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THE EFFICIENT ANALYSIS OF ELECTROMAGNETIC SCATTERING FROM COMPOSITE STRUCTURES USING HYBRID CFIE-IEFIE

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Abstract—The efficient algorithm is presented for the analysis of electromagnetic scattering from composite structures with coexisting open and closed conductors. A hybrid combined-field integral equation-the improved electric-field integral equation (CFIE-IEFIE) formulation with the incomplete LU factorization (ILU) preconditioner is proposed. Numerical results are given to demonstrate that the efficiency of our algorithm can be significantly improved when compared with the conventional EFIE formulation and the hybrid CFIE-EFIE formulation.

1. INTRODUCTION

The radar cross section (RCS) for a target is of great importance in the area of electromagnetic scattering [1–10]. The present study aims to develop an efficient algorithm to analyze the composite-geometry problems involving both open and closed conducting surfaces. For the electromagnetic modeling of problems involving only closed conducting surfaces, the combined-field integral equation (CFIE) [11] is preferred mainly because it is free of the internal resonance problems of both electric-field integral equation (EFIE) and magnetic-field integral equation (MFIE). In addition, it generates considerably better-conditioned linear systems compared to the EFIE. However, in the simulations of practical electromagnetic problems, the thin and thick conducing parts of the objects are usually modeled with open and closed surfaces, respectively. These problems are traditionally solved by the EFIE due to the presence of the open surfaces. However, the EFIE has internal-resonance problems and also leads to ill-conditioned matrix equations that deteriorate the performances of the iterative solvers, especially when the problem size becomes large. In [12], the hybrid CFIE-EFIE formulation for composite geometries is presented to significantly improve the efficiency of the solutions. In this formulation, the EFIE is adopted for thin conductors that are modeled with open surfaces and the CFIE is taken with the closed parts in order to improve the efficiency of the iterative solutions.

To solve discretized surface integral equations, which give rise to large, dense, complex systems, direct methods based on the Gaussian elimination are prohibited due to their $\mathrm{O}(n^2)$ memory and $\mathrm{O}(n^3)$ computational complexity for n unknowns. On the other hand, by making use of the multilevel fast multipole algorithm (MLFMA) [13–16], the dense matrix-vector products required in each step of the iterative solvers can be performed in $\mathrm{O}(n \log n)$ time and $\mathrm{O}(n \log n)$ memory, rendering these solvers very attractive for large problems. However, the iterative solver may not converge, or convergence may require too many iterations for the hybrid CFIE-EFIE formulation when the number of unknowns of the EFIE parts becomes large. In this paper, we propose a novel technique denoted as the hybrid CFIE-IEFIE to accelerate the convergence rate, which replaces the EFIE part in the hybrid CFIE-EFIE techniques with the improved electric field integral equation (IEFIE) [17]. To further accelerate the solution, the incomplete LU preconditioning technique in [18, 19] is proposed to solve the hybrid CFIE-IEFIE.

Incomplete LU (ILU) preconditioners are widely used and available in several solver packages. There are two popular drop strategies for ILU factorization: the level based drop strategy and the threshold based drop strategy, the former is denoted ILU(p), where $p \geq 0$ is an integer denoted as the level of fill-in, and the latter is ILUT. The ILU-class preconditioners have been tested for electromagnetic problems in [19]. It can be found that ILU(0) preconditioner produces highly unstable and hence useless factorizations for the EFIE as the number of unknowns increases. However, the iteration numbers obtained with ILU(0) for the CFIE are very close to those of the exact solution of the near-field matrix preconditioned iterative solvers. It is also observed that the ILU(0) and ILUT preconditioners produce very similar convergence rate for the CFIE. In this paper, the condition number of our proposed hybrid CFIE-IEFIE formulation is similar to that of the CFIE. The ILU(0) preconditioner is chosen to accelerate the convergence rate of hybrid CFIE-IEFIE equation, since it is of lower computational complexity and easier to be implemented than ILUT. The efficiency of the ILU(0) preconditioned hybrid CFIE-IEFIE formulation is demonstrated by numerical experiments.

