Progress In Electromagnetics Research Letters, Vol. 4, 183–190, 2008

COMPARISON OF THE RADIATION PATTERN OF FRACTAL AND CONVENTIONAL LINEAR ARRAY ANTENNA

R. T. Hussein and F. J. Jibrael

Electrical and Electronic Engineering Department
University of Technology
Baghdad, Iraq

Abstract—The purpose of this paper is to introduce the concept of fractals and its use in antenna arrays for obtaining multiband property. One type of fractals namely, Cantor set is investigated. Cantor set is used in linear array antenna design. Therefore, this array know fractal Cantor linear array antenna. A comparison with conventional non-fractal linear array antenna is made regarding the beamwidth, directivity, and side lobe level. MATLAB programming language version 7.2 (R2006a) is used to simulate the fractal and conventional non-fractal linear array antenna and their radiation pattern.

1. INTRODUCTION

In many applications, it is necessary to design antennas with very directive characteristics (very high gains) to meet the demands of long distance communication; this can be accomplished by antenna array [1]. The increasing range of wireless telecommunication services and related applications is driving the attention to the design of multifrequency (multiservice) and small antennas. The telecom operators and equipment manufacturers can produce variety of communications systems, like cellular communications, global positioning, satellite communications, and others, each one of this systems operates at several frequency bands. To give service to the users, each system needs to have an antenna that has to work in the frequency band employed for the specific system. The tendency during last years had been to use one antenna for each system, but this solution is inefficient in terms of space usage, and it is very expensive. The variety of communication systems suggests that there is a need for multiband antennas. The use of fractal geometry is a new solution to the design of multiband antennas and arrays. Fractal geometries have found an intricate place in science as a representation of some of the unique geometrical features occurring in nature. Fractal was first defined by Benoit Mandelbrot [2] in 1975 as a way of classifying structures whose dimensions were not whole numbers. These geometries have been used previously to characterized unique occurrences in nature that were difficult to define with Euclidean geometries, including the length of coastlines, the density of clouds, and branching of trees [3]. Fractals can be divided into many types, as shown in Fig. 1.

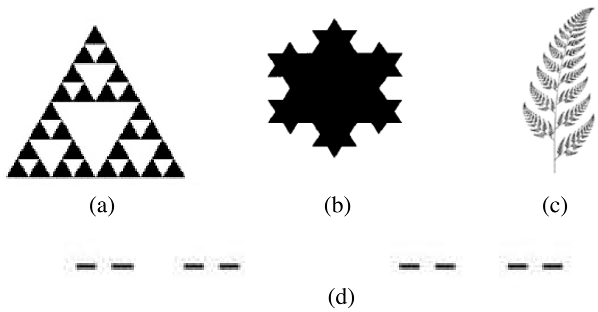

Figure 1. Three fractal examples. (a) Sierpinski gasket. (b) Koch snowflake. (c) Tree. (d) Cantor set.

2. CONVENTIONAL LINEAR ARRAY ANTENNA

An array is usually comprised of identical elements position in a regular geometrical arrangement. A linear array of isotropic elements N, uniformly spaced a distance d apart along the z-axis, is shown in Fig. 2 [4].

The array factor corresponding to this linear array may be expressed in the form [1, 5]

$$AF(\psi) = \begin{cases} a_0 + 2\sum\limits_{n=1}^{N} a_n \cos(n\psi) & \text{for } (2N+1) \text{ elements} \\ 2\sum\limits_{n=1}^{N} a_n \cos\left(\left(\frac{2n-1}{2}\right)\psi\right) & \text{for } (2N) \text{ elements} \end{cases} \quad (1)$$

$$\psi = kd\cos\theta + \alpha \quad (2)$$

$$k = \frac{2\pi}{\lambda} \quad (3)$$

where,

$AF(\psi)$ = the array factor
d = spacing between adjacent elements in the array
α = the progressive phase shift between elements
$k = \frac{2\pi}{\lambda}$ = the wave number
θ = elevation angle

3. FRACTAL LINEAR ARRAY ANTENNA

Fast recursive algorithms for calculating the radiation patterns of fractal arrays have recently been developed in [6–8]. These algorithms are based on the fact that fractal arrays can be formed recursively through the repetitive application of a generating array. A generating array is a small array at level one ($P = 1$) used to recursively construct larger arrays at higher levels (i.e., $P > 1$). In many cases, the generating subarray has elements that are turned on and off in a certain pattern. A set formula for copying, scaling, and translating of the generating array is then followed in order to produce a family of higher order arrays.

