MODAL ANALYSIS AND DISPERSION CURVES OF A BRAGG FIBER HAVING ASYMMETRIC LOOP BOUNDARY

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Abstract—An analysis of the modal propagation characteristics of a Bragg fiber having asymmetric loop boundary is made, using a simple matrix method. The boundary condition is replaced by matrix equation and the modal eigen value equation is obtained under weak guidance condition. The computed results are shown in the form of dispersion curves and cutoff frequencies and are compared with the dispersion curves of a standard Bragg fiber having circular core cross section. It is seen that the proposed Bragg fiber with a small number of claddings (two of four) shows comparable or even better performance than the standard Bragg fiber with respect to a few mode-guidance properties.

1. INTRODUCTION

The rapid developments in the fields such as fiber optics communication engineering and integrated optical electronics have expanded the interest and increased expectations about guided-wave optics, in which optical waveguides play a very important role. Optical lightguides for optical fiber and optical integrated circuits utilize a wave phenomenon that trap the light locally and guides it in any direction. In order to develop some new optical communication systems or optical devices, we need some optical waveguides having some new structure and geometry whose eigen characteristics must be truly known.

Recently Bragg waveguides and fibers have been investigated with great attention and interest especially due to their novel applications in communication engineering, integrated optical electronics and sensor technology [1–13]. A comparative analysis of Bragg fibers have been given by Guo et al. [14]. Pal et al. designed Bragg fibers for transparent metro networks and for dispersion compensation [15]. Recently Singh et al. [16] have studied the Bragg fiber using a very simple technique and it was shown that by using only a small number of cladding layers, a Bragg fiber is almost as good as a conventional standard fiber under weak guidance condition with an additional advantage that there will be very little loss of energy. More recently Bragg waveguides having unconventional structure and geometry [17, 18] have been studied by Prajapati et al. using this simple technique. Very recently the analytical and numerical aspects of Bragg fiber designing was estimated by Prokovich et al. [19]. Here in our proposed paper we will study a new asymmetrical Bragg Waveguide having six layers shown in Fig. 1(b). Using a simple matrix method and replacing the boundary condition by matrix equation, the eigen value equation for the said waveguide has been obtained. Next, this eigen value equation is used to get dispersion curves and cutoff frequencies of proposed unconventional Bragg waveguide. The computed results are shown in the form of dispersion curves and cutoff frequencies and are compared with the dispersion curves of a standard Bragg fiber having circular core cross section [16]. It is observed that the proposed unconventional Bragg fiber shows good performance regarding limiting modes of small number. Our main motivation is to show the influence of eventual fabrication defects modifying a ring core fiber, already well studied if is perfect. Such topics are interesting for a better knowledge of the imperfect ring core fibers [20–24]. In this way the objective of the proposed analysis is two fold: (1) to solve the complicated problems relatively on the basis of simple analytical lines of the nature of propagation of EM waves in new types of noncircular lightguides and

photonic band gap structures and (2) to get as many new results as possible with the help of such analysis made in papers [20–24], so new technological application possibilities may arise, leading to the people of our globe and in particular, our nation to the nano-technological age.

The present article is organized in the following manner: Section 2 deals with the derivation of eigen value equation and cutoff frequencies. The results and discussions are described in Section 3. Finally conclusion is presented in Section 4.

2. THE CHARACTERISTICS EIGEN VALUE EQUATION FOR ASYMMETRIC LOOP BRAGG WAVEGUIDE

The general equation for boundary of the proposed unconventional waveguide can be written as [20]

$$r = \xi e^{(1/2)\sin\theta} \tag{1}$$

where ξ is a size parameter. We choose new coordinates (ξ, η, z) instead of (r, θ, z), and assuming that the propagation is along the z direction. Here we will use matrix method to compute the modal characteristics of a asymmetric loop Bragg waveguide. The basic idea is to replace the boundary condition by a matrix equation. The cross-sectional view of six-layered Bragg waveguide is shown in Fig. 1(b). It has low refractive index (n_a) in central region and higher refractive indices n_1 and n_2 $(n_1 > n_2)$ in the cladding regions around it. Thereby we

