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WAVELET PACKET TRANSFORM-BASED LEAST MEAN SQUARE BEAMFORMER WITH LOW COMPLEXITY

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Abstract—A low complexity wavelet packet transform-based least mean square (LMS) adaptive beamformer is presented in this paper. This beamformer uses wavelet packet transform as the preprocessing, reduces the signal dimension in wavelet packet domain for low complexity and denoising, and employs least mean square algorithm to implement adaptive beamformer. Theoretical analysis and simulations demonstrate that this algorithm with better beamforming performance converges faster than the conventional adaptive beamformer and the wavelet transform-based beamformer. Finally, our proposed algorithm has the low complexity, and it can be easy to implement.

1. INTRODUCTION

Antenna array has been used in many fields such as radar, sonar, communications, seismic data processing, and so on [1–6]. Adaptive beamforming technique [7–12] is the key technique of array signal processing. A beamformer is a spatial filter that operates on the output of an array of sensors in order to enhance the amplitude of a coherent wavefront relative to background noise and directional interference [13]. Least Mean Square (LMS) adaptive beamforming is a simple and practical beamforming technique. LMS algorithm has many advantages, such as low complexity, simply implementation and high stability, but its remarkable disadvantage is slow convergence, which limits its application. It has been proved that the main factor affecting the LMS adaptive beamforming convergence is the ratio of maximal eigenvalue to minimal eigenvalue $\lambda_{\max}/\lambda_{\min}$ of input signal. The convergence become faster as the reduction of $\lambda_{\max}/\lambda_{\min}$ [14, 15]. In order to increase the convergence speed, the frequency-domain filtering method has been extended to yield the transform domain filtering method, which finds applications in many field including beamforming [16, 17], and so on.

Wavelet theory provides a new way for transform-domain adaptive filtering [18]. Wavelet is characterized by good time-frequency characteristic [19, 20]. Wavelet transformed signal decreases autocorrelation, and show a special band distribution [21], which leads to an increase in convergence speed. Wavelet transform has been used in adaptive beamforming [22–26], and the algorithms can be still improved further. The wavelet packet method is a generalization of wavelet decomposition, and it offers a rich range of possibilities for signal analysis. In the wavelet analysis, the signal is split into an approximation and a detail. The approximation is then itself split into a second-level approximation and detail, and the process is repeated. In wavelet packet analysis, the details as well as the approximations can be split repeatedly, so wavelet packet transformed signal has faster convergence. White noise often pollutes the received signal, and is hard to denoise in the time domain, but it is easy to denoise in the transform domain [27].

In this paper, the received signal of array antenna is analyzed, which shows that different Directions of Arrival (DOA) correspond to different multi-solution in the fixed array spacing. The received signal of array antenna has multi-resolution from DOA angle, and is taken as the foundation of wavelet packet application. Wavelet packet transform-based LMS adaptive beamformer is proposed. This algorithm has the faster convergence speed than wavelet transform-based LMS beamformer and conventional LMS beamformer. This algorithm has low complexity, and it can be easy to implement.

This paper is structured as follows. Section 2 develops multi-resolution characteristics of received signal. Wavelet packet transform-based LMS adaptive beamformer is presented in Section 3. Section 4 deals with algorithm performance analysis. Section 5 presents simulation results and Section 6 summarizes our conclusions.

Denote: We denote by $(.)^*$ the complex conjugation, by $(.)^T$ the matrix transpose, and by $(.)^H$ the matrix conjugate transpose. The notation $(.)^+$ refers to the Moore-Penrose inverse (pseudo inverse). $\|\|_F$ stands for the matrix conjugate transpose.

2. MULTI-RESOLUTION CHARACTERISTICS OF RECEIVED SIGNAL

In applying wavelets to an array signal processing problem, a key issue is how to understand the scale in an array signal. According to Ref. [28], the difference between the received signals at two adjacent sensors is contained in the phase term. Basically, this phase term determines the oscillation rate in a spatial signal. This can be further understood from the spatial sampling criterion. To avoid phase ambiguity, a constraint is imposed on the phase difference between the received signals at two adjacent sensors:

$$|kd\sin\theta| = \left|\frac{2\pi}{\lambda}d\sin\theta\right| \leq \pi \tag{1}$$

where λ is wavelength, d is the spacing between adjacent sensors, and θ is DOA. From Eq. (1), the spatial sampling interval depends on the signal direction.