The array factor for a fractal antenna array may be expressed in the general form [6–8]

$$AF_P = \prod_{p=1}^{P} GA\left(\delta^{p-1}\psi\right) \tag{4}$$

where $GA(\psi)$ represents the array factor associated with the generating array. The parameter δ is a scaling or expansion factor that governs how large the array grows with each successive application of the generating array and P is a level of iteration.

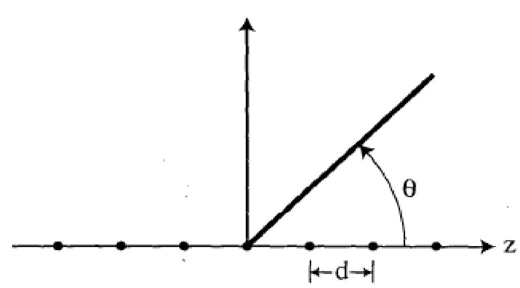

Figure 2. Linear array geometry of uniformly spaced isotropic sources.

This arrays become fractal-like when appropriate elements are turned off or removed, such that

$$a_n = \begin{cases} 1 & \text{if element } n \text{ is turned on} \\ 0 & \text{if element } n \text{ is turned off} \end{cases}$$

One of the simplest schemes for constructing a fractal linear array follows the recipe for the cantor set [9]. Cantor arrays own also multiband properties, so it has multi frequencies (F_n):

$$F_n = \frac{F_0}{\delta^n} \qquad n = 0, 1, 2, \ldots, P - 1 \tag{5}$$

where F_0 is the design frequency.

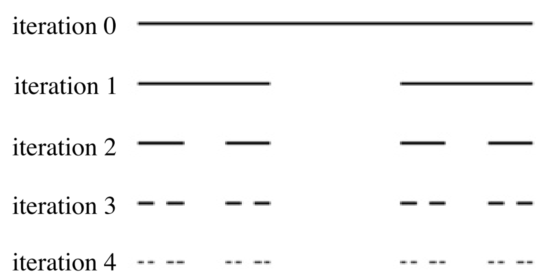

Figure 3. The first four iterations in the construction of the Cantor set array.

The basic Cantor array, as shown in Fig. 3 may be created by starting with a three element generating subarray, and then applying it repeatedly over P scales of growth. The generating subarray in this case has three uniformly spaced elements, with the center element turned off or removed, i.e., 101. The Cantor array is generated recursively by replacing 1 by 101 and 0 by 000 at each level of the construction. Table 1 provides the array pattern for the first four levels of the Cantor array.

The array factor of the three element generating subarray with the representation 101 is

$$GA(\psi) = 2\cos(\psi) \tag{6}$$

which may be derived from Eq. (1) by setting $N = 1$, $a_0 = 0$. Substituting Eq. (6) into Eq. (4) and choosing an expansion factor of three ($\delta = 3$), the results in an expression for the Cantor array factor given by

$$AF_P(\psi) = \prod_{p=1}^{P} GA\left(3^{p-1}\psi\right) = 2\prod_{p=1}^{P} \cos\left(3^{p-1}\psi\right) \tag{7}$$

Table 1. First four levels of the fractal Cantor linear array.