(a)   (b)
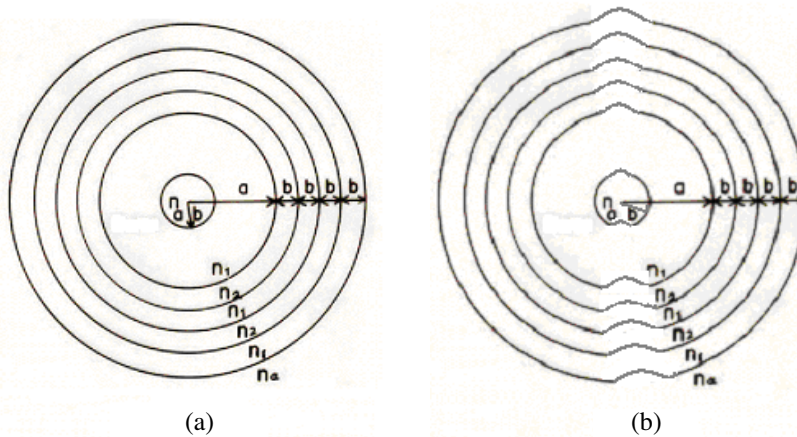
Figure 1. (a) The cross-sectional view of the standard Bragg fiber, (b) The cross-sectional view of the proposed Bragg fiber.

have suitably designed alternating claddings of high and low refractive indices. Fig. 1(a) is drawn for comparison purposes only [16]. The index profile is then given [16] by

$$n(\xi) = \begin{pmatrix} n_a; & 0 < \xi < b \\ n_1; & b < \xi < a \\ n_2; & a < \xi < a+b \\ n_1; & a+b < \xi < a+2b \\ n_2; & a+2b < \xi < a+3b \\ n_1; & a+3b < \xi < a+4b \\ n_a; & \xi > a+4b \end{pmatrix} \tag{2}$$

Using the new coordinates and Maxwell's equations, we can obtain the expressions for the field E and H in terms of the new coordinates. We also assume that as the electromagnetic wave propagates along the z-axis, the electric and magnetic field vectors take the form

$$\psi(\xi, \eta, z) = \psi(\xi)e^{j\nu\theta}e^{i(\beta z - \omega t)} \tag{3}$$

where $\psi(\xi)$ can be $E_z, H_z, E_r, H_r, E_\theta, H_\theta$ and ω is the angular frequency and β is the propagation constant. This means the fields are harmonic in the time t and the coordinate z. From the waveguide theory we know that the transverse field components can be expressed in terms of E_z and H_z.

$$E_\xi = \frac{i\beta}{\omega^2\mu\varepsilon - \beta^2} \left[\frac{\partial E_z}{\partial \xi} + \frac{\omega\mu}{\beta} \frac{1}{\xi} \frac{\partial H_z}{\partial \eta} \right] \tag{4}$$

$$E_\eta = \frac{i\beta}{\omega^2\mu\varepsilon - \beta^2} \left[\frac{1}{\xi} \frac{\partial E_z}{\partial \eta} - \frac{\omega\mu}{\beta} \frac{\partial H_z}{\partial \xi} \right] \tag{5}$$

$$H_\xi = \frac{i\beta}{\omega^2\mu\varepsilon - \beta^2} \left[\frac{\partial H_z}{\partial \xi} - \frac{\omega\mu}{\beta} \frac{1}{\xi} \frac{\partial E_z}{\partial \eta} \right] \tag{6}$$

$$H_\eta = \frac{i\beta}{\omega^2\mu\varepsilon - \beta^2} \left[\frac{1}{\xi} \frac{\partial H_z}{\partial \eta} + \frac{\omega\mu}{\beta} \frac{\partial E_z}{\partial \xi} \right] \tag{7}$$

Here $E_z(\xi, \eta)$ and $H_z(\xi, \eta)$ satisfy the wave equation

$$\left[\nabla_i^2 + \left(\omega^2\mu\varepsilon - \beta^2 \right) \right] \begin{bmatrix} E_z \\ H_z \end{bmatrix} = 0 \tag{8}$$

where $\nabla_i^2 = \nabla^2 - \frac{\partial^2}{\partial z^2}$ is the transverse Laplacian operator. The details of this procedure are given in our previous papers [16–18]. The general solution for the cladding regions is given as

$$\left. \begin{array}{l} E_z = [A_i J_1 (u_i\xi) + B_i Y_1 (u_i\xi)] \, e^{j\nu\theta} e^{j(\omega t - \beta z)} \\ H_z = [C_i J_1 (u_i\xi) + D_i Y_1 (u_i\xi)] \, e^{j\nu\theta} e^{j(\omega t - \beta z)} \end{array} \right\} \tag{9}$$