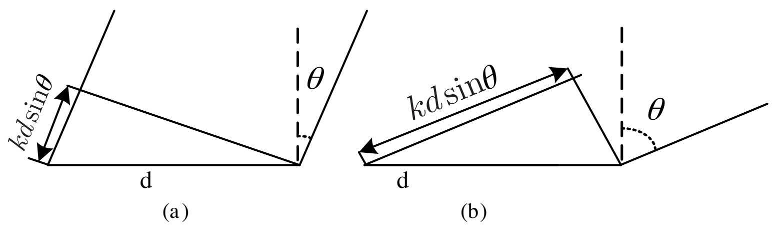

Figure 1. Spatial multi-resolution of different DOAs.

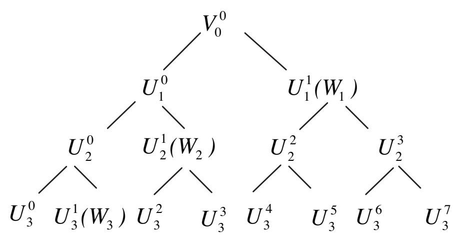

Figure 2. DOA-spatial frequency resolution rectangle.

In the condition of the fixed array spacing, we assume that DOAs ping at array from DOA [0, $\pi/2$]. When DOA changes from 0 to $\pi/2$, the spatial sampling interval $kd\sin\theta$ changes, as shown in Fig. 1. When DOA is small, the spatial sampling interval is small and the spatial resolution is high (Fig. 1(a)). When DOA is large, the spatial sampling interval is large and the spatial resolution is low (Fig. 1(b)). Therefore, different DOAs correspond to different spatial resolutions in the fixed array spacing. Assume that DOA is different; then the resolution is different. When the DOA increase between [0, $\pi/2$], its resolution degrades, which is shown in Fig. 2. Hence, the received signal of array antenna is regarded as the addition of multi-resolution signals. If the received signal is under wavelet pack transform, the different resolutions can be detected through wavelet packet decomposition; thus the detection of the different DOA signals can be implemented.

3. WAVELET PACKET TRANSFORM-BASED LMS BEAMFORMER

An illustration of the wavelet packet transform-based LMS adaptive beamforming is shown in Fig. 3. First, the wavelet packet transform is made, secondly reduce dimension is processed, and then the LMS algorithm is employed to implement the adaptive beamforming.

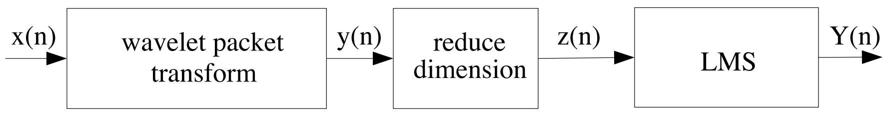

Figure 3. Wavelet packet transform-based LMS adaptive beamformer.

3.1. Wavelet Packet Transform

Wavelet packet has the ability to further decompose the wavelet space as follows:

$$\begin{aligned} V_0 &= V_1 \oplus W_1 = U_1^0 \oplus U_1^1 = U_2^0 \oplus U_2^1 \oplus U_2^2 \oplus U_2^3 \\ &= \ldots = U_N^0 \oplus U_N^1 \oplus \ldots \oplus U_N^{2^N-1} \end{aligned} \tag{2}$$

where $\mathbf{V}_0$ is the signal space; $\mathbf{V}_1$ and $\mathbf{W}_1$ are the approximation and detail of $\mathbf{V}_0$, respectively; $U_N^0, U_N^1, \ldots, U_N^{2^N-1}$ are the subspaces of $\mathbf{V}_0$; N is decomposition series.