2. THE HYBRID CFIE-IEFIE FORMULATION

For the analysis of electromagnetic scattering from the composite conductors S involving both closed and open surfaces, the hybrid CFIE-EFIE shows a higher efficiency than the traditional EFIE [12]. In the hybrid CFIE-EFIE technique, the linear system with N unknowns is written as:

$$\sum_{n=1}^{N} Z_{mn}^{CE} I_n = v_m^{CE}, \quad m = 1, 2, \ldots N \tag{1}$$

The corresponding elements of the hybrid CFIE-EFIE are linearly combined as

$$Z_{mn}^{CE} = \alpha_m Z_{mn}^{E} + (1 - \alpha_m) \cdot \eta \cdot Z_{mn}^{M} \tag{2}$$

$$v_m^{CE} = \alpha_m v_m^{E} + \eta \,(1 - \alpha_m)\, v_m^{M} \tag{3}$$

where η is the wave impendence and α_m a variable combination parameter with its value between 0 and 1. α_m is set to be 1 when Λ_m locates on the open surface parts, and $0 < \alpha_m < 1$ when Λ_m resides in the closed surface parts. α_m is customarily taken as 0.2–0.3 for the closed surface to achieve the fast convergence rate and remove the inner resonance phenomenon. I_n is the unknown coefficient of the nth basis function. The EFIE and the MFIE impedance matrix elements are calculated by:

$$Z_{mn}^{E} = jk_0\eta \iint_S \Lambda_m \cdot \iint_S \bar{\mathbf{G}}_0 \cdot \Lambda_n dS' dS \tag{4}$$

$$Z_{mn}^{M} = \frac{1}{2} \iint_S \Lambda_m \cdot \Lambda_n dS - \iint_{S_0} \Lambda_m \cdot \hat{n} \times \nabla \times \iint_S G_0 \Lambda_n dS' dS \tag{5}$$

where j is the imaginary unit, Λ_m, Λ_n the Rao-Wilton-Glisson (RWG) test and basis functions [20], $\bar{\mathbf{G}}_0$ the three dimensional free space dyad Green's function, k_0 the wavenumber of the incident wave, $\hat{n}$ the unit outward normal vector of the closed parts of the surfaces. The right-hand side vectors of the EFIE and the MFIE are

$$v_m^{E} = \iint_S \Lambda_m \cdot \mathbf{E}^{inc} dS \tag{6}$$

$$v_m^{M} = \iint_S \Lambda_m \cdot \hat{n} \times \mathbf{H}^{inc} dS \tag{7}$$

where $\mathbf{E}^{inc}$, $\mathbf{H}^{inc}$ are the electric field and magnetic field vectors of the incident plane wave, respectively.

The EFIE parts are replaced with the IEFIE in [17] to further speed up the solution. In this case, the hybrid CFIE-IEFIE solution utilizes an updated process for $i = 1, 2, \ldots, Nstep$ to solve

$$\sum_{n=1}^{N} Z_{mn}^{CI} I_n^i = v_m^{CE} + \alpha_m \beta_m \sum_{n=1}^{N} \tilde{Z}_{mn}^{M} \cdot I_n^{i-1} \tag{8}$$

where β_m is the IEFIE combination factor, and it is set to be 0 when Λ_m locates on the closed surface parts and $0 < \beta_m < 1$ for the open surface parts. *Nstep* is the number of iteration steps that has the neighbor approximations satisfying

$$\frac{\|I_{Nstep} - I_{Nstep-1}\|_2}{\|I_{Nstep}\|_2} \le \varepsilon \tag{9}$$

where $\|.\|_2$ denotes the 2-norm of a vector and ε is a given small positive real number. The matrix elements in the hybrid CFIE-IEFIE formulation are

$$Z_{mn}^{CI} = Z_{mn}^{CE} + \alpha_m \beta_m \cdot \tilde{Z}_{mn}^{M} \tag{10}$$

where $\tilde{Z}^M$ is the principle value terms of the MFIE and with the elements

$$\tilde{Z}_{mn}^{M} = \frac{1}{2} \iint_{S_0} \Lambda_m \cdot \Lambda_n dS \tag{11}$$

Since $\tilde{Z}^M$ is a sparse symmetric matrix with a maximum five elements for each row, the cost for building $\tilde{Z}^M$ is very little.

