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CA-CFAR DETECTION PERFORMANCE OF RADAR TARGETS EMBEDDED IN "NON CENTERED CHI-2 GAMMA" CLUTTER

M. A. Habib

INRS-EMT, University of Quebec
Montréal, Qc., Canada

M. Barkat

American University of Sharjah
Sharjah, UAE

B. Aïssa and T. A. Denidni

INRS-EMT, University of Quebec
Montréal, Qc., Canada

Abstract—This work is about evaluating radar performance in detection of targets embedded in a clutter following Non centered chi-2 Gamma distribution model. This model, also called NG-distribution model, is able to fit high resolution sea radar clutter. In this paper, NG model is described. The performances of CA-CFAR radar, namely probability of detection and probability of false alarm, are calculated and closed forms of these probabilities are achieved. In order to evaluate the obtained results, simulation and analytical results are compared. Good matching between these results is achieved.

1. INTRODUCTION

In radar detection, prediction of clutter level highly affects detection performances, in terms of probability of detection and probability of false alarm. Gaussian model has been widely considered but it showed limitations when considering high resolution radars [1–6]. Many other models have been considered such Lognormal law, Weibull model and the K-distribution [7, 8]. K-distribution is the most suitable model to fit sea clutter in high resolution radars and quantity of work [9–14] has been related to this area. However, a main drawback is related to this clutter model; it is not possible, with K-distribution clutter, to achieve a closed form expression when calculating the probability of detection of the CA-CFAR radar (Cell-Averaging Constant False Alarm Radar) [15].

Shnidman has proposed a new model to describe high resolution sea radar clutter, named "Non-centered chi-2 Gamma" or more simply NG-distribution [16]. He compared this model both Weibull and Lognormal ones and also compared it to real data in order to confirm that this model is able to fit the high resolution sea radar clutter. Hence, it is worth to evaluate performance of radar in detection of targets embedded is such kind of clutter.

In this paper, we propose to study the NG-distribution. We calculate the performance of a CA-CFAR detector within the case of NG clutter. We achieve a closed form expression for both probability of detection and probability of false alarm. Simulation results are then compared to theoretical results.

2. NG-DISTRIBUTION

While illuminating the sea surface by high resolution radar, received signal can be assumed to be a non-centered gaussian random process. The received signal is assumed to be a complex variable when considering both the in phase and in quadrature phase components [16]. Hence, we have in the kth cell is

$$r_k = r_{ki} + j r_{kq} \tag{1}$$

where r_k is the kth return in the kth cell. The mean of each component can be expressed by

$$\begin{cases} m_{ki} = m_k \cos\theta \\ m_{kq} = m_k \sin\theta \end{cases} \tag{2}$$

where θ is the relative phase between the received signal and the oscillator. It is assumed to be a uniformly distributed random variable. The quadratic detection leads to the following signal

$$Z_k = |r_k|^2 = (r_{ki})^2 + (r_{kq})^2, \tag{3}$$

and the function density of Z_k is given by [16]

$$f_{Z_k}(z_k) = \frac{1}{2\sigma^2} \exp\left(-\frac{z_k + m_k^2}{2\sigma^2}\right) I_0\left(\frac{\sqrt{z_k m_k^2}}{2\sigma^2}\right) \tag{4}$$

$I_N(\cdot)$ is the Nth order modified Bessel function, Z_k is a $\chi - 2$ non centered process with 2 degrees of freedom, and $(m_k)^2$ is the non-centrality parameter. Supposing the existence of N_c returns yields to a new variable

$$Z = \sum_{k=1}^{N_c} Z_k \tag{5}$$

Let define the following parameter

$$\kappa = \sum_{k=1}^{N_c} m_k^2; \tag{6}$$

the density function of Z becomes

$$f_Z(z\,|2N_c, \kappa, \sigma) = \frac{1}{2\sigma^2}\left(\frac{z}{\kappa}\right)^{N_c-1} \exp\left(-\frac{z+\kappa}{2\sigma^2}\right) I_{N_c-1}\left(\frac{\sqrt{z\kappa}}{2\sigma^2}\right) \tag{7}$$