The wavelet packet decomposition for $N = 3$ is shown in Fig. 4.

In order to analyze the wavelet packet transform characteristic, the orthogonal matrices of wavelet packet transform are derived according to the lowpass filter and highpass filter of wavelet. The lowpass filer and highpass filter of the wavelet packet transform are $\mathbf{h} = [h_0, h_1, \ldots, h_{2N-1}]$ and $\mathbf{g} = [g_0, g_1, \ldots, g_{2N-1}]$, respectively. The filter tag length is $2N$. For simplicity N is supposed to be integer exponent 2.

When $1 \leq i \leq J - \log_2 2N + 1$, the $2^{J-i+1} \times 2^{J-i+1}$ matrix $\mathbf{W}_i$

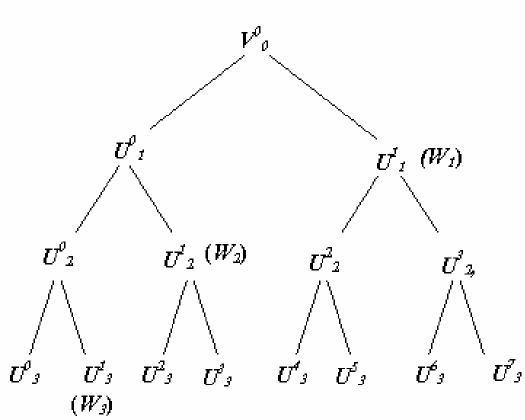

Figure 4. Wavelet packet decomposition.

for the i-th series wavelet decomposition is

$$\mathbf{W}_i = \begin{bmatrix} h_N & \cdots & h_{2N-1} & 0 & \cdots & 0 & h_0 & \cdots & h_{N-1} \\ h_{N-2} & \cdots & h_{2N-3} & h_{2N-2} & \cdots & 0 & 0 & \cdots & h_{N-3} \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ h_{N+2} & \cdots & 0 & 0 & \cdots & h_1 & h_2 & \cdots & h_{N+1} \\ g_N & \cdots & g_{2N-1} & 0 & \cdots & 0 & g_0 & \cdots & g_{N-2} \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ g_{N+2} & \cdots & 0 & 0 & \cdots & g_1 & g_2 & \cdots & g_{N+1} \end{bmatrix} \tag{3}$$

where the number of 0 in every row is $2^{J-i+1} - 2N$.

Similarly, when $J - \log_2 2N + 1 < i \leq J$, the $2^{J-i+1} \times 2^{J-i+1}$ orthogonal matrix $\mathbf{W}_i$ for the i-th series wavelet decomposition is

$$\mathbf{W}_i = \begin{bmatrix} h'_0 & h'_1 & h'_2 & \cdots & h'_{2^{J-I+1}-2} & h'_{2^{J-I+1}-1} \\ h'_{2^{J-I+1}-2} & h'_{2^{J-I+1}-1} & h'_0 & \cdots & h'_{2^{J-I+1}-4} & h'_{2^{J-I+1}-3} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ h'_2 & h'_3 & h'_4 & \cdots & h'_0 & h'_1 \\ g'_0 & g'_1 & g'_2 & \cdots & g'_{2^{J-I+1}-2} & g'_{2^{J-I+1}-1} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ g'_2 & g'_3 & g'_4 & \cdots & g'_0 & g'_1 \end{bmatrix} \tag{4}$$

where $h'_m = h_m + h_{2^{J-I+1}+m} + \cdots + h_{2N-2^{J-I+1}+m}$, $g'_m = g_m + g_{2^{J-I+1}+m} + \cdots + g_{2N-2^{J-I+1}+m}$, $0 \leq m < 2^{J-i+1}$.

It is proved that $\mathbf{W}_i$ is also an orthogonal matrix.