where $u_i = \sqrt{k^2 n_i^2 - \beta^2}$, $i = 1, 2$ corresponding to refractive indices n_1 and n_2. The solution for the central region and outermost cladding region can be written as

$$\left.\begin{array}{l} E_z = \left[EI_1(w\xi) + FK_1(w\xi)\right] e^{j\nu\theta} e^{j(\omega t - \beta z)} \\ H_z = \left[GI_1(w\xi) + HK_1(w\xi)\right] e^{j\nu\theta} e^{j(\omega t - \beta z)} \end{array}\right\} \tag{10}$$

where $w = \sqrt{\beta^2 - k^2 n_a^2}$, n_a being the common refractive index of these regions. Here A, B, C, D, E, F, G and H are unknown constants and J_1 and Y_1 are the Bessel functions of first and second kind while I_1 and K_1 are the modified Bessel functions of order one respectively. Also d is a number $(0.2)^{\frac{1}{2}}$ which emerges in the analysis because of the peculiarity of the geometrical shape [20].

The boundary conditions at $r = r_i$ are that E_z, H_z, E_θ and H_θ are continuous at the interfaces [25, 26]. Thus we get a set of equations having twenty-two unknown constants. The nontrivial solution will exist only when the determinant formed by the coefficients of the unknown constants is equal to zero. Calling this 12×12 determinant Δ, we have characteristic equation

$$\Delta = 0 \tag{11}$$

The element in the rows and columns of this determinant can be identified readily.

We also define (see Fig. 1(b)) that

$$\begin{aligned} \Delta n &= n_1 - n_2 \\ \Delta n' &= n_1 - n_a \\ V &= k_0(a-b)(n_1^2 - n_a^2)^{\frac{1}{2}} = k_0(a-b)\left[2n(\Delta n + \Delta n')\right]^{\frac{1}{2}} \quad (12) \end{aligned}$$

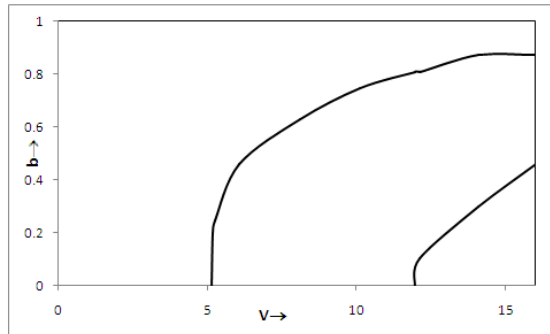where k_0 is vacuum wavenumber. We define the usual normalized propagation parameter

$$b' = \frac{\beta^2 - k_0^2 n_a^2}{k_0^2 \left(n_1^2 - n_a^2\right)} \approx \frac{\beta - k_0 n_a}{k_0 \left(\Delta n + \Delta n'\right)} \quad \text{(weakly guidance case)} \tag{13}$$

The dimensionless V-parameter is introduced to incorporate the parameters n_a, n_1, n_2, a, b and k_0 which may possibly have an effect on the propagation. One may choose other alternative ways to define the quantities V and b', but as an illustrative case, the present definitions are adequate.

3. NUMERICAL RESULT AND DISCUSSION

The eigen value Equation (11) has all of the information that we can obtain from our modal analysis and it gives the central results of this investigation. We now proceed to some numerical computation in order to have the modal dispersion curves for the proposed unconventional Bragg waveguide. It is convenient to plot the normalized propagation constant $b' \approx \frac{\beta - k_0 n_a}{k_0(\Delta n + \Delta n')}$ against the V-parameter defined by $V = k_0(a - b)[2n(\Delta n + \Delta n')]^{\frac{1}{2}}$. Now we choose $n_a = 1.0002$, $n_1 = 1.45$, $n_2 = 1.50$, $b = 0.01\,\mu\text{m}$, $0.1\,\mu\text{m}$, $1.0\,\mu\text{m}$, an operating wavelength $\lambda_0 = 1.55\,\mu\text{m}$ and various values of dimensional parameter a in a regular increasing order. For each value of a we obtain the V-parameter and also compute the values of β from the characteristic Equation (11) by graphical method. It means that the left hand side of eigen value equation is plotted against β for the assumed value of a and the zero crossings of the graph with the β axis are noted. These values are the solutions of the characteristic equation for the different modes. From the β values of the guided modes we can obtain b' and then plot the b' versus V graphs.

