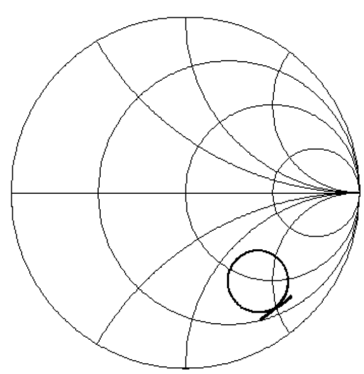

**Figure 8.** Measurement $Z_i$ of the cylindrical cavity in Smith Chart.

$Q_0$ versus the potential rotation angle are shown in Fig. 7. Figs. 8–10 show the measured and calculated curves of a high Q hollow cylindrical cavity, which further demonstrate the performance of the proposed method. In this example, the wanted rotation angle is $-42°$. And, $f_0 = 10.2377\,\text{GHz}$ and $Q_0 = 16915$. The curves of $f_0$ and $Q_0$ versus the potential rotation angle of the two resonators are similar to the curves in theory as shown in Fig. 4. It can be seen from the curves that if a wrong rotation angle were chosen or no rotation angle were used at all, there would be large errors in the results of $f_0$ and $Q_0$.

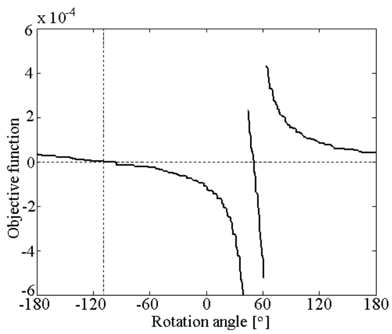


**Figure 9.** Objective function versus rotation angle of the cylindrical cavity.

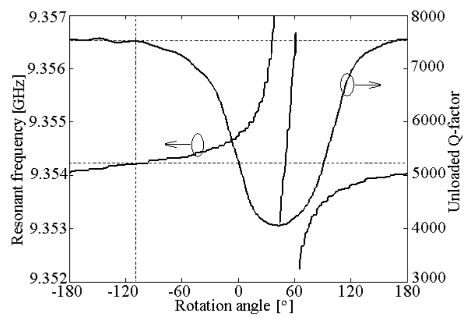


**Figure 10.** Resonant frequency and unloaded Q-factor versus rotation angle of the cylindrical cavity.

## 5. CONCLUSION

A practical method has been presented for de-embedding feed-line phase shift in the measurement of unloaded Q-factor by the critical-points method. In this method, an objective function is introduced to find the proper rotation angle caused by the feed line which impairs the performance of measurement. Another advantage of the proposed method lies in the fact that no special attention is needed to distinguish magnetic coupling and electric coupling. The reason is that the two kinds of coupling change into each other with an extra one-quarter-wavelength transmission line. And the transmission line can be unified to the rotation angle found by objective function. One set of simulation data and two experimental examples of a low Q dielectric resonator and a high Q hollow cylindrical cavity were given, which demonstrated the performance of the proposed method.

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