P	Elements array Pattern	Active Elements	Total Elements
1	101	2	3
2	101000101	4	9
3	101000101000000000101000101	8	27
4	101000101000000000 101000101000000000 00000000000000000010 1000101000000000101000101	16	81

4. COMPUTER SIMULATION RESULTS

In this work, MATLAB programming language version 7.2 (R2006a) used to simulate and design the conventional and fractal linear array antenna and their radiation pattern. Let, a linear array will be design and simulate at a frequency F_0 equal to 8100 MHz, (then the wavelength $\lambda_0 = 0.037\,\mathrm{m}$), with quarter-wavelength ($d = \lambda_0/4$) spacing between array elements and 16 active elements in the array and progressive phase shift between elements (α) equal to zero. The level four of Cantor linear array (101) have the number of active elements of 16 and the total elements number of 81. This array will operate at four frequencies depending on the Eq. (5). These frequencies are 8100 MHz, 2700 MHz, 900 MHz, and 300 MHz. Depending on the frequencies of the fractal Cantor linear array will be design and simulate of conventional linear array antenna then compare the radiation field pattern properties for them. The array factor for fractal and linear array antenna is plotted with uniformly amplitude distribution which they are feeding to active elements. The field patterns are illustrating as shown in Fig. 4 and Fig. 5. While, the values of the side lobe level, half power beamwidth, and directivity are illustrating in Table 2 and Table 3.

Table 2. SLL, D, and $HPBW$ for fractal linear array antenna.

F (MHz)	D (dB)	$HPBW$ (degree)	SLL max (dB)
8100	12.0436	2.0233	−5.451
2700	9.1969	6.0721	−5.446
900	6.204	18.2852	−5.446
300	3.1848	56.9372	$-\infty$

Table 3. SLL, D, and $HPBW$ for conventional linear array antenna.

F (MHz)	D (dB)	$HPBW$ (degree)	SLL max (dB)
8100	9.1202	12.7372	−13.148
2700	4.6369	38.8742	−13.593
900	0.893	—	$-\infty$
300	0.106	—	$-\infty$

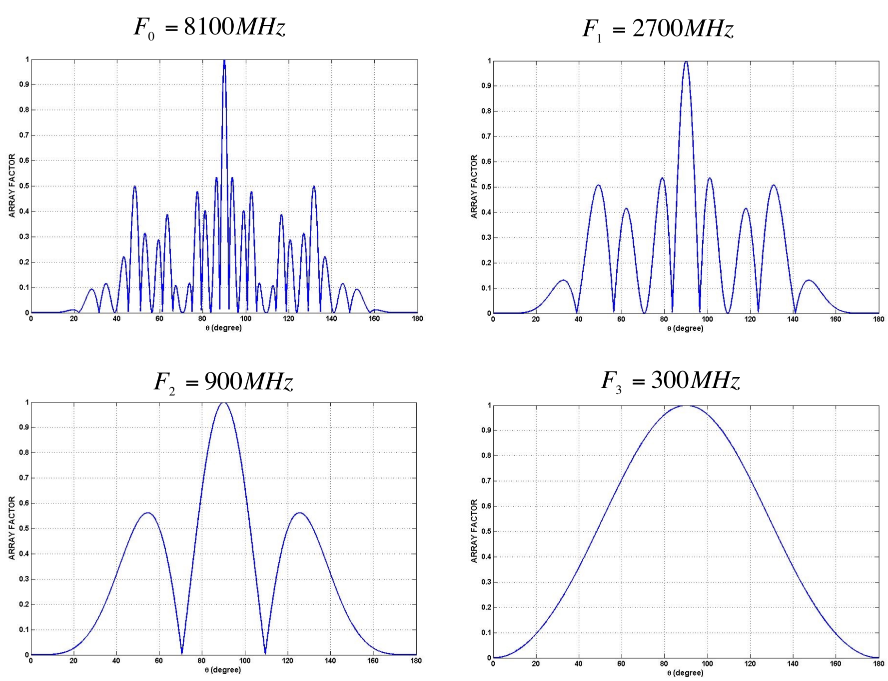

Figure 4. Array factor of a fractal Cantor linear array antenna.