3. ILU PRECONDITIONER

Equation (8) can be written in the following form:

$$\mathbf{Z}^{CI}\mathbf{I}^i = \mathbf{V}^i \text{ and } i = 1,\, 2,\, \ldots,\, Nstep \tag{12}$$

where $V_m^i = v_m^{CE} + \alpha_m \beta_m \sum_{n=1}^{N} \tilde{Z}_{mn}^{M} \cdot I_n^{i-1}$. Instead of solving the original linear system in (9), one may solve a left preconditioned system

$$\mathbf{M}^{-1}\mathbf{Z}^{CI}\mathbf{I}^i = \mathbf{M}^{-1}\mathbf{V}^i \tag{13}$$

via the two steps

$$b = \mathbf{M}^{-1}\mathbf{V}^i \text{ and } \mathbf{M}^{-1}\mathbf{Z}^{CI}\mathbf{I}^i = b \tag{14}$$

which are equivalent to solving the original system so long as the preconditioner matrix $\mathbf{M}$ is nonsingular. The preconditioner matrix $\mathbf{M}$ often approximates $\mathbf{Z}^{CI}$, thus the product matrix $\mathbf{M}^{-1}\mathbf{Z}^{CI}$ has a much better condition number. As a consequence, the number of iterations is greatly reduced. Since the operator $\mathbf{M}^{-1}$ has to be applied at each step of the iterative linear solver, its computation should be high efficient in order to reduce the cost (computing time) of applying the $\mathbf{M}^{-1}$ operation. Incomplete factorization methods start from a factorization method such as $\mathbf{LU}$ or Cholesky decomposition that factorizes a matrix into a lower triangular matrix $\mathbf{L}$ and an upper triangular matrix $\mathbf{U}$. And $\mathbf{LU}$ factorization is modified into its incomplete version to reduce the construction cost and memory requirement for the sparse matrix. Usually let $\bar{\mathbf{L}}\bar{\mathbf{U}} \approx \mathbf{LU} = \mathbf{M}$, where $\bar{\mathbf{L}} \approx \mathbf{L}$, $\bar{\mathbf{U}} \approx \mathbf{U}$. The basic idea to construct $\bar{\mathbf{L}}$ and $\bar{\mathbf{U}}$ is to keep the factors artificially sparse, for instance by dropping some elements in the prescribed non-diagonal positions during the standard Gaussian elimination algorithm. Then the final preconditioning matrix is $\mathbf{M}' = \bar{\mathbf{L}}\bar{\mathbf{U}}$ and the preconditioning operation $\mathbf{z} = (\mathbf{M}')^{-1}\mathbf{y}$ is computed by solving the linear system $\bar{\mathbf{L}}\bar{\mathbf{U}}\mathbf{z} = \mathbf{y}$, which can be performed in two distinct steps: solve $\bar{\mathbf{L}}\mathbf{w} = \mathbf{y}$ and $\bar{\mathbf{U}}\mathbf{z} = \mathbf{w}$ successively and they are much easier to solve than the original preconditioning operation $\mathbf{z} = \mathbf{M}^{-1}\mathbf{y}$.