Thus the b' versus V curves known as dispersion curves have been shown in Fig. 2 to Fig. 10. These curves have been studied for change in number of cladding layers and also for change in thickness of the cladding strips. Some interesting features are noticed from these plots. The cutoff frequencies (V-values) and their dependence on the thickness b of the cladding strip for different cladding layers (two, four and six) are given in Table 1 and Table 2 respectively for the
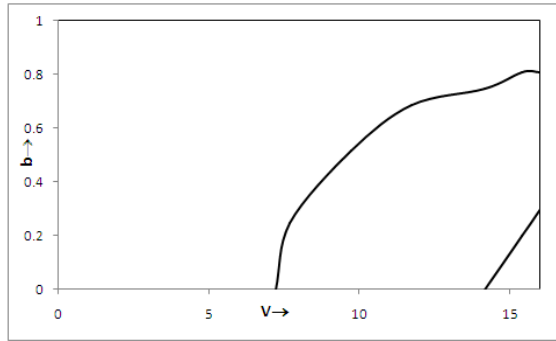

Figure 2. Dispersion curves of normalized frequency V versus normalized propagation constant b' for the two layered cladding regions with its thickness $b = 1.0\,\mu\text{m}$.

conventional circular core Bragg fiber and proposed unconventional asymmetric Bragg fiber. For comparison purpose Table 1 is taken directly from our earlier paper [16]. Considering Table 1 and Table 2, it is evident that as the thickness b is increased from $b = 0.01\,\mu\text{m}$ to $b = 1.00$, the cutoff value decreases for LP_{11} and LP_{12} modes in both waveguides. This decrease is, however, considerably larger in the

Table 1. Cutoff frequencies (V-values)for some modes in standard Bragg fiber [16] for three different thicknesses of the cladding strips.

Mode No.	Cut off frequencies of various modes in Bragg fiber with thickness of cladding strip b=0.01μm. 0<V<16			Cut off frequencies of various modes in Bragg fiber with thickness of cladding strip b=0.10μm. 0<V<16			Cut off frequencies of various modes in Bragg fiber with thickness of cladding strip b=1.00μm. 0<V<16		
LP_{1m}	Six layered	Four layered	Two layered	Six layered	Four layered	Two layered	Six layered	Four layered	Two layered
LP_{11}	4.55	4.71	4.98	3.02	3.93	4.77	-	-	4.57
LP_{12}	7.80	7.98	8.06	6.05	6.97	7.86	-	3.09	8.10
LP_{13}	10.89	11.06	11.21	9.24	10.10	10.94	0.54	6.51	11.18
LP_{14}	13.92	14.11	14.00	12.31	13.22	13.98	3.18	8.87	14.18
LP_{15}	-	-	-	15.33	-	-	6.50	11.96	-
LP_{16}	-	-	-	-	-	-	9.22	13.99	-
LP_{17}	-	-	-	-	-	-	13.01	-	-

Table 2. Cutoff frequencies (V-values)for some modes in asymmetric Bragg fiber for three different thicknesses of the cladding strips.

| Mode No.	Cut off frequencies of various modes in Asymmetric Bragg fiber with thickness of cladding strip b=0.01μm. 0<V<16			Cut off frequencies of various modes in Asymmetric Bragg fiber with thickness of cladding strip b=0.10μm. 0<V<16			Cut off frequencies of various modes in Asymmetric Bragg fiber with thickness of cladding strip b=1.00μm. 0<V<16		
LP_{1m}	Six layered	Four layered	Two layered	Six layered	Four layered	Two layered	Six layered	Four layered	Two layered
LP_{11}	7.33	7.41	7.58	5.51	6.34	7.21	-	-	5.15
LP_{12}	14.3	14.52	14.50	12.56	13.3	14.18	-	0.0	11.95
LP_{13}	-	-	-	-	-	-	0.72	3.64	-
LP_{14}	-	-	-	-	-	-	8.1	8.88	-
LP_{15}	-	-	-	-	-	-	8.91	8.91	-
								12.1	

case of proposed unconventional Bragg fiber compared to the standard
Bragg fiber. We also observe that LP_{13} and LP_{14} modes are present
is the case of standard Bragg fiber but these modes are absent in the
proposed waveguide. Thus it is clear that the proposed waveguide acts
as a mode filter. We also notice that for the fixed thickness b, as the
number of cladding layers increases the cutoff frequency decrease s in
both cases.