When considering the following notations

$$y = \frac{z}{2\sigma^2}, \;\; \kappa_n = \frac{\kappa}{2\sigma^2} \tag{8}$$

then we obtain

$$f_Y\left(y\,|2N_c, \kappa_n\right) = \left(\frac{y}{\kappa_n}\right)^{N_c-1} \exp\{-\left(y+\kappa_n\right)\} I_{N_c-1}\left(\sqrt{y\kappa_n}\right) \tag{9}$$

m_k variables are not supposed to be constant; for a generalized approach, they will be assumed random. Instead of treating each m_k separately, their quadratic sum κ_n is supposed to be a Gamma random process. This allows describing correctly the texture effect of high resolution sea radar clutter. Hence, it is possible to write

$$f(\kappa_n) = \frac{\kappa_n^{L-1} L^L}{\Gamma(L) X_c^L} \exp\left\{-\left(\frac{L\kappa_n}{X_c}\right)\right\} \tag{10}$$

where X_c is the mean parameter and x is the fluctuation parameter that can define the returns correlation parameter. From Equations (9) and (10), we can write the non-centered chi-2 Gamma law to be

$$\begin{aligned} f_{NG}\left(y\,|N_c, X_c, L\right) &= \int_0^{+\infty} f_Y\left(y\,|2N_c, \kappa_n\right) f(\kappa_n) d\kappa_n \\ &= \left(\frac{\dfrac{L}{X_c}}{1+\dfrac{L}{X_c}}\right)^L \exp(-y) \frac{y^{N_c-1}}{\Gamma(L)} \end{aligned}$$

$$\sum_{k=0}^{\infty}\frac{\Gamma(L+k)}{\Gamma(N_c+k)\Gamma(k+1)}\left(\frac{y}{1+\dfrac{L}{X_c}}\right)^k \tag{11}$$

or simply,

$$f_{NG}(y\,|N_c,X_c,L)=\left(\frac{\dfrac{L}{X_c}}{1+\dfrac{L}{X_c}}\right)^L\exp(-y)\frac{y^{N_c-1}}{\Gamma(N_c)}M\left(L,N_c,\frac{y}{1+\dfrac{L}{X_c}}\right) \tag{12}$$

with

$$M(a,b,z)={}_1F_1(a,b,z)=\frac{\Gamma(b)}{\Gamma(a)}\sum_{k=0}^{\infty}\frac{\Gamma(a+k)}{\Gamma(b+k)}\frac{z^k}{k!} \tag{13}$$

$M(a,b,z)$ is the Kummer function or hypergeometric confluent function [17, 18]. The distribution function can then be written as

$$\begin{aligned}F_{NG}\left(y\,|N_c,X_c,L\right) &= \int_0^y f_{NG}\left(x\,|N_c,X_c,L\right)dx\\ &= \sum_{m=N_c}^{\infty}\exp(-y)\frac{y^m}{\Gamma(m+1)}\\ &\quad\left[\sum_{k=0}^{m-N_c}\frac{\Gamma(L+k)}{\Gamma(L)\Gamma(k+1)}\frac{\left(\dfrac{\dfrac{X_c}{L}}{1+\dfrac{X_c}{L}}\right)^k}{\left(1+\dfrac{X_c}{L}\right)^L}\right]\end{aligned} \tag{14}$$

or simply,

$$\begin{aligned}F_{NG}\left(y\,|N_c,X_c,L\right) &= \sum_{k=0}^{\infty}\frac{\Gamma(L+k)}{\Gamma(L)\Gamma(k+1)}\frac{\left(\dfrac{\dfrac{X_c}{L}}{1+\dfrac{X_c}{L}}\right)^k}{\left(1+\dfrac{X_c}{L}\right)^L}\\ &\quad\left[1-\sum_{m=0}^{N_c-1+k}\exp(-y)\frac{y^m}{\Gamma(m+1)}\right]\end{aligned} \tag{15}$$

The characteristic function is given by the following expression

$$\Phi(j\omega) = \frac{1}{(1+j\omega)^{N_c}} \left(\frac{(L/X_c)\,(1+j\omega)}{j\omega + (L/X_c)\,(1+j\omega)} \right)^L \tag{16}$$