The key step to build an ILU preconditioner is the selection of the non-zero pattern of triangular factors since the triangular factors $\mathbf{L}$ and $\mathbf{U}$ can often be fairly dense, even when the matrix is sparse. A simple way is to take the non-zero pattern of factors ($\bar{\mathbf{L}}$ and $\bar{\mathbf{U}}$) to be the same as the sparsified matrix of $\mathbf{Z}$, which is called ILU factorization with no fill-ins and denoted by ILU(0) [21, 22]. Since ILU(0) prescribes the non-zero pattern in advance, its constructing cost is quite small. Therefore, the preconditioning matrix for the IEFIE can be constructed as

$$\mathbf{M} = \alpha_m \cdot \tilde{\mathbf{Z}}^{\mathbf{EN}} + (1 - \alpha_m)\,\beta_m \cdot \eta \cdot \tilde{\mathbf{Z}}^{\mathbf{MN}} + \alpha_m\beta_m \cdot \tilde{\mathbf{Z}}^{\mathbf{M}} \tag{15}$$

where $\tilde{\mathbf{Z}}^{\mathbf{EN}}$, $\tilde{\mathbf{Z}}^{\mathbf{MN}}$ are the near-field matrices of EFIE and MFIE in the context of MLFMA as in [13]. To reduce the both computational cost and memory requirement, the preconditioner $\mathbf{M}$ is further sparsified by maintaining only the entries with the largest 100 moduli for each row.

4. NUMERICAL EXPERIMENTS

In this section, several numerical results are provided to illustrate the efficiency of the proposed ILU(0) preconditioned hybrid CFIE-IEFIE for the analysis of the electromagnetic scattering of composite objects

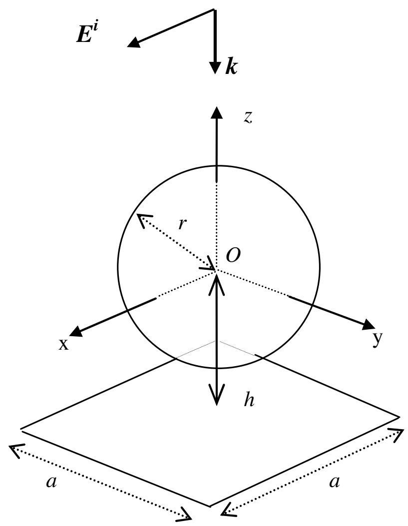

Figure 1. The geometry of examples 1–3: A sphere above a square patch with an incident plane wave $\mathbf{E}^{\mathbf{i}} = \hat{x}\exp(jk_0 z)$.

containing both close and open surfaces. As shown in Fig. 1, examples 1–3 are involved in a closed geometry of perfectly conducting sphere and an open geometry of square patch. The geometries are solved at various dimensions and the different meshes and numbers of unknowns are required for the closed surfaces and the open surfaces. How the number of the unknowns in the EFIE parts is demonstrated to have the influence on the ILU(0) preconditioner for the hybrid CFIE-EFIE formulation and the CFIE-IEFIE formulation, respectively.

Example 1 involves a sphere of radius $r = 5\,\mathrm{m}$ and a rectangular patch of edge length $a = 10\,\mathrm{m}$. The patch is located below the center of the sphere with a distance of $h = 7\,\mathrm{m}$.

Example 2 is with the sphere of radius $r = 4\,\mathrm{m}$ and the rectangular patch of edge length $a = 12\,\mathrm{m}$. The patch is located at a distance of $h = 6\,\mathrm{m}$ from the center of the sphere.

Example 3 has the sphere of radius 3 m and the rectangular patch of edge length $a = 14\,\mathrm{m}$. The patch is located at a distance of $h = 5\,\mathrm{m}$ below the center of the sphere.

