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DIFFRACTION OF A PLANE ELECTROMAGNETIC WAVE BY A SLOT IN A CONDUCTING SCREEN OF FINITE THICKNESS PLACED IN FRONT OF A HALF-INFINITE DIELECTRIC

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Abstract—The problem of diffraction of a plane electromagnetic wave by a slot in a planar perfectly conducting screen of arbitrary thickness in the presence of a half-infinite dielectric arranged at a distance from a screen is solved rigorously on the bases of eigenfunction expansion and mode matching technique. The calculation algorithm for various components of the electric and magnetic field vectors in the entire space is presented, and a simple computation method for corresponding diffraction integrals is described. Just one more method of field visualization is demonstrated, which utilizes a picture of the energy-flux lines.

1. INTRODUCTION

Slotted diffraction elements are widely used for electromagnetic field transformation in various optical and microwave devices [1–7]. That is why the problem of simulation of a plane wave diffraction by slots in conducting screens, as well the like problem of diffraction by a complementary screen in the form of a strip, is of great interest [2, 3, 8–14]. The classic statement of these problems proceeds from the premise of a conducting screen having infinitesimal thickness [1, 2, 8, 9]. Their rigorous solutions as a limiting case of the solution for an elliptical cylinder were obtained more than half a century ago [15]. However, they are too complex and inconvenient for applications. Therefore, rather successfully attempts to construct an approximate theory were made (see, for example, [8]). At the same time, one should recognize as necessary that the abstract premise about screens having infinitesimal thickness does not quite correspond to the real situation. That is why the other theoretical approach was developed. It takes into account finite thickness of a screen and finite extent of the region inside a slot, passing by radiation. This approach is based on the mode-matching method [2], which looks like the method used in the rigorous diffraction theory [1, 8, 9]. It provided the possibility to obtain the solutions [3–6, 10–13], which may be unwieldy, but is greatly simple than the rigorous solution for a screen of infinitesimal thickness [15] and is convenient for computer programming. However, the solutions obtained by the mode-matching method, are applicable only in the cases when thickness of a screen with a slot is in the order of the wavelength of diffracted radiation or more. For small values of thickness a computation algorithm is unstable and yields false results [10, 11] since the system of amplitude equations on interfaces becomes ill-conditioned. To avoid this disadvantage, in [16] it was proposed to apply additionally the Tikhonov regularization procedure [17] to such a system. It provided the possibility to improve largely the computation algorithm and to extend the field of theory application to the cases of arbitrary screen thickness. As a result, this approach received attributes of a perfectly rigorous diffraction theory being independent of any constraint or approximation related to parameters of a diffraction system.

Just one more significant merit of the mode-matching approach is that it provides the possibility to consider additional dielectric inclusions arranged nearly a slot. For example, in the work [18] the problem of a plane wave diffraction by a slot which is passed by a plane infinite dielectric layer was solved rigorously. Its solution formed the basis of the theoretical model of a cylindrical cavity resonator with a transverse circular slot using for measuring of permittivity of a plane dielectric [7]. Other possible geometry of mutual arrangement of a slot and a dielectric is its placing behind a screen. In the given work, the solution of a diffraction problem is obtained for such geometry, when a plane electromagnetic wave is incident on a slot in a perfectly conducting screen and on a half-infinite dielectric arranged at a distance from it. A similar geometry corresponds to assemblage of practically important problems. For instance, it can considered as a more really simulation of a diaphragm process of radiation, or a simulation of a process of photosensitive material exposition through an opaque template with passing openings.

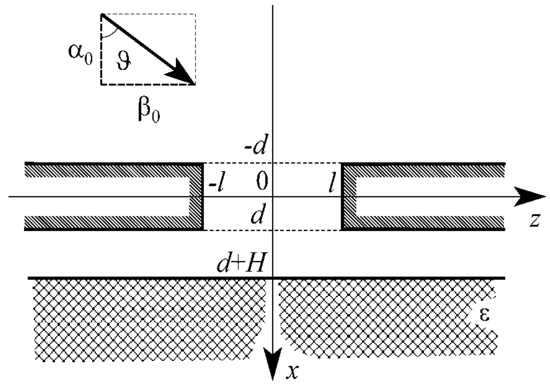

Figure 1. Diffraction of a plane wave by a slot in a screen, which is placed in front of a dielectric. l is the half-width of a slot, d is the half-thickness of a screen, H is the distance between a screen and a dielectric.

