ELECTROMAGNETIC SCATTERING OF TWO OR MORE INCIDENT PLANE WAVES BY A PERFECT ELECTROMAGNETIC CONDUCTOR CYLINDER COATED WITH A METAMATERIAL

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Abstract—Electromagnetic scattering of two or more incident plane waves has been investigated for a perfect electromagnetic conductor (PEMC) circular cylinder, coated with a metamaterial having negative index of refraction. The incident waves are considered for both the TM and TE cases in the analysis. The scattered fields are calculated by the application of appropriate boundary conditions at the interfaces between the different media. It is assumed that both the PEMC cylinder and the coating layer are infinite along the cylinder axis. The numerical results are compared with the published literature, and are found to be in good agreement.

1. INTRODUCTION

Perfect electromagnetic conductor (PEMC) material is a generalized form of the PEC and PMC materials introduced by Lindell and Sihvola [1]. The behavior of the PEMC material in response to the incident electromagnetic plane waves has been studied by many researchers [2–19]. As the PEC boundary may be defined by the conditions

$$\vec{n} \times \vec{E} = 0, \qquad \vec{n} \cdot \vec{B} = 0 \tag{1}$$

While PMC boundary may be defined by the boundary conditions

$$\vec{n} \times \vec{H} = 0, \qquad \vec{n} \cdot \vec{D} = 0 \tag{2}$$

The PEMC boundary conditions are of the more general form

$$\vec{n} \times \left(\vec{H} + M\vec{E} \right) = 0 \tag{3}$$

$$\vec{n} \cdot \left(\vec{D} - M\vec{B} \right) = 0 \qquad (4)$$

where $\vec{D} = \epsilon \vec{E}$ and $\vec{B} = \mu \vec{H}$, ϵ and μ being the permittivity and the permeability of the host medium and M denotes the admittance of the PEMC boundary. It is obvious that PMC corresponds to $M = 0$, while PEC corresponds to $M = \pm\infty$.

Scattering of electromagnetic waves from coated cylinder, has been extensively studied [20–27]. Metamaterial with both negative permittivity and permeability is known as backward (BW) medium or double negative (DNG) medium. Scattering of an electromagnetic plane waves by a conducting cylinder coated with DNG metamaterial is investigated by Li [25]. The problem of scattering of two plane electromagnetic waves from a dielectric coated circular cylinder is investigated by Mushref [26].

In this paper, electromagnetic scattering of two or more plane waves from an infinite PEMC circular cylinder coated with homogeneous DNG metamaterial layer is derived. In the case of a PEMC material, both co-polarized and cross-polarized components of field are produced [6, 12]. For the sake of simplicity, we have taken an infinite PEMC circular cylinder covered with a homogeneous layer of metamaterial of uniform thickness. Both the parallel and perpendicular polarizations of the incident plane waves are considered in the formulation. The method of eigenfunction expansion is used in the theoretical analysis. The problem is formulated with two regions, i.e., free space and the DNG metamaterial layer. Appropriate boundary conditions are applied at the interfaces to calculate the unknown scattering coefficients. The far scattered field patterns are calculated, using the the large argument approximation of Hankel function. The numerical results presented in this paper contain comparisons with the published work under some special conditions. It is found that the results obtained here are in an excellent agreement with the previously published work, which shows the correctness of the analytical formulation and the numerical code.

2. FORMULATION

The geometry of the scattering problem is shown in Figure 1, where the inner cylinder is a PEMC cylinder while the outer cylinder is a linear, isotropic and uniform layer of the DNG metamaterial of uniform thickness. The cylinder is supposed to be infinite along the axis and of circular cross section, with a and b, be the radii of the PEMC cylinder and the coating layer respectively. The external medium mentioned as Region 0 in Figure 1, is free space (μ_0, ϵ_0), with wave
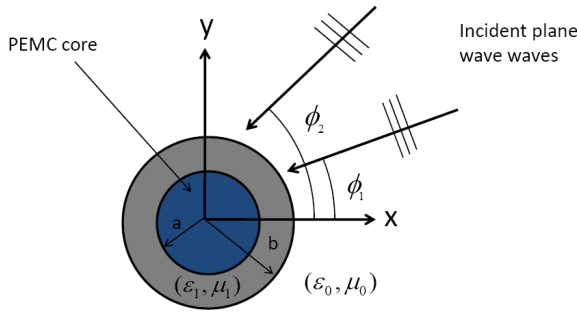
Figure 1. Coated PEMC circular cylinder coated by a Metamaterial.