Example 4 is to analyze the scattering from China's lunar orbiter Chang'E1. Chang'E1 is simplified to a model consisted of a conducting cuboid and two symmetric conducting rectangular patches as shown in Fig. 2. The cuboid is with side lengths $a = 1.7\,\mathrm{m}$, $b = 2\,\mathrm{m}$, $c = 2.2\,\mathrm{m}$, and the rectangular patch on the XOY plane with edge

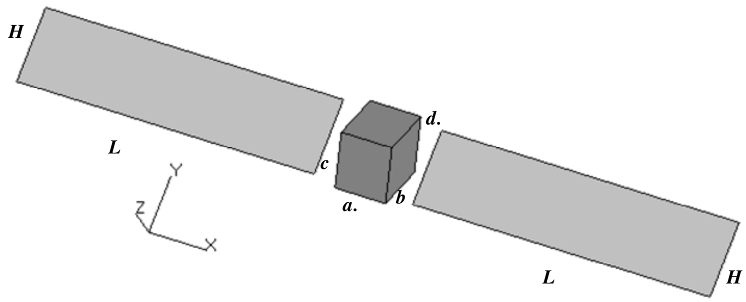

Figure 2. The geometry of example 4: China's lunar orbiter Chang'E1 with an incident plane wave $\mathbf{E^i} = \hat{x}\exp(jk_0 z)$.

length $L = 10\,\mathrm{m}$ and $H = 2\,\mathrm{m}$, and the distance between the rectangular patch and the cuboid $d = 0.8\,\mathrm{m}$. The cuboid is built as a cuboid centering at the coordinate origin with edges length 1.7 m, 2 m, 2.2 m along the x-, y-, z-direction respectively, then rotate 45 degrees around the x axis.

Table 1. The number of unknowns of CFIE parts and EFIE parts in the hybrid CFIE-EFIE formulation for the examples 1–4.

Example	CFIE	EFIE
1	70563	34165
2	53532	49308
3	36291	67217
4	21948	76338

In the analysis, the incident electric field is $\mathbf{E^i} = \hat{x}\exp(jk_0 z)$ and the frequency of the incident plane wave is 300 MHz for examples 1–3 and 550 MHz for example 4. The number of unknowns of CFIE parts and EFIE parts in the hybrid CFIE-EFIE formulation for the four examples are listed in Table 1. In our calculation, $\alpha_m = 0.2$, $\beta_m = 0$ are set for the closed surfaces, and $\alpha_m = 1$, $\beta_m = 0.3$ for the open surfaces if not explicitly given. The size of smallest clusters in MLFMA is set to be 0.25λ. The iterative solver is the generalized minimal residual algorithm (GMRES) [6, 18, 23] restarted every 30 steps. The iterative solver starts with the zero initial guess in the first update step and adopts the latest approximate solution as initial guess for other update

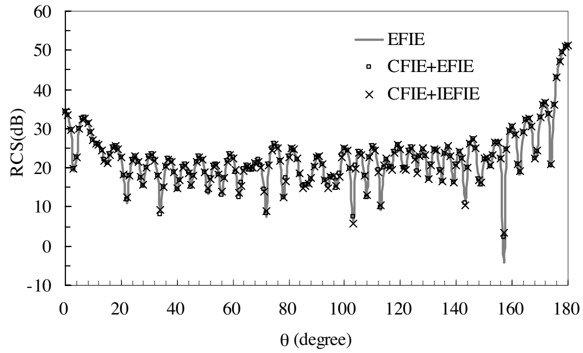

Figure 3. Bistatic RCS in example 1 by EFIE, hybrid CFIE-EFIE, hybrid CFIE-IEFIE. The incident wave is $\mathbf{E^i} = \hat{x}\exp(jkz)$ with frequency 300 MHz.