2. FORMULATION OF THE DIFFRACTION PROBLEM AND ITS SOLUTION

Let us consider the field which is formed by passing of the plane wave $\exp[i\alpha_0(x + d) + i\beta_0 z]$ through a slot in a plane screen in the direction of a half-infinite dielectric (Fig. 1). It is supposed that a screen has the finite thickness $2d$ and is perfectly conducting. We shall solve a stationary diffraction problem for the monochromatic field with time dependence determined by the factor $\exp(-i\omega t)$ which is hereafter omitted. In the Cartesian coordinate system, one usually describes two-dimensional diffraction field in terms of two independent polarizations having the following components of the electric and magnetic vectors [1, 2, 8, 9, 19]

$$E_y = u \qquad H_x = \frac{i}{k}\frac{\partial u}{\partial z} \qquad H_z = -\frac{i}{k}\frac{\partial u}{\partial x} \tag{1a}$$

for H polarization, and

$$E_x = -\frac{i}{k}\frac{\partial \bar{u}}{\partial z} \qquad E_z = \frac{i}{k}\frac{\partial \bar{u}}{\partial x} \qquad H_y = \bar{u}\varepsilon(x) \tag{1b}$$

for E polarization, where $i = \sqrt{-1}$ is the imaginary unite, $k = \omega/c$ is the wave number, c is the speed of light, u and $\bar{u}$ are the complex scalar functions of the coordinates x and z. They should satisfy the wave equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k^2\varepsilon(x)\right)\left\{\begin{matrix} u \\ \bar{u} \end{matrix}\right\} = 0 \qquad (2)$$

Here, $\varepsilon(x)$ is the piecewise constant function equal to the permittivity of a dielectric ε inside the region of its placing and unity outside it.

The boundary conditions for the functions u and $\bar{u}$ follow from the known boundary conditions for the electric and magnetic vectors (1) [1, 2, 8, 9]: at a conducting surface, the value u and the normal derivative of $\bar{u}$ should vanish, and on the surface of a dielectric the continuity conditions should be enforced on the values u and $\bar{u}\varepsilon(x)$ as well as on the normal derivatives of u and $\bar{u}$.

In all of the regions of the field space (in front of the screen, in the interior of the slot and behind the screen), we shall seek the scalar field functions u and $\bar{u}$ as superpositions of standing modes of the coordinate z. Following [16], let us write the representation for the field in front of the screen (at $x \leq -d$)

$$\begin{aligned} u &= (\cos\beta_0 z + i\sin\beta_0 z)\left(e^{i\alpha_0(x+d)} - e^{-i\alpha_0(x+d)}\right) \\ &\quad + \int_0^{+\infty}[A_{(s)}(\beta)\cos\beta z + iA_{(a)}(\beta)\sin\beta z]e^{-i\alpha(x+d)}d\beta \qquad (3a) \\ \bar{u} &= (\cos\beta_0 z + i\sin\beta_0 z)\left(e^{i\alpha_0(x+d)} + e^{-i\alpha_0(x+d)}\right) \\ &\quad - k\int_0^{+\infty}\alpha^{-1}[\bar{A}_{(s)}(\beta)\cos\beta z + i\bar{A}_{(a)}(\beta)\sin\beta z]e^{-i\alpha(x+d)}d\beta \qquad (3b) \end{aligned}$$

where the first terms on the right-hand sides of (3) represent a sum of the incident and reflected plane waves, and the integral terms are Fourier expansions of the diffracted field with their separation on symmetric (index “s”) and antisymmetric (index “a”) parts in z,

$$\alpha = \sqrt{k^2 - \beta^2} \qquad (4)$$

Inside the slot ($-d \leq x \leq d$, $-l \leq z \leq l$), one should take into consideration the boundary conditions on the conducting surfaces $z = -l$ and $z = l$. Then, instead of a continuous distribution in mode parameter (3) we have a sum over an infinite discrete series of mode propagation parameters [10, 11, 16]