For the nth moment, we have

$$E\left[Y^n\right] = \left(\frac{\dfrac{L}{X_c}}{1+\dfrac{L}{X_c}} \right)^L \frac{\Gamma(N_c+n)}{\Gamma(N_c)} F\left(N_c+n, L, N_c, \frac{1}{1+\dfrac{L}{X_c}} \right) \tag{17}$$

where $F(a,b,c,z)$ is the Gauss hypergeometric function [12, 13] given by

$$F\left(a,b,c,z\right) = {}_2F_1(a,b,c,z) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{k=0}^{\infty} \frac{\Gamma(a+k)\Gamma(b+k)}{\Gamma(c+k)} \frac{z^k}{k!} \tag{18}$$

Shnidman has shown that the NG-distribution can fit many common models that are usually used for high resolution sea radar clutter as the Weibull law and the Lognormal distribution [10]. He showed also that the shape of real situation for both horizontal and vertical polarization can be obtained just by varying the NG-distribution parameters X_c and L.

In [7, 14], K-distribution has been presented as a model to describe clutter model. This model presents a Gaussian variable, namely speckle, with a variance as gamma distributed variable (namely texture). Hence, centrality parameter for the gamma distribution represents for this distribution clutter density.

Comparatively, NG-distribution is defined as quadratic sum of non-centred Gaussian processes (Equation (5)). Their cumulative non-centrality parameter (defined κ in Equation (6)) is gamma distributed, and this texture has a non-centrality parameters X_c (as for K-distribution) which represents clutter spectral density level. Parameter L can be compared to shape parameter in K-distribution; its role is to define the shape of the random process, it is defined from 1 to infinite. A small value close to 1 corresponds to highly correlated cells; on the other hand, if L is equal to number of average cells, this permits to model uncorrelated cells variables. The limit case where L tends to infinites converges toward simple Gaussian variables.

It is interesting to observe how these two parameters, namely X_c and L, affect density function f_{NG} defined in Equation (12) for a specific number of observations N_c. Figures 1 to 4 depict these effects

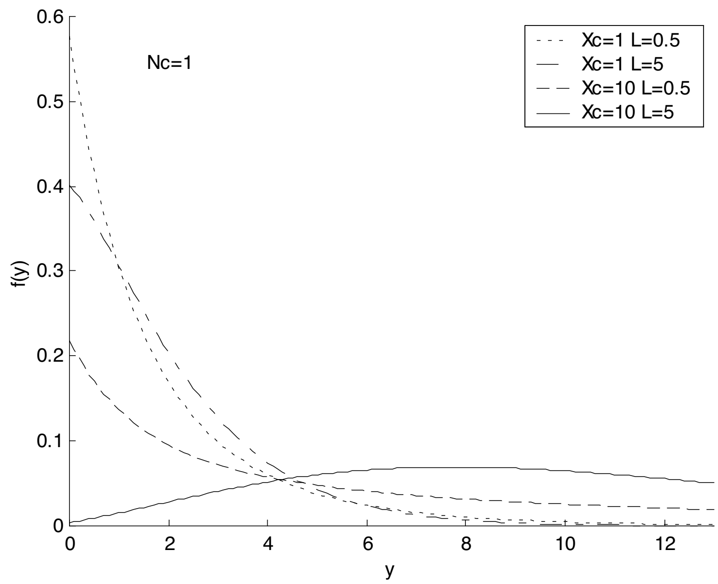

Figure 1. Density function shape for $N_c = 1$.

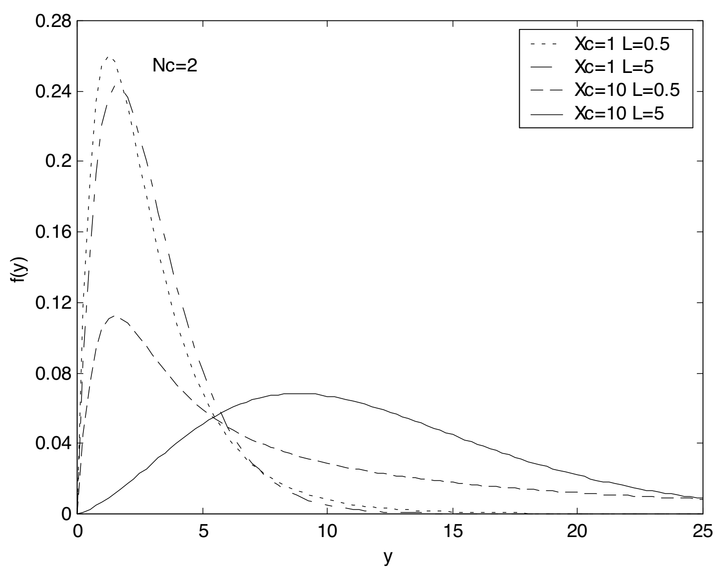

Figure 2. Density function shape for $N_c = 2$.