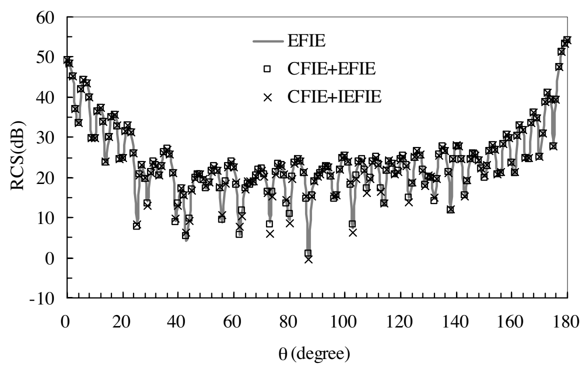

Figure 4. Bistatic RCS in example 2 by EFIE, hybrid CFIE-EFIE, hybrid CFIE-IEFIE. The incident wave is $\mathbf{E^i} = \hat{x}\exp(jkz)$ with frequency 300 MHz.

steps. The solver terminates if the normalized backward error is less than 10^{-3} or the number of iterations reaches 10000. The parameter ε in eq. (9) is taken to be 10^{-3}. All experiments are performed on a Core-2 6300 with 1.86 GHz CPU and 2 GB RAM in single precision. As shown in Figs. 3–6, the curves of bistatic RCS of the four examples are given for the EFIE, the hybrid CFIE-EFIE and the hybrid CFIE-IEFIE formulations, respectively. It can be found that there is a good agreement between them.

To demonstrate the efficiency of our proposed hybrid CFIE-IEFIE formulation, the total number of iterations of GMRES, the

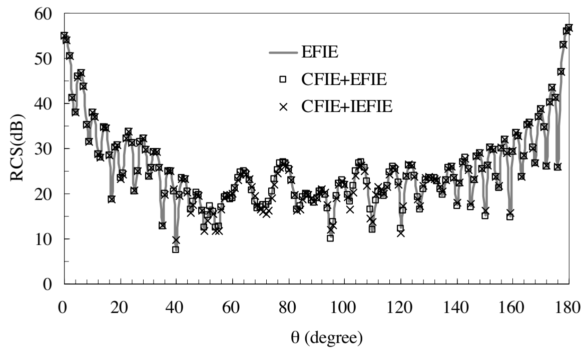

Figure 5. Bistatic RCS in example 3 by EFIE, hybrid CFIE-EFIE, and hybrid CFIE-IEFIE. The incident wave is $\mathbf{E^i} = \hat{x}\exp(jkz)$ with frequency 300 MHz.

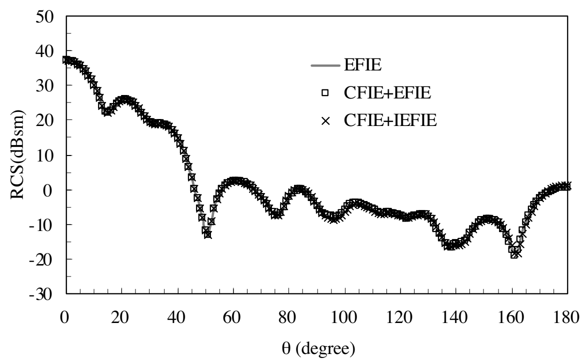

Figure 6. Bistatic RCS in examples 4 by EFIE, hybrid CFIE-EFIE, and hybrid CFIE-IEFIE. The incident wave is $\mathbf{E^i} = \hat{x}\exp(-jkz)$ with frequency 550 MHz.

solution-time, and the update step *Nsteps* are listed in Table 2 and the comparison is made among EFIE, hybrid CFIE-EFIE and hybrid CFIE-IEFIE without/with ILU(0) for our four examples. The total iteration number of GMRES denotes the summation of the number of iterations of every update step. It can be observed for example 1 that the ILU(0) preconditioned hybrid CFIE-EFIE and hybrid CFIE-IEFIE reduce the solution time by a factor of about 15.8 and 24.8 respectively when compared with the traditional EFIE solution.

For example 2, it can be observed from Table 2 that the ILU(0) preconditioned CFIE-EFIE and hybrid CFIE-IEFIE reduce the

Table 2. Comparison of the total number of iterations of GMRES (denoted as N), update steps of the hybrid CFIE-IEFIE formulation (denoted as n) and the sol-time (denotes as t in seconds) in various schemes for three examples, * denotes no convergence achieved in 10000 iterations. Superscript ' stands for the case with ILU(0) preconditioner.