$$\begin{aligned} u = \sum_{n=1}^{+\infty}\Big\{&\left[a_{(\mathrm{s})n}\exp[i\sigma_{(\mathrm{s})}(d+x)] + b_{(\mathrm{s})n}\exp[i\sigma_{(\mathrm{s})}(d-x)]\right]\cos\xi_{(\mathrm{s})n}z \\ &+ i\left[a_{(\mathrm{a})n}\exp[i\sigma_{(\mathrm{a})}(d+x)] + b_{(\mathrm{a})n}\exp[i\sigma_{(\mathrm{a})}(d-x)]\right]\sin\xi_{(\mathrm{a})n}z\Big\} \qquad (5a) \end{aligned}$$

$$\bar{u} = k\sum_{n=1}^{+\infty}\Big\{\bar{\sigma}_{(\mathrm{s})n}^{-1}\left[\bar{a}_{(\mathrm{s})n}\exp[i\bar{\sigma}_{(\mathrm{s})}(d+x)]-\bar{b}_{(\mathrm{s})n}\exp[i\bar{\sigma}_{(\mathrm{s})}(d-x)]\right]\cos\bar{\xi}_{(\mathrm{s})n}z$$
$$+i\bar{\sigma}_{(\mathrm{a})n}^{-1}\left[\bar{a}_{(\mathrm{a})n}\exp[i\bar{\sigma}_{(\mathrm{a})}(d+x)]-\bar{b}_{(\mathrm{a})n}\exp[i\bar{\sigma}_{(\mathrm{a})}(d-x)]\right]\sin\bar{\xi}_{(\mathrm{a})n}z\Big\} \quad (5b)$$

where

$$\xi_{(\mathrm{s})n} = \bar{\xi}_{(\mathrm{a})n} = (\pi/l)(n-1/2) \quad \xi_{(\mathrm{a})n} = \pi n/l \quad \bar{\xi}_{(\mathrm{s})n} = (\pi/l)(n-1) \quad (6a)$$

$$\sigma_{(\mathrm{s,a})n} = \sqrt{k^2-\xi^2_{(\mathrm{s,a})n}} \quad \bar{\sigma}_{(\mathrm{s,a})n} = \sqrt{k^2-\bar{\xi}^2_{(\mathrm{s,a})n}} \quad (6b)$$

are the propagation parameters of symmetric and antisymmetric modes on the coordinates z and x, $a_{(\mathrm{s,a})n}$, $\bar{a}_{(\mathrm{s,a})n}$ and $b_{(\mathrm{s,a})n}$, $\bar{b}_{(\mathrm{s,a})n}$ are the amplitudes of the normal slot modes, propagating in the opposite directions of the x axis.

In the region behind the screen ($x \geq d$), we should take into account reflection and refraction at the plane boundary of a dielectric $x = d + H$. Then at $d \leq x \leq d + H$ (outside a dielectric) we get

$$\begin{aligned} u &= \int_0^{+\infty}[B_{(\mathrm{s})}(\beta)\cos\beta z + iB_{(\mathrm{a})}(\beta)\sin\beta z] \\ &\quad \left(e^{i\alpha(x-d)} + Re^{i\alpha(d+2H-x)}\right)d\beta \\ \bar{u} &= k\int_0^{+\infty}[\bar{B}_{(\mathrm{s})}(\beta)\cos\beta z + i\bar{B}_{(\mathrm{a})}(\beta)\sin\beta z] \\ &\quad \left(e^{i\alpha(x-d)} + \bar{R}e^{i\alpha(d+2H-x)}\right)\frac{d\beta}{\alpha} \end{aligned} \quad (7)$$

and in a dielectric (at $x \geq d + H$)

$$\begin{aligned} u &= \int_0^{+\infty}[B_{(\mathrm{s})}(\beta)\cos\beta z + iB_{(\mathrm{a})}(\beta)\sin\beta z]Te^{i\alpha H+i\gamma(x-d-H)}d\beta \\ \bar{u} &= k\int_0^{+\infty}\alpha^{-1}[\bar{B}_{(\mathrm{s})}(\beta)\cos\beta z+i\bar{B}_{(\mathrm{a})}(\beta)\sin\beta z]\bar{T}e^{i\alpha H+i\gamma(x-d-H)}d\beta \end{aligned} \quad (8)$$