Now we consider the dispersion curves (Fig. 2 to Fig. 7) for
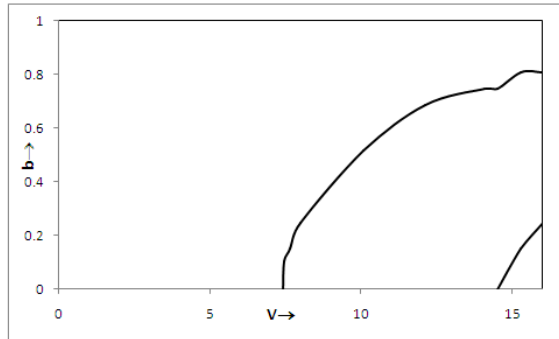thickness $b = 0.01\,\mu\text{m}$ and $0.1\,\mu\text{m}$. We see that all curves are in

Figure 3. Dispersion curves of normalized frequency V versus
normalized propagation constant b' for the two layered cladding regions
with its thickness $b = 0.10\,\mu\text{m}$.
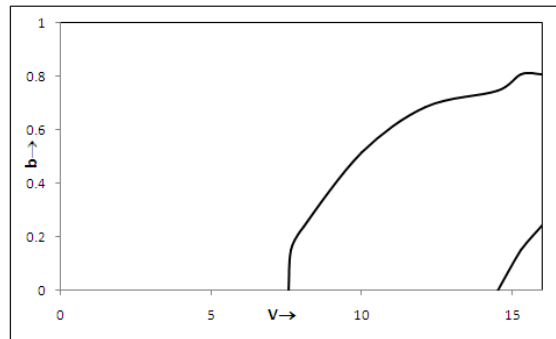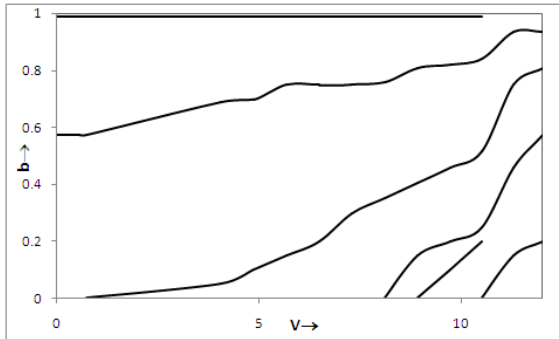

Figure 4. Dispersion curves of normalized frequency V versus
normalized propagation constant b' for the two layered cladding regions
with its thickness $b = 0.01\,\mu\text{m}$.

expected standard form except one mode in Fig. 5. We also notice that the proposed waveguide sustains only two modes (except Fig. 5) for different cladding layers chosen, whereas in same condition Bragg waveguide [16] sustains four modes. Further, we observe that the cutoff values for the lowest order mode are somewhat greater in the case of proposed Bragg fiber compared to the standard Bragg fiber [16]. For example in Figs. 4, 7 and 10, for LP_{11} mode and $b = 0.01\,\mu m$, the cutoff values for the lowest order mode for the proposed Bragg waveguide are at $V = 7.33$, 7.41 and 7.58 respectively, which are considerably
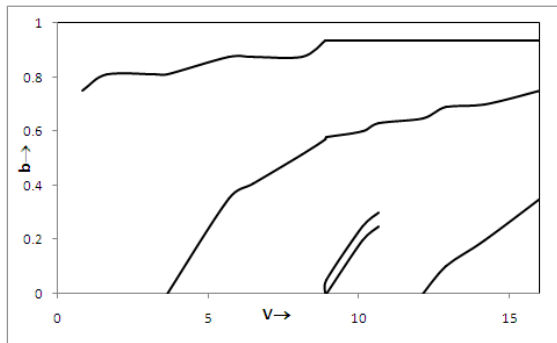
Figure 5. Dispersion curves of normalized frequency V versus normalized propagation constant b' for the four layered cladding regions with its thickness $b = 1.0\,\mu m$.
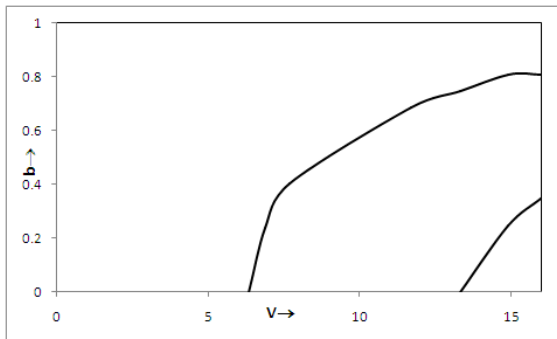
Figure 6. Dispersion curves of normalized frequency V versus normalized propagation constant b' for the four layered cladding regions with its thickness $b = 0.10\,\mu m$.