The $2^J \times 2^J$ matrix $\mathbf{W}_T$ for the J series wavelet decomposition and the $2^J \times 2^J$ matrix $\mathbf{W}_P$ for the J series wavelet packet decomposition are as follow:

$$\mathbf{W}_T = \begin{bmatrix} \mathbf{W}_J & 0 & & 0 & \cdots & 0 \\ 0 & \mathbf{I} & & & \cdots & 0 \\ \vdots & & \ddots & & \cdots & \vdots \\ 0 & 0 & & 0 & \cdots & \mathbf{I} \end{bmatrix} \times \cdots \times \begin{bmatrix} \mathbf{W}_2 & 0 \\ 0 & \mathbf{I} \end{bmatrix} \times \mathbf{W}_1 \tag{5}$$

$$\mathbf{W}_P = \begin{bmatrix} \mathbf{W}_J & 0 & & 0 & \cdots & 0 \\ 0 & \mathbf{W}_J & & & \cdots & 0 \\ \vdots & & \ddots & & \cdots & \vdots \\ 0 & 0 & & 0 & \cdots & \mathbf{W}_J \end{bmatrix} \times \cdots \times \begin{bmatrix} \mathbf{W}_2 & 0 \\ 0 & \mathbf{W}_2 \end{bmatrix} \times \mathbf{W}_1 \tag{6}$$

It is proved that $\mathbf{W}_T$ and $\mathbf{W}_P$ are orthogonal matrices.

3.2. Reduce Dimension and Denoise

Different DOAs correspond to different spatial resolutions, and the useful signal is in low frequency part. We think the high frequency part in the wavelet packet transform signal includes the noise, and we can remove high frequency part for the wavelet packet transform signal, and that is also called reduce-dimension. The reduce-dimension for the he wavelet packet transform signal is to denoise and reduce the complexity of LMS algorithm. In general, we reduce dimension into half of it. Reduce dimension will be reduce the algorithm complexity.

3.3. Wavelet Packet Transform-based LMS Adaptive Beamformer

Perform wavelet packet transform for the received signal $\mathbf{x}$. The transformed signal is denoted by

$$\mathbf{y}(n) = \mathbf{W}_P \mathbf{x}(n) \tag{7}$$

$\mathbf{y}(n)$ is a vector with $M \times 1$, the wavelet packet transform signal.

We reduce dimension for the wavelet packet transform signal

$$\mathbf{z}(n) = P\mathbf{y}(n) \tag{8}$$

where P is reduce-dimension matrix.

LMS algorithm is used

$$\mathbf{r}(n) = \mathbf{v}^{\mathrm{T}}(n)\mathbf{z}(n) \tag{9}$$

where $\mathbf{v}$ is the weighed vector of LMS algorithm; $\mathbf{r}$ is the output signal of LMS. The error in the nth iteration is shown

$$e(n) = d(n) - r(n) \tag{10}$$

where $d(n)$ is the training sequence; e is the error.

$$\mathbf{v}(n+1) = \mathbf{v}(n) + 2ae(n)\mathbf{r}^*(n) \tag{11}$$

where a is the learning step.

4. ALGORITHM PERFORMANCE ANALYSIS

4.1. Optimal Solution

The cross-correlation vector $\mathbf{P}_y$ between the desired signal and the wavelet packet transform signal is

$$\mathbf{P}_y = E(d(n)\mathbf{y}(n)) = E(d(n)\mathbf{W}_P\mathbf{x}(n)) = \mathbf{W}_P\mathbf{P}_x \tag{12}$$

where $\mathbf{P}_x$ is the cross-correlation between the input signal $\mathbf{x}(n)$ and desired signal.