Here,

$$\gamma = \sqrt{k^2\varepsilon - \beta^2} \quad (9)$$

is the mode propagation parameter on the coordinate x in a dielectric,

$$R = \frac{\alpha-\gamma}{\alpha+\gamma} \quad \bar{R} = \frac{\varepsilon\alpha-\gamma}{\varepsilon\alpha+\gamma} \quad T = \frac{2\alpha}{\alpha+\gamma} \quad \bar{T} = \frac{2\alpha}{\varepsilon\alpha+\gamma} \quad (10)$$

are the reflection and refraction coefficients for every mode on the plane dielectric boundary $x = d + H$.

The field functions (3), (7), (8) should have finite values at infinity of x, hence one needs to choose the roots (4), (9) with the nonnegative imaginary parts ($\mathrm{Im}\alpha \geq 0$, $\mathrm{Im}\gamma \geq 0$).

In each of the field existence regions, mode amplitudes are determined by field matching at their boundaries $x = -d$ and $x = d$. It is convenient to use the following procedure: at first, to express the amplitudes of the modes outside a slot $A_{(\mathrm{s,a})}(\beta)$, $B_{(\mathrm{s,a})}(\beta)$ and $\bar{A}_{(\mathrm{s,a})}(\beta)$, $\bar{B}_{(\mathrm{s,a})}(\beta)$ in terms of the slot-mode amplitudes $a_{(\mathrm{s,a})n}$, $b_{(\mathrm{s,a})n}$ and $\bar{a}_{(\mathrm{s,a})n}$, $\bar{b}_{(\mathrm{s,a})n}$. Corresponding equations can be obtained by means of enforcing the boundary conditions at $x = \pm d$ on the function u and on the normal derivative $\partial\bar{u}/\partial x$, and using the property of mutual orthogonality of the mode functions of z in the expansions (3), (5). Then, one should substitute the obtained expressions into the boundary conditions for the normal derivative $\partial u/\partial x$ and for the function $\bar{u}$. It provides the possibility to reduce the problem to solving a system of linear algebraic equations in the slot-mode amplitudes. In this way, we can find the following expressions for the amplitudes of the modes outside a slot

$$
\begin{aligned}
A_{(\mathrm{s,a})}(\beta) &= \frac{l}{\pi}\sum_{n=1}^{\infty}[a_{(\mathrm{s,a})n} + b_{(\mathrm{s,a})n}\exp(2i\sigma_{(\mathrm{s,a})n}d)]Q_n^{(\mathrm{s,a})}(\beta) \\
B_{(\mathrm{s,a})}(\beta) &= \frac{l}{\pi[1 + R\exp(2i\alpha H)]} \\
&\quad \sum_{n=1}^{\infty}[a_{(\mathrm{s,a})n}\exp(2i\sigma_{(\mathrm{s,a})n}d) + b_{(\mathrm{s,a})n}]Q_n^{(\mathrm{s,a})}(\beta)
\end{aligned}
\tag{11}
$$

where

$$
\begin{aligned}
Q_n^{(\mathrm{s})}(\beta) &= \frac{1}{l}\int_{-l}^{l}\cos\beta z\cos\xi_{(\mathrm{s})n}z\,dz \\
&= \mathrm{sinc}[(\beta - \xi_{(\mathrm{s})n})l] + \mathrm{sinc}[(\beta + \xi_{(\mathrm{s})n})l] \\
Q_n^{(\mathrm{a})}(\beta) &= \frac{1}{l}\int_{-l}^{l}\sin\beta z\sin\xi_{(\mathrm{a})n}z\,dz \\
&= \mathrm{sinc}[(\beta - \xi_{(\mathrm{a})n})l] - \mathrm{sinc}[(\beta + \xi_{(\mathrm{a})n})l]
\end{aligned}
$$