number $k_0 = \omega\sqrt{\mu_0\epsilon_0}$, and the metamaterial region (μ_1, ϵ_1) is named as Region 1, with wave number $k_1 = \omega\sqrt{\mu_1\epsilon_1}$. In this paper we have used $e^{-j\omega t}$, time dependence and it is suppressed throughout the analysis. Following two subsections describe the analysis of the scattering of incident plane waves from the proposed configuration, for both the TM and TE polarizations.

2.1. TM Polarization

In the case of TM polarization of the incident plane waves the electric vector of all the waves is parallel to the axis of the cylinders, the incident electric fields in terms of the cylindrical coordinates (ρ, ϕ), can be expressed as

$$E_{0z}^{inc(l)}(\rho, \phi) = E_{0l}e^{jk_0\rho cos(\phi - \phi_l)} \tag{5}$$

where $l = 1, 2, 3, \ldots$, which represents the number of incident plane waves. Therefore the total incident wave is the sum of the all the incident plane waves, using the wave transformation, the total incident field may be written in terms of an infinite Fourier-Bessel series as

$$E_{0z}^{inc}(\rho, \phi) = \sum_{n=-\infty}^{\infty} j^n J_n(k_0\rho)e^{jn\phi} \sum_{l=1}^{N} E_{0l}e^{-jn\phi_l} \tag{6}$$

Using Maxwell equation, the corresponding ϕ component of the magnetic field, is obtained as

$$
\begin{aligned}
H_{0\phi}^{inc}(\rho, \phi) &= \frac{1}{-j\eta_0 k_0}\frac{\partial E_{0z}^{inc}(\rho, \phi)}{\partial \rho} \\
&= -\frac{1}{j\eta_0}\sum_{n=-\infty}^{\infty} j^n J_n'(k_0\rho)e^{jn\phi} \sum_{l=1}^{N} E_{0l}e^{-jn\phi_l} \tag{7}
\end{aligned}
$$

where $\eta_0 = \sqrt{\mu_0 \epsilon_0}$ is the free space impedance, $J_n(.)$ is the Bessel function of first kind and prime denotes the derivative with respect to the argument. As the inner cylinder is a PEMC cylinder so the scattered field must contain a cross-polarized field component in addition to the co-polarized field component. Hence the scattered fields in Region 0 using Maxwell's equation, can be written as

$$E_{0z}^s = \sum_{n=-\infty}^{\infty} j^n a_n^E H_n^{(1)}(k_0\rho) e^{jn\phi} \tag{8}$$

$$H_{0\phi}^s = -\frac{1}{j\eta_0} \sum_{n=-\infty}^{\infty} j^n a_n^E H_n^{(1)\prime}(k_0\rho) e^{jn\phi} \tag{9}$$

$$H_{0z}^s = -\frac{j}{\eta_0} \sum_{n=-\infty}^{\infty} j^n b_n^E H_n^{(1)}(k_0\rho) e^{jn\phi} \tag{10}$$

$$E_{0\phi}^s = -\sum_{n=-\infty}^{\infty} j^n b_n^E H_n^{(1)\prime}(k_0\rho) e^{jn\phi} \tag{11}$$