The autocorrelation matrix $\mathbf{R}_y$ of the wavelet packet transform signal is

$$\mathbf{R}_y = \mathbf{E}\left(\mathbf{y}(n)\mathbf{y}(n)^{\mathrm{H}}\right) = E\left(\mathbf{W}_P\mathbf{x}(n)\mathbf{x}(n)^{\mathrm{H}}\mathbf{W}_P\right) = \mathbf{W}_P\mathbf{R}_x\mathbf{W}_P \tag{13}$$

where $\mathbf{R}_x$ is the autocorrelation matrix of input signal $\mathbf{x}(n)$. Then, the wiener optimal solution of wavelet packet domain beamforming is

$$\begin{aligned} \mathbf{V}_{\mathrm{opt}} &= \mathbf{R}_y^{-1}\mathbf{P}_y = \mathbf{R}_y^{-1}\mathbf{W}_P\mathbf{P}_x \\ &= \mathbf{R}_y^{-1}\mathbf{W}\mathbf{R}_x\mathbf{R}_x^{-1}\mathbf{P}_x = \mathbf{R}_y^{-1}\mathbf{W}_P\mathbf{R}_x\mathbf{w}_{\mathrm{opt}} \end{aligned} \tag{14}$$

where $\mathbf{w}_{\mathrm{opt}}$ is the time domain beamforming Wiener solution.

The minimal mean square error (MSE) of wavelet packet domain beamforming is

$$\begin{aligned} e_{\min}^{y} &= E\left(d^2(n)\right) - \mathbf{P}_y^H\mathbf{R}_y^{-1}\mathbf{P}_y = E\left(d^2(n)\right) - \mathbf{P}_y^H\mathbf{R}_y^{-1}\mathbf{W}_P\mathbf{P}_x \\ &= e_{\min} - \mathbf{P}_y^H\left(\mathbf{W}_P\mathbf{R}_y^{-1}\mathbf{W}_P - \mathbf{R}_x^{-1}\right)\mathbf{P}_x \end{aligned} \tag{15}$$

where $e_{\min}$ is the time domain LMS minimal mean square error.

4.2. Convergence Analysis

The algorithm convergence speed is determined by the ratio of maximal eigenvalue to minimal eigenvalue $\lambda_{\max}/\lambda_{\min}$. The convergence speed becomes faster with the reduction of $\lambda_{\max}/\lambda_{\min}$. According to Ref. [29], the autocorrelation matrix of the orthogonal transform signal has a diagonal tendency, and the remarkable decrease in $\lambda_{\max}/\lambda_{\min}$ results in the improvement of convergence performance of the algorithm.

Assume the input signal is a real signal $\mathbf{x}(n)$ and its autocorrelation matrix is $\mathbf{R}_{xx}$, and the autocorrelation matrix of wavelet packet transform signal $\mathbf{y}(n)$ is $\mathbf{R}_{yy}$. Then, $\mathbf{R}_{xx}$ and $\mathbf{R}_{yy}$ are real symmetrical matrices, and the orthogonal matrices $\mathbf{Q}_x$ and $\mathbf{Q}_y$ exist, such that

$$\mathbf{R}_{xx} = \mathbf{Q}_x \Lambda_x \mathbf{Q}_x^{-1}; \quad \mathbf{R}_{yy} = \mathbf{Q}_y \Lambda_y \mathbf{Q}_y^{-1} \tag{16}$$

where Λ_x and Λ_y are eigenvalue diagonal matrices of $\mathbf{R}_{xx}$ and $\mathbf{R}_{yy}$, respectively.

According to Eq. (7)

$$\begin{aligned} \mathbf{R}_{yy} &= \mathbf{Q}_y \Lambda_y \mathbf{Q}_y^{-1} = E\left[\mathbf{y}(n)\mathbf{y}^T(n)\right] \\ &= E\left[\mathbf{W}_P \mathbf{X}(n) \mathbf{X}^T(n) \mathbf{W}_P^T\right] = \mathbf{W}_P \mathbf{R}_{xx} \mathbf{W}_P^T \end{aligned} \tag{17}$$