are the overlap integrals for the modes outside a slot and the slot modes, sinc is the conventional notation for the function $\mathrm{sinc}x = \sin x/x$. The same expressions will be valid for the amplitudes of E polarization if parameters of H polarization are replaced with corresponding parameters of E polarization. On substitution all these expressions into the boundary conditions for the normal derivative of u, we obtain the following equations in the slot-mode amplitudes of H polarization, which are analogous with the equations of [16]

$$\begin{aligned}
&\sum_{m=1}^{\infty}\left(V_{nm}^{(\mathrm{s,a})}\Gamma_m^{(\mathrm{s,a})}p_m^{(\mathrm{s,a})}+W_{nm}^{(\mathrm{s,a})}\Delta_m^{(\mathrm{s,a})}q_m^{(\mathrm{s,a})}\right)\\
&\qquad+\sigma_{(\mathrm{s,a})n}\Delta_n^{(\mathrm{s,a})}p_n^{(\mathrm{s,a})}=\alpha_0 Q_n^{(\mathrm{s,a})}(\beta_0)\\
&\sum_{m=1}^{\infty}\left(W_{nm}^{(\mathrm{s,a})}\Gamma_m^{(\mathrm{s,a})}p_m^{(\mathrm{s,a})}+V_{nm}^{(\mathrm{s,a})}\Delta_m^{(\mathrm{s,a})}q_m^{(\mathrm{s,a})}\right)\\
&\qquad+\sigma_{(\mathrm{s,a})n}\Gamma_n^{(\mathrm{s,a})}q_n^{(\mathrm{s,a})}=\alpha_0 Q_n^{(\mathrm{s,a})}(\beta_0)
\end{aligned}\tag{12}$$

where

$$\begin{aligned}
p_n^{(\mathrm{s,a})}&=(a_{(\mathrm{s,a})n}+b_{(\mathrm{s,a})n})/2 \qquad q_n^{(\mathrm{s,a})}=(a_{(\mathrm{s,a})n}-b_{(\mathrm{s,a})n})/2\\
\Gamma_n^{(\mathrm{s,a})}&=1+\exp(2i\sigma_{(\mathrm{s,a})n}d) \qquad \Delta_n^{(\mathrm{s,a})}=1-\exp(2i\sigma_{(\mathrm{s,a})n}d)\\
V_{nm}^{(\mathrm{s,a})}&=\frac{l}{\pi}\int_0^{+\infty}U_{nm}^{(\mathrm{s,a})}(\beta)d\beta \quad W_{nm}^{(\mathrm{s,a})}=\frac{l}{\pi}\int_0^{+\infty}U_{nm}^{(\mathrm{s,a})}(\beta)Re^{2i\alpha H}d\beta
\end{aligned}$$

$$U_{nm}^{(\mathrm{s,a})}(\beta)=\frac{\alpha Q_n^{(\mathrm{s,a})}(\beta)Q_m^{(\mathrm{s,a})}(\beta)}{1+R\exp(2i\alpha H)}\tag{13}$$

The form of the corresponding equations for E polarization is distinguished from (12) by the presence of the factor $1/k$ instead of the constant factor α_0. Besides, in the matrix element (13), the coefficient α will go from the numerator to the denominator. Because of symmetry of the diffraction structure about the x axis (Fig. 1), the obtained system of equations (12) splits into two subsystems for the symmetric and antisymmetric modes which is independent one from the other. However, the presence of a dielectric disturbs symmetry of the structure about the z axis, therefore, the system does not further split into independent subsystems for the amplitude values $p_n^{(\mathrm{s,a})}$ and $q_n^{(\mathrm{s,a})}$, as it occurred in the case of the absence of a dielectric [16].

One should carry out the reduction of the infinite-dimensional system (12) to a finite-dimensional system [20], bounding number of the slot modes in all expressions by the finite value N. Then, the either system (12) with the index "s" or "a" can be solved by one of the well-known methods [21], using previously the Tikhonov regularization procedure, as it was described in [16]. This procedure provides the possibility to obtain a stable mathematical solution for the general case of arbitrary screen thickness.