And Region 1 which is a coating layer made up of metamaterial, with uniform thickness, is bounded by two interfaces at $\rho = a$ and $\rho = b$, the fields in this region can be expressed in the form

$$E_{1z} = \sum_{n=-\infty}^{\infty} j^n \left[c_n^E H_n^{(2)}(k_1\rho) + d_n^E H_n^{(1)}(k_1\rho) \right] e^{jn\phi} \tag{12}$$

$$H_{1\phi} = -\frac{1}{j\eta_1} \sum_{n=-\infty}^{\infty} j^n \left[c_n^E H_n^{(2)\prime}(k_1\rho) + d_n^E H_n^{(1)\prime}(k_1\rho) \right] e^{jn\phi} \tag{13}$$

$$H_{1z} = -\frac{j}{\eta_1} \sum_{n=-\infty}^{\infty} j^n \left[e_n^E H_n^{(2)}(k_1\rho) + f_n^E H_n^{(1)}(k_1\rho) \right] e^{jn\phi} \tag{14}$$

$$E_{1\phi} = -\sum_{n=-\infty}^{\infty} j^n \left[e_n^E H_n^{(2)\prime}(k_1\rho) + f_n^E H_n^{(1)\prime}(k_1\rho) \right] e^{jn\phi} \tag{15}$$

where $J_n(x)$ and $N_n(x)$ are the Bessel functions of first and second kinds, and $H_n^{(1)}(x) = J_n(x) + iN_n(x)$ and $H_n^{(2)}(x) = J_n(x) - iN_n(x)$. The constants $a_n^E, b_n^E, c_n^E, d_n^E, e_n^E$, and f_n^E are the six unknown scattering coefficients in case of the TM incident plane waves which are to be determined. These unknowns can be found by using appropriate boundary conditions at the interfaces $\rho = a$ and $\rho = b$. The boundary conditions at the interface $\rho = a$ are

$$H_{1z} + ME_{1z} = 0, \qquad \rho = a, \qquad 0 \le \phi \le 2\pi \tag{16}$$

$$H_{1\phi} + ME_{1\phi} = 0, \qquad \rho = a, \qquad 0 \le \phi \le 2\pi \tag{17}$$

And the boundary conditions at the interface $\rho = b$, are

$$E_{0z}^{inc} + E_{0z}^{s} = E_{1z}, \qquad \rho = b, \qquad 0 \leq \phi \leq 2\pi \qquad (18)$$

$$H_{0\phi}^{inc} + H_{0\phi}^{s} = H_{1\phi}, \qquad \rho = b, \qquad 0 \leq \phi \leq 2\pi \qquad (19)$$

$$H_{0z}^{s} = H_{1z}, \qquad \rho = b, \qquad 0 \leq \phi \leq 2\pi \qquad (20)$$

$$E_{0\phi}^{s} = E_{1\phi}, \qquad \rho = b, \qquad 0 \leq \phi \leq 2\pi \qquad (21)$$

Applying these boundary conditions at $\rho = a$ and $\rho = b$, we obtain a linear matrix equation for the unknown scattering coefficients. Solution of this matrix equation yields the values of all the unknowns scattering coefficients. Using the values of a_n^E and b_n^E in Equations (8) and (11), we get the scattered co- and cross-polarized fields radiated by the coated PEMC cylinder, respectively. Far-zone scattered fields result when the asymptotic form of Hankel functions is used, and these fields are given as

$$E_{0z}^{s} = P(\phi)e^{-j(k_0\rho - \pi/4)}\sqrt{\frac{2}{\pi k_0 \rho}} \qquad (22)$$

$$E_{0\phi}^{s} = Q(\phi)e^{-j(k_0\rho - \pi/4)}\sqrt{\frac{2}{\pi k_0 \rho}} \qquad (23)$$

where

$$P(\phi) = \sum_{n=-\infty}^{\infty} j^n a_n^E e^{jn\phi} \qquad (24)$$

$$Q(\phi) = \sum_{n=-\infty}^{\infty} j^n b_n^E e^{jn\phi} \qquad (25)$$

The formulation for the TE polarization case can be obtained by using the duality theorem.

3. NUMERICAL RESULTS AND DISCUSSION

This section presents some numerical results for co- and cross-polarized scattered field patterns in the far-zone by a PEMC circular cylinder coated with metamaterial. To verify the analytical formulation and the numerical code, few examples are given under some special conditions, and the results obtained are compared with the published literature. The comparisons are given in Figures 2(a) and 2(b), and are found to be in excellent agreement with the compared published results. In Figure 2(a), we have taken $M\eta_1 = \pm\infty$, $k_0 a = 5, k_0 b = 5.1, \epsilon_r =$