Formulation	Example 1		Example 2		Example 3		Example 4	
	N (n)	t	N (n)	t	N (n)	t	N (n)	t
EFIE	1931	33831	1334	17544	1371	14699	1042	16015
CFIE-EFIE	239	4197	295	3887	324	3485	150	2318
CFIE-EFIE'	119	2138	125	1700	*	*	*	*
CFIE-IEFIE	153(5)	2822	204 (5)	2836	207(5)	2164	80(2)	1293
CFIE-IEFIE'	65(5)	1365	66 (5)	1018	72 (5)	833	25(2)	381

solution time by a factor of 10.3 and 17.2 respectively when compared with the EFIE. They reduce the solution time by a factor of 2.3 and 2.8 respectively when compared with the hybrid CFIE-EFIE and the hybrid CFIE-IEFIE without the preconditioning technique.

For examples 3 and 4, it can be observed that the ILU(0) preconditioned hybrid CFIE-EFIE does not reach the convergence over 10000 iterations since the number of unknowns of open parts becomes so large that ILU(0) produces highly unstable and hence useless factorizations for the EFIE part. However, the ILU(0) preconditioned hybrid CFIE-IEFIE reaches the convergence with a reduction of the solution time by a factor of 17.6 for example 3 and 42.0 for example 4 when compared to the traditional EFIE and a factor of 4.2 for example 3 and 6.1 for example 4 when compared to the hybrid CFIE-EFIE.

Table 3 shows the influence of the IEFIE combination factor β on the convergence of the hybrid CFIE-IEFIE formulation in example 3. It can be found that the total number of iterations decreases as β_m increases for open surface parts without ILU(0) preconditioner. However, the total number of iterations is almost constant after the ILU(0) preconditioner is used.

From our numerical results, it can be observed that the hybrid CFIE-IEFIE is more efficient than the hybrid CFIE-EFIE. The ILU(0) preconditioners can accelerate the convergence of both the CFIE-EFIE and the CFIE-IEFIE for smaller number unknowns of EFIE as in examples 1 and 2. However, the ILU(0) preconditioning algorithm becomes unstable for the hybrid CFIE-EFIE when the number of

Table 3. The influence of the IEFIE combination factor β for the open surface parts on the total number of iterations and update steps *Nstep* in example 3. N_1 and N_2 stand for the total number of iterations of the CFIE-IEFIE formulation and the ILU(0) preconditioned CFIE-IEFIE formulation, respectively.

parameter	β										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.
Nstep	1	4	5	5	6	6	7	7	7	7	8
N_1	324	278	233	207	180	175	160	152	142	135	132
N_2	*	77	72	72	76	74	70	77	76	72	80

unknowns for the open surface parts increases as in examples 3 and 4. But this phenomenon is not true for the ILU(0) preconditioned CFIE-IEFIE and it is stable and highly efficient.

5. CONCLUSIONS

In this paper, the ILU(0) preconditioned hybrid CFIE-IEFIE is presented to efficiently solve the electromagnetic scattering from composite objects involving both open and closed conducting surfaces. The instability of the incomplete LU factorization is avoided for the EFIE part of the hybrid CFIE-IEFIE. The significant reductions for both the number of iterations and solution time are obtained without compromising the accuracy.

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REFERENCES

1. Xu, L., Y.-C. Guo, and X.-W. Shi, "Dielectric half space model for the analysis of scattering from objects on ocean surface," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 15, 2287–2296, 2007.