So, the calculation of diffracted field is reduced to the following. At first, the slot amplitudes are determined as the solutions of two systems (12), then the mode amplitudes outside of a screen are calculated by means of expressions (11), and afterwards one can determine the scalar field functions in each of the field existence regions using expressions (3), (5), (7) and (8). The components of the electric and magnetic vectors in these regions are computed with the help of Equations (1), into which the given expressions should be substituted. The integral matrix elements (13) and the integral expressions for the scalar field functions (3), (7) and (8) can be calculated using quadrature formulas on uniform grid for argument β by the mid-ordinate rule [22]. In that way, the continuous spectrum of the modes outside a slot is approximated by a discrete finite spectrum like the slot-mode spectrum. One should bear in mind that far from a slot (at large $|x|$ or $|z|$) sinusoidal and complex exponential functions in the integrands of (3), (7), (8) will be fast oscillating, therefore, for the integrals, a simple mid-ordinate rule can cause to be in great error. In order to decrease such errors, one can utilize the more general rule for integral computation over every interval of a grid (see Appendix).

Figure 2 demonstrates the spatial distributions of two different electric field components of H and E polarizations. They were computed according to the described above algorithm for the case of normal incidence of a plane wave on the slot with the width $2l = 1.15\lambda$ (i.e., when $kl = 3.6$), cut in the perfectly conducting screen with the thickness $2d = 0.573\lambda$, which is placed within the distance $H = 0.955\lambda$ of the self-infinite dielectric with the complex index of refraction $n = 1.85 + 0.022i$.

One more method of field visualization for two-dimensional diffraction problems is indication of energy-flux lines, i.e., lines having the Poynting vector [1, 9, 19]

$$\mathbf{S} = (c/8\pi)\mathrm{Re}(\mathbf{E} \times \mathbf{H}^*) \tag{14}$$

as a tangential one, where the asterisk denotes complex conjugation. These lines show direction of energy propagation and also provide the possibility to evaluate energy flux value using their density in a given field region. For the convenience of computer calculation of such lines, one can consider a scalar function, which is called conventionally the energy potential [16]

$$U = U_0 + \int_{\mathbf{r}_0}^{\mathbf{r}} (\mathbf{e}_y \times \mathbf{S}) d\mathbf{r} \tag{15}$$

where $\mathbf{e}_y$ is the ort of the y direction, i.e., of the direction of translational invariance, $U_0 = U(\mathbf{r}_0)$, $\mathbf{r}_0$ is the radius vector of an

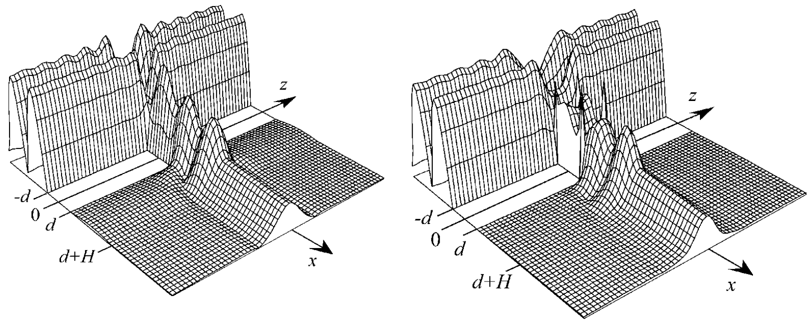

Figure 2. Spatial distribution of one electric field component for normal propagation of a plane wave through a slot in a conducting screen in the direction of a self-infinite dielectric. The case of H polarization (the component E_y) is left, and the case of E polarization (the component E_z) is right.

arbitrary chosen fixed point in space. Gradient of the energy potential (15) is orthogonal to the Poynting vector $\mathbf{S}$ (14), hence every such a vector directs along equipontential line of (15) in some point of space, and a set of these lines will be energy-flux ones.

Figure 3 demonstrates the energy-flux lines of H and E polarizations computed by means of the energy potential (15) for the same case as the amplitude distributions in Fig. 2. Because of symmetry about the x axis, this picture is displayed only for the half-space $z \geq 0$. In a dielectric, the energy-flux lines are drew as dotted lines, since in absorbing media these lines are not vortex as in transparent media. Here, according to the known equation of energy balance [9, 19], there is continuous distribution of sources of electromagnetic field absorption over the whole dielectric volume.