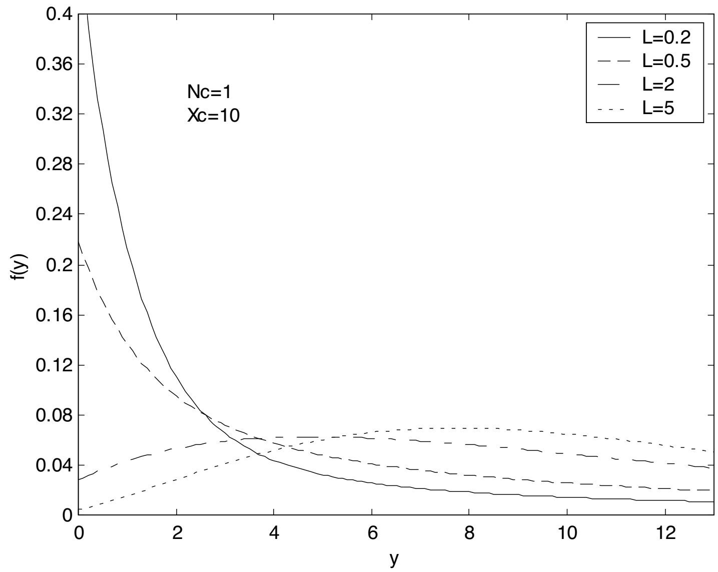

Figure 3. Density function shape with respect to L for $N_c = 1$.

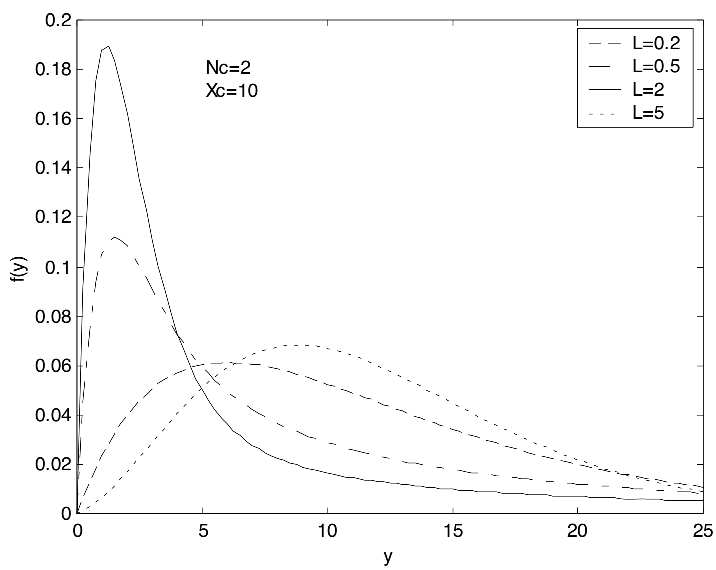

Figure 4. Density function shape with respect to L for $N_c = 2$.

on the density function. In the next section, NG-distribution clutter is considered for evaluating performance of CA-CFAR radar.

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3. CA-CFAR DETECTION

We aim to calculate probability of detection and probability of false alarm of CA-CFAR radar using this model of clutter. CA-CFAR radar or cell-averaging constant false alarm radar uses the principle of evaluating clutter level to maintain a constant rate of false alarm. An estimated value is obtained by averaging received signal over a certain number of cells. This estimation, weighted with a threshold, is then compared to signal from cell under test. A decision of detection is then obtained. Considering a CA-CFAR detector with N cells, the received signal received by each cell is assumed to be N_d times integrated signal; the target is assumed to follow Swerling II model. The detection hypotheses are as follow

$$\begin{cases} H_0 : r(t) = c(t) \\ H_1 : r(t) = s(t) + c(t) \end{cases} \tag{19}$$

$c(t)$ is the clutter and $s(t)$ is the target return, r is assumed to be a non centered gaussian process.