greater than the cutoff values $V = 4.55$, 4.71 and 4.98 for the standard Bragg fiber. It is well known that the greater the cutoff values the fewer the number of mode sustained. This striking feature of the proposed waveguide may offer important advantages in mode filtering techniques. Next, considering the Figs. 8, 9 and 10, for the cladding thickness $b = 1$ mm, it is clear that all curves are in expected standard shape except the curves shown in Fig. 8. Here, in the case of Fig. 9 and Fig. 10, we see that the proposed waveguide sustain only two modes for different cladding layers chosen, whereas in same condition standard Bragg waveguide [16] sustain more than four modes. We note that the
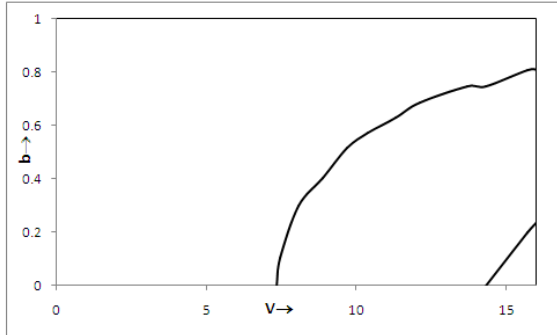

Figure 7. Dispersion curves of normalized frequency V versus normalized propagation constant b' for the four layered cladding regions with its thickness $b = 0.01 \, \mu$m.

Figure 8. Dispersion curves of normalized frequency V versus normalized propagation constant b' for the six layered cladding regions with its thickness $b = 1.0 \, \mu$m.
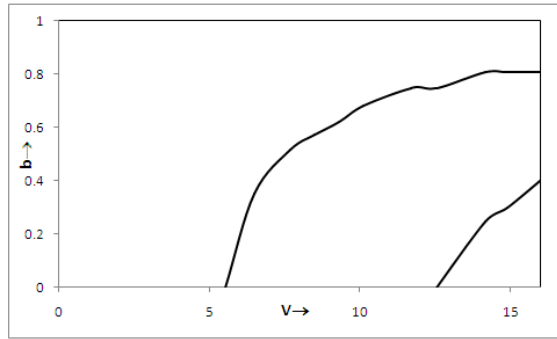
Figure 9. Dispersion curves of normalized frequency V versus normalized propagation constant b' for the six layered cladding regions with its thickness $b = 0.10\,\mu$m.

Figure 10. Dispersion curves of normalized frequency V versus normalized propagation constant b' for the six layered cladding regions with its thickness $b = 0.01\,\mu$m.

other important feature is that when we make the thickness of cladding layers of the proposed waveguide smaller, we get higher cutoff values, even when the number of cladding layers is only two. This fact is also evident from Table 2 and from all dispersion curves except the curve shown in Fig. 5 and Fig. 8. Considering the Fig. 5 and Fig. 8 it is clear that we have anomalous curves for greater thickness ($b = 1$ mm) with large number of cladding layers (four and six). This anomalous behaviour may be due to some kind of radiation loss. The radiation loss in optical waveguide results from mode coupling caused by random microbends of the optical fiber [27, 28]. Here microbend is a small scale

fluctuations in the radius of curvature of the fiber axis as illustrated in Fig. 1(b). It is clear that we have anomalous curves only in Fig. 8 for $b = 1.0\,\mu$m. This thickness of strip is much greater than $b = 0.01\,\mu$m and $b = 0.1\,\mu$m.

4. CONCLUSION

In this article a new type of asymmetric Bragg fiber is proposed and analyzed for the first time in our knowledge. Using simple matrix method, the modal eigen value equation is obtained under the weak guidance approximation. This eigen value equation comprises the dispersion relation and cutoff frequencies which are the main result of the paper. Since our aim is essentially to obtain insight into the modal properties, we adopt a simple analytical method using the scalar wave approximation. An attempt has been made to see how the modal dispersion curves and cutoff frequencies of a standard Bragg fiber changes as its circular loop is changed to an asymmetric loop. It is observed that proposed Bragg waveguide shows good performance regarding limiting modes of small number. Therefore, such Bragg waveguides may be used in many applications where mode filtering is required.

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