2. Strifors, H. C. and G. C. Gaunaurd, "Bistatic scattering by bare and coated perfectly conducting targets of simple shape," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 8, 037–1050, 2006.
3. Li, Y. L., J. Y. Huang, and M. J. Wang, "Scattering cross section for airborne and its application," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 15, 2341–2349, 2007.
4. Collard, B., B. Fares, and B. Souny, "A new formulation for scattering by impedant 3D bodies," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 10, 1291–1298, 2006.
5. Illahi, A., M. Afzaal, and Q. A. Naqvi, "Scattering of dipole field by a perfect electromagnetic conductor cylinder," *Progress In Electromagnetics Research Letters*, Vol. 4, 43–53, 2008.
6. Carpentieri, B., "Fast large RCS calculation using the boundary element method," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 14, 1959–1968, 2007.
7. Huang, E. X. and A. K. Fung, "An application of sampling theorem to moment method simulation in surface scattering," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 4, 531–546, 2006.
8. Varmazyar, S. H. and M. N. Moghadasi, "An integral equation modeling of electromagnetic scattering from the surfaces of arbitrary resistance distribution," *Progress In Electromagnetics Research B*, Vol. 3, 157–172, 2008.
9. Mittra, R. and K. Du, "Characteristic basis function method for iteration-free solution of large method of moments problems," *Progress In Electromagnetics Research B*, Vol. 6, 307–336, 2008.
10. Hatamzadeh-Varmazyar, S., M. Naser-Moghadasi, and Z. Masouri, "A moment method simulation of electromagnetic scattering from conducting bodies," *Progress In Electromagnetics Research*, PIER 81, 99–119, 2008.
11. Mautz, J. R. and R. F. Harrington, "H-field, E-field, and combined field solutions for conducting bodies of revolution," *AEÜ*, Vol. 32, 157–164, 1978.
12. Ergul, O. and L. Gurel, "Hybrid CFIE-EFIE solution of composite geometries with coexisting open and closed surfaces," *IEEE Transactions on Electromagnetic Compatibility*, Vol. 5, 289–292, 2005.
13. Chew, W. C., J. M. Jin, E. Midielssen, and J. M. Song, *Fast and Efficient Algorithms in Computational Electromagnetics Boston*, Artech House, MA, 2001.

14. Pan, X.-M. and X.-Q. Sheng, "A highly efficient parallel approach of multi-level fast multipole algorithm," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 8, 1081–1092, 2006.
15. Li, L. and Y.-J. Xie, "Efficient algorithm for analyzing microstrip antennas using fast-multipole algorithm combined with fixed real-image simulated method," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 15, 2177–2188, 2006.
16. Wang, P. and Y.-J. Xie, "Scattering and radiation problem of surface/surface junction structure with multilevel fast multipole algorithm," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 15, 2189–2200, 2006.
17. Hu, J. and Z. Nie, "Improved electric field integral equation (IEFIE) for analysis of scattering from 3-D conducting structures," *IEEE Transactions on Electromagnetic Compatibility*, Vol. 49, 644–648, 2007.
18. Saad, Y., *Iterative Methods for Sparse Linear Systems*, PWS Publishing Company, 1996.
19. Malas, T. and L. Gurel, "Incomplete LU preconditioner with multilevel fast multipole algorithm for electromagnetic scattering," *SIAM Journal of Scientific Computing*, Vol. 29, 1476–1494, 2007.
20. Rao, S. M., D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Trans. Antennas Propagat.*, Vol. 30, 409–418, 1982.
21. Sertel, K. and J. L. Volakis, "Incomplete LU preconditioner for FMM implementation," *Microwave Opt. Technol. Lett.*, Vol. 26, 265–267, 2000.
22. Rui, P. L., R. S. Chen, Z. H. Fan, J. Hu, and Z. P. Nie, "Perturbed incomplete ILU preconditioner for efficient solution of electric field integral equations," *IET Microwave Antennas Propagation*, Vol. 1, 1059–1063, 2007.
23. Rui, P. L., R. S. Chen, Z.W. Liu, and Y. N. Gan, "Schwarz-Krylov subspace method for MLFMM analysis of electromagnetic wave scattering problems," *Progress In Electromagnetics Research*, PIER 82, 51–63, 2008.