So we get

$$\mathbf{R}_{yy} = \mathbf{Q}_y \Lambda_y \mathbf{Q}_y^{-1} = \mathbf{W}_P \mathbf{Q}_x \Lambda_x \mathbf{Q}_x^{-1} \mathbf{W}_P^T \tag{18}$$

$$\Lambda_y = \mathbf{Q}_y^T \mathbf{W}_P \mathbf{Q}_x \Lambda_x \mathbf{Q}_x^{-1} \mathbf{W}_P^T \mathbf{Q}_y = \mathbf{A} \mathbf{\Lambda}_x \mathbf{A}^T \tag{19}$$

where

$$\mathbf{A} = \mathbf{Q}_y^T \mathbf{W}_P \mathbf{Q}_x \tag{20}$$

According to Eq. (19), we have

$$\lambda_k^y = \sum_{i=1}^{N} a_{ki}^2 \lambda_k^x, \quad k = 1, 2, \ldots, N \tag{21}$$

where λ_k^y and λ_k^x are the eigenvalues of $\mathbf{R}_{xx}$ and $\mathbf{R}_{yy}$, respectively; a_{ki} is element (k, i) of $\mathbf{A}$.

It is seen that eigenvalue λ_k^y is related to $\mathbf{W}_P$ closely. The following conclusions are made according to characteristics of orthogonal matrices $\mathbf{W}_T$ and $\mathbf{W}_P$:

(1) Different wavelet bases correspond to different orthogonal matrices, which lead to different eigenvalue distribution. Therefore, different wavelet bases lead to different convergence performances of wavelet packet transform-based adaptive beamforming.

(2) With wavelet packet decomposition series increasing, the autocorrelation matrix of wavelet packet transform signal has the further diagonal tendency, so the convergence performance of our proposed algorithm is improved.

(3) When the decomposition series are the same, wavelet packet transform based beamforming algorithm has a better convergence than wavelet transform based beamforming algorithm. (This will be demonstrated experimentally in Section 5).

4.3. Complexity Analysis

This beamforming algorithm uses wavelet packet transform as the preprocessing, and then wavelet packet transform signal uses LMS algorithm to implement the adaptive beamforming. Using wavelet packet transform requires an extra but small computation. Suppose that the number of antennas is M, and the decomposition series is J. The computational complexity of wavelet packet transform is $O(JM \log_2 M)$. The computational complexity of a conventional LMS iteration is $O(2M)$. When $M = 32$, $J = 3$, the computational complexity of wavelet packet transform is equivalent to several iterations of LMS. Our proposed adaptive beamforming algorithm uses the reduce-dimension technique to reduce the complexity. The reduced dimension is $M/2$, so and the computational complexity of LMS in our proposed algorithm is $O(M)$. The autocorrelation matrix of wavelet packet transform signal has the diagonal tendency, which leads to a quick convergence speed of LMS. Our proposed algorithm has the low complexity, and it can be easy to implement.

5. SIMULATION RESULTS

We present simulations to test the performance of wavelet packet transform-based adaptive beamforming algorithm. The 32-element-uniform-linear array with spacing $\lambda/2$ is used in the simulation. The array antenna receives 6 signals with different DOAs of 10°, 20°, 30°, 40°, 50° and 60°, respectively. The DOA of the expected signal is 30°, and Daubechies wavelet base series is chosen. We use MATLAB software and computer with Pentium 4, CPU 3.2 GHz, memory 504 Mb as simulation environment. For our proposed algorithm with SNR=20 and 600 iterations, our proposed algorithm method requires 12 s.

Simulation 1. Wavelet Packet transform-based Adaptive BeamForming algorithm (WP-ABF) is compared with Wavelet Transform based Adaptive BeamForming (WT-ABF) and LMS Adaptive BeamForming algorithm (LMS-ABF), respectively. db5 is

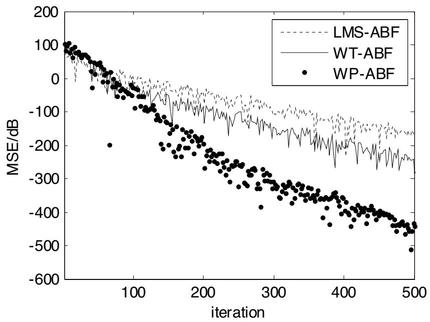

Figure 5. Convergence performance comparison without noise.