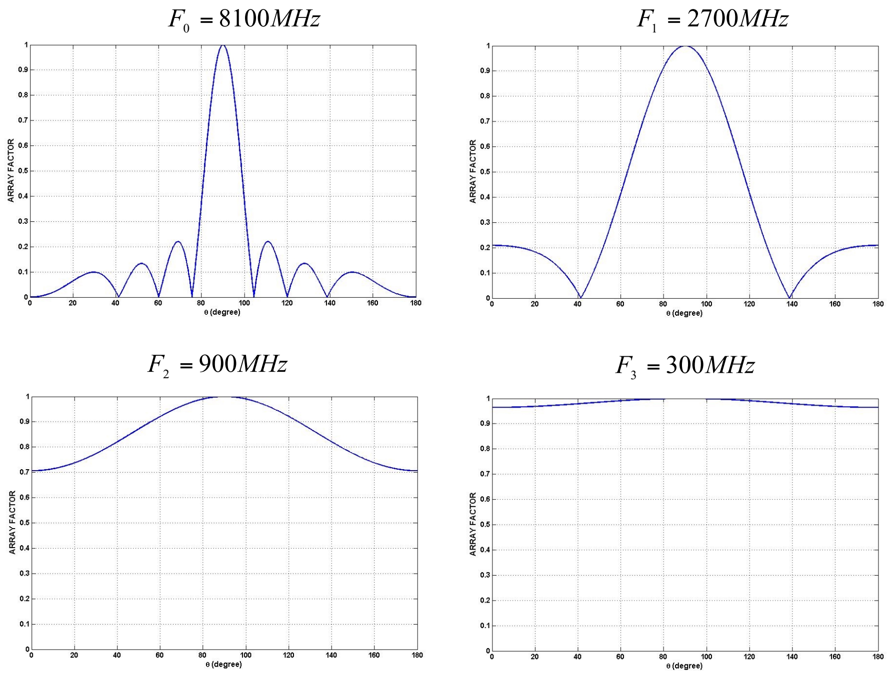

Figure 5. Array factor of a conventional non-fractal linear array antenna.

5. CONCLUSION

At design frequency $F_0 = 8100$ MHz, the field pattern for conventional linear array antenna has the side lobes and narrow beam width, in other word, the system work as a normal array antenna. But at frequencies very low from the design frequency such as 300 MHz, the array antenna operates as a point source. While fractal linear array antenna at all frequencies not operates as a point source so we conclude that the fractal Cantor linear array have capable to operating in multiband while, the conventional linear array have not capable to operate in multiband. Also the field pattern of the fractal linear array antenna have high side lobe level, lower half power beam width and high directivity, while, the conventional linear array antenna have lower side lobe level, high half power beam width and lower directivity.

REFERENCES

1. Balanis, C. A., *Antenna Theory: Analysis and Design*, 2nd edition, Wiley, 1997.
2. Mandelbrot, B. B., *The Fractal Geometry of Nature*, W. H. Freeman and Company, New York, 1983.
3. Gianvittorio, J., "Fractal antennas: Design, characterization and application," Master's Thesis, University of California, Los Angeles, 2000.
4. Werner, D. H., R. L. Haupt, and P. L. Werner, "Fractal antenna engineering: The theory and design of fractal antenna arrays," *IEEE Antennas and Propagation Magazine*, Vol. 41, No. 5, 37–59, October 1999.
5. Stutzman, W. L. and G. A. Thiele, *Antenna Theory and Design*, 2nd edition, John Wiley & Sons, New York, 1998.
6. Baliarda, C. P. and R. Pous, "Fractal design of multi-band and low side-lobe arrays," *IEEE Transactions on Antennas and Propagation*, Vol. 44, No. 5, 730–739, May 1996.
7. Werner, D. H. and R. L. Haupt, "Fractal construction of linear and planar arrays," *Proceedings IEEE Antennas Propagation Soc. Int. Symp.*, Vol. 3, 1968–1971, July 1997.
8. Werner, D. H. and R. Mittra, *Frontiers in Electromagnetics*, IEEE Press, 2000.
9. Peitgen, H. O., H. Jurgens, and D. Saupe, *Chaos and Fractals: New Frontiers of Science*, Springer-Verlag, Inc., New York, 1992.