Notice that substitution of Equations (1) into (14) yields the following expression for the Poynting vector in terms of the scalar field function

$$\mathbf{S} = (c/8\pi k)|u|^2\nabla(\arg u)$$

both for H and for E polarization, where $\arg u$ is the argument of the complex field function u. From that, it is clear why the energy-flux lines of H polarization are pushed from a conducting screen, but conversely, the energy-flux lines of E polarization are as if attracted by it [16] (see Fig. 3). Indeed, the field function value for H polarization should vanish on conducting surface, consequently appropriate flux lines should be sparse near it. On the contrary, for E polarization, on such a surface not the scalar field function but its normal derivative

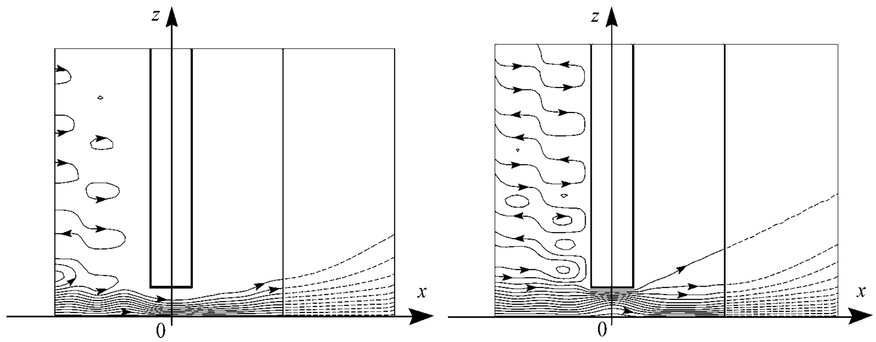

Figure 3. Energy-flux lines for normal propagation of a plane wave through a slot in a conducting screen in the direction of a self-infinite dielectric. The case of H polarization is left, and the case of E polarization is right.

specifies zero value. It corresponds to maximum value of this function hence there is dense distribution of flux lines near conducting surface.

3. CONCLUSION

We obtained the solution of the problem of plane wave diffraction by a slot in a plane screen and a self-infinite dielectric placed behind a screen. Just as the similar solution obtained for more simple case of the absence of a dielectric [16], this solution can regarded as quite rigorous, since it corresponds to a stable calculation algorithm at any thickness of a screen, and its utilization is not conditioned on some geometrical parameter of a problem. The solution can be easily extended on the case when a slot is filled in by a dielectric with the permittivity ε_s; for this purpose, one needs to substitute $k^2\varepsilon_s$ for k^2 in Equations (6b). The obtained solution can be generalized also on the case when a more complex dielectric structure is placed behind a screen, which is formed by plane layers with various dielectric permittivity. Then, in Equations (7) and (13), the reflection coefficients for one boundary (10) should be replaced with appropriate reflection coefficients calculated for a layered dielectric structure as a whole.

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APPENDIX A. A SIMPLE NUMERICAL METHOD FOR COMPUTATION OF FIELD PROPAGATION INTEGRALS

In the diffraction theory, one should often evaluate an integral of the form

$$\int F(\beta)\exp[i(\alpha x+\beta z+\gamma y)]d\beta \tag{A1}$$

where

$$\alpha=\sqrt{k^2-\beta^2}\qquad \gamma=\sqrt{k^2\varepsilon-\beta^2} \tag{A2}$$

and x, z and y can specify rather great values. For example, the integrals determining the scalar field function (3), (7), (8) of our diffraction problem in regions outside a slot, are such integrals. In [16, 18], the brief qualitative instruction for deduction of corresponding quadrature formula for these integrals was given. This formula should stand duty as a simple generalization of the ordinary mid-ordinate rule [22] to fast oscillating functions. We present below the specific form of such a formula and its deduction.

Following the usual procedure of integral computation, we should split the interval of integration into a set of small segments. Then the integral (1) will be present itself a sum of elementary integrals

$$I=\frac{1}{b-a}\int_a^b F(\beta)\exp[i(\alpha x+\beta z+\gamma y)]d\beta$$

over all such small segments. For the function F which change slowly within a small segment, one can use the ordinary mid-ordinate approximation [22], when a function value in the middle of a segment is used as a value of F over a whole segment

$$I\approx\frac{1}{b-a}F\left(\frac{a+b}{2}\right)\int_a^b \exp[i(\alpha x+\beta z+\gamma y)]d\beta \tag{A3}$$

For the remaining integral of exponent, one can write a simple expression using a linear approximation for the parameters α and γ (A2) as functions of β. Really, far from zero, a minor portion of the square root function is well approximated by a linear function. More exactly, let $0\le\beta\le 0.7k$ or $\beta\ge 1.3k(\mathrm{Re}\varepsilon)^{1/2}$ for all values of β ranges from a to b. For definiteness, we assume that $\mathrm{Re}\varepsilon>1$, but this supposition has not influence on a final result. Then, one can use a linear approximation for the functions (A2)

$$\begin{aligned}\alpha(\beta) &\approx [(b\alpha_a - a\alpha_b) + (\alpha_b - \alpha_a)\beta]/(b-a)\\ \gamma(\beta) &\approx [(b\gamma_a - a\gamma_b) + (\gamma_b - \gamma_a)\beta]/(b-a)\end{aligned} \tag{A4}$$

where we introduced the brief designations

$$\alpha_a = \alpha(a) \qquad \alpha_b = \alpha(b) \qquad \gamma_a = \gamma(a) \qquad \gamma_b = \gamma(b)$$

Substituting Equations (A4) into the exponent (A3) provides the possibility to express easily the corresponding integral in the explicit form of

$$\begin{aligned}I &\approx -iF\left(\frac{a+b}{2}\right)\frac{\exp(iw_b)-\exp(iw_a)}{w_b-w_a}\\ &= F\left(\frac{a+b}{2}\right)\exp[i(w_a+w_b)/2]\mathrm{sinc}[(w_a-w_b)/2]\end{aligned} \tag{A5}$$

where

$$\begin{aligned}w(\beta) &= \alpha x + \beta z + \gamma y\\ w_a &= w(a) = \alpha_a x + az + \gamma_a y\\ w_b &= w(b) = \alpha_b x + bz + \gamma_b y\end{aligned}$$

Let now the small section $[a,\, b]$ falls wholly within the range $0.7k \leq \beta \leq 1.3k$. Here, not far from zero of the function α (A2), the linear approximation of this function is in great error, but its argument β in itself differs from zero appreciably. Hence, in this section it is expediently to convent to the new integration variable α

$$I \approx -\frac{1}{b-a}F\left(\frac{a+b}{2}\right)\int_{\alpha_a}^{\alpha_b}\frac{\alpha}{\beta}\exp[i(\alpha x + \beta z + \gamma y)]d\alpha \tag{A6}$$

and to consider the parameters β and γ as functions of α applying to them the linear approximation

$$\begin{aligned}\beta(\alpha) &\approx [(\alpha_b a - \alpha_a b) + (b-a)\alpha]/(\alpha_b - \alpha_a)\\ \gamma(\alpha) &\approx [(\alpha_b\gamma_a - \alpha_a\gamma_b) + (\gamma_b - \gamma_a)\alpha]/(\alpha_b - \alpha_a)\end{aligned}$$

Substituting the above expressions into the exponent (A6) and replacing the ratio of α and β with its approximate value $(\alpha_a+\alpha_b)/(a+b)$, we obtain the same result (A5) as in the first case. Similarly, one can consider the case when the interval of integration $[a,\, b]$ falls within the range about zero of the function γ (A2).

So, approximate computation of the diffraction integral (A1) reduces to summation of the elementary integrals (A5) over all small intervals, such that a general form of these integrals does not depend on position of corresponding interval. Comparative calculations carried out for various diffraction problems show that even at very great values of the coordinates x and z Equation (A5) provides a good accuracy for diffraction integrals. More complex approximations (for instance, computation of the integrals (A3) in terms of the Fresnel integrals) do not provide appreciable gain in accuracy but essentially complicate a computation algorithm.

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