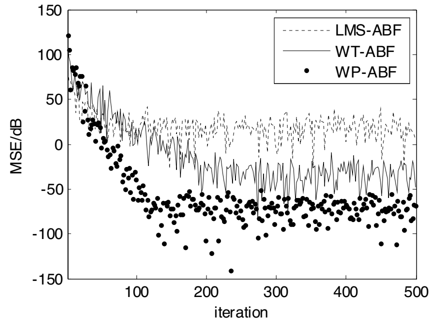

Figure 6. Convergence performance comparison with SNR = 20.

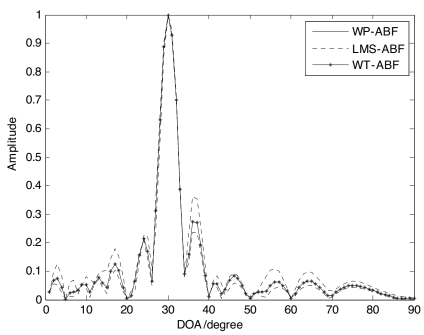

Figure 7. Beamforming performance comparison.

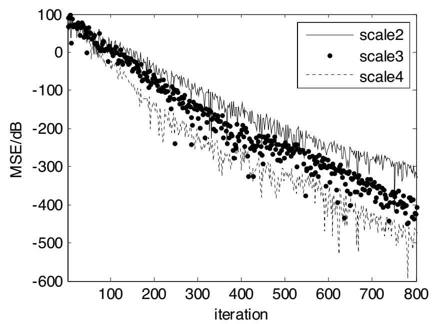

Figure 8. Convergence comparison under different composition series.

chosen as the wavelet base, and decomposition series is 4. The simulation results are shown in Fig. 5–Fig. 7. Different algorithms are compared without noise in Fig. 5 and with SNR = 20 in Fig. 6. From Fig. 5 and Fig. 6, we find that WP-ABF algorithm has much better convergence performance than WT-ABF, and the convergence performance of LMS-ABF is the worst among the three algorithms. Different algorithm beamforming performances with 200 iterations are shown in Fig. 7; it is seen that WP-ABF has lower sidelobe and better beamforming performance than LMS-ABF.

Simulation 2. The effect of different series under the same wavelet base on the convergence speed of WP-ABF is studied. db3 is chosen as the wavelet base. The simulation results are shown in Fig. 8,

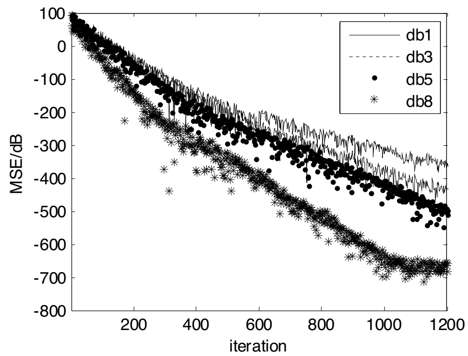

Figure 9. Convergence comparison under different wavelet bases.

from which we find that the convergence performance of the algorithm becomes better with series increasing. That is because the wavelet packet transform signal correlation decreases with series increasing.

Simulation 3. The effect of different wavelet bases on the convergence speed of WP-ABF under the same series is investigated. Daubechies wavelet base series, including db1, db3, db5 and db8, are chosen. Simulation results are shown in Fig. 9. It is seen that db8 has the best convergence performance among the four wavelet bases, db5 is better than db3, and db3 is better than db1. That is because the convergence performance gets better with wavelet base regularity increasing. Among the four wavelet bases, db8, which has the highest regularity, is the best in convergence performance; and db1, which has the worst regularity, has the lowest convergence performance.

6. CONCLUSIONS

A low complexity wavelet packet transform-based LMS adaptive beamforming algorithm is presented. Theoretical analysis and simulations demonstrate that this algorithm converges faster than the conventional LMS adaptive beamforming algorithm and the wavelet transform-based beamforming algorithm. The convergence of the algorithm is closely related to wavelet base and series: the convergence gets better with the increasing of series, and under the same series of wavelet base the convergence gets better with the increasing of regularity. Our proposed algorithm has the low complexity, and it can be easy to implement.

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