Let write

$$S = \frac{\sigma_s^2}{\sigma^2} \tag{20}$$

σ_s^2 is the variance of the target return and σ^2 is related to the clutter. After passing through a quadratic detector, density function of the signal under hypothesis H_1 is then

$$\begin{aligned} f_{NG}(y\,|N_d, X_d, L_d, S, H_1) \;=\; & \frac{\exp\left(-\dfrac{y}{1+S}\right)}{1+S} \cdot \left(\frac{\dfrac{L_d}{X_d}}{\dfrac{1}{1+S}+\dfrac{L_d}{X_d}}\right)^{L_d} \\ & \cdot \frac{\left(\dfrac{y}{1+S}\right)^{N_d-1}}{\Gamma(L_d)} \cdot \sum_{k=0}^{\infty} \frac{\Gamma(L_d+k)}{\Gamma(N_d+k)\Gamma(k+1)} \\ & \left(\frac{\dfrac{y}{(1+S)^2}}{\dfrac{1}{1+S}+\dfrac{L_d}{X_d}}\right)^{k} \end{aligned} \tag{21}$$

Density function under hypothesis H_0 is deduced by pushing S to be equal to 0 in (21) to obtain

$$f_{NG}(y\,|N_d, X_d, L_d, H_0) = \exp(-y)\cdot\left(\frac{\frac{L_d}{X_d}}{1+\frac{L_d}{X_d}}\right)^{L_d}\cdot\frac{(y)^{N_d-1}}{\Gamma(L_d)} \cdot\sum_{k=0}^{\infty}\frac{\Gamma(L_d+k)}{\Gamma(N_d+k)\Gamma(k+1)}\left(\frac{y}{1+\frac{L_d}{X_d}}\right)^{k} \quad (22)$$

In each cell, the density function is given by the same density function as in Equation (22). Clutter power estimation in CA-CFAR detection is given by considering all cells returns. New process is obtained as follow

$$W=\sum_{i=1}^{N}X_i \quad (23)$$

Total number of returns N_c is given by

$$N_c = N_d\cdot N \quad (24)$$

So we obtain

$$f_{NG}(w\,|N_c, X_c, L) = \exp(-w)\left(\frac{\frac{L}{X_c}}{1+\frac{L}{X_c}}\right)^{L}\frac{(w)^{N_c-1}}{\Gamma(L)} \cdot\sum_{k=0}^{\infty}\frac{\Gamma(L+k)}{\Gamma(N_c+k)\Gamma(k+1)}\left(\frac{w}{1+\frac{L}{X_c}}\right)^{k} \quad (25)$$

All the cells are supposed to be independent. X_c and L can be derived from X_d and L_d as for N_d. Considering the decision rule

$$Y \underset{H_0}{\overset{H_1}{\gtrless}} TW \quad (26)$$

where T is the threshold multiplier, both of the probability of false alarm P_{fa} and the probability of detection P_D can be calculated.

$$P_{fa} = \int_{-\infty}^{+\infty} \int_{Tw}^{+\infty} f_{NG}(y|N_d, X_d, L_d, H_0) dy f_{NG}(w|N_c, X_c, L) dw$$

$$= 1 - \left(\frac{\frac{L}{X_c}}{1 + \frac{L}{X_c}} \right)^L$$

$$\cdot \sum_{m=N_d}^{\infty} \sum_{n=0}^{\infty} \frac{B(m) \left(\frac{T}{1+T} \right)^m}{\beta(L, n) \beta(m, n+N_c) nm \left(1 + \frac{L}{X_c} \right)^n (1+T)^{N_c+n}} \quad (27)$$

$$P_D = \int_{-\infty}^{+\infty} \int_{Tw}^{+\infty} f_{NG}(y|N_d, X_d, L_d, S, H_1) dy f_{NG}(w|N_c, X_c, L) dw$$

$$= 11 - \left(\frac{\frac{L}{X_c}}{1 + \frac{L}{X_c}} \right)^L$$

$$\cdot \sum_{m=N_d}^{\infty} \frac{B_S(m)}{m} \sum_{n=0}^{\infty} \frac{\left(\frac{T}{1+S+T} \right)^m \left(\frac{1+S}{1+S+T} \right)^{N_c+n}}{\beta(L, n) \beta(m, n+N_c) n \left(1 + \frac{L}{X_c} \right)^n} \quad (28)$$

where,

$$B_S(m) = \sum_{k=0}^{m-N_d} \frac{\Gamma(L_d + k)}{\Gamma(L_d) \Gamma(k+1)} \left(\frac{1+S}{1+S+\frac{X_d}{L_d}} \right)^{L_d} \left(\frac{\frac{X_d}{L_d}}{1+S+\frac{X_d}{L_d}} \right)^k \quad (29)$$

and $B(m) = B_{s=0}(m)$. Expressions (27) and (28) show that closed mathematical expressions giving probability of detection and probability of false have been achieved.

4. DISCUSSION AND RESULTS

In order to verify theoretical forms of both the probability of detection and the probability of false alarm, Monte Carlo simulation are carried out [19, 20].

Table 1. Adaptive threshold T.

L/X_c	N_d \ N	6	10	12	16
0.5	1	12.23205	4.1083	2.97992	1.88366
	5	1.35249	0.80432	0.56642	0.40628
	10	0.78588	0.43330	0.35347	0.25873
1	1	8.99999	2.98324	2.16227	1.37137
	5	1.08706	0.57057	0.45996	0.33103
	10	0.66386	0.36818	0.30084	0.22014
2	1	7.41530	2.45934	1.78811	1.13902
	5	0.96356	0.50874	0.41080	0.29627
	10	0.60610	0.33749	0.27604	0.20514

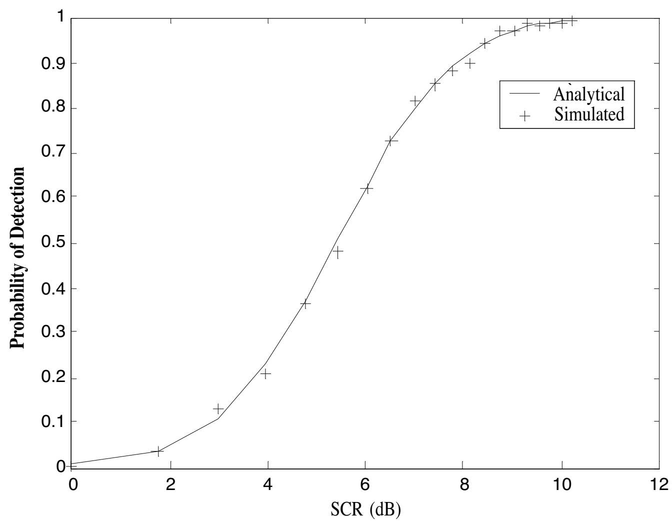

Figure 5. Theoretical and simulated probability of detection.

Table 1 gives adaptive thresholds obtained from theoretical formulas for a probability of false alarm 10^{-6}. These values of thresholds have been verified with Monte Carlo simulations in CA-CFAR detector with NG-clutter. The cells are considered independent and no additive gaussian noise is considered. These values of the adaptive threshold have been calculated from Equation (27) and have been confirmed by the simulations. As known, when integrating the echoes in each cell, the adaptive threshold is much lower, this increases the detection and permits to improve the CA-CFAR detector efficiency. Also, a higher value of the ratio giving L decreases the threshold. This can be explained by the fact that a high value of this ratio converges to a simple gaussian process, this is synonym of low texture effect and leads to low threshold.

Figure 5 compares the theoretical and the simulated probability of detection. It can be seen that both of the curves match. Since a great agreement is observed, this permits to validate analytical expressions of both probability of detection and probability of false alarm with NG-distribution.

5. CONCLUSION

In this work, the non centered chi-2 Gamma distribution has been considered for sea clutter in high resolution radar. When calculating the CA-CFAR detector performances, namely probability of detection and probability of false alarm, closed forms of these probabilities have been achieved. This feature is not achieved when considering K-distribution clutter.

In order to validate our theoretical equations, Monte Carlo simulations have been carried out. Great agreements between theoretical approach and simulations results have been observed.

Hence, NG-distribution can be a real substitute to K-distribution for describing the high resolution sea radar clutter. It offers many advantages, and the most important is that it achieves a closed form for calculating the probability of detection in CA-CFAR detection. This main result can exploited to study decentralized CFAR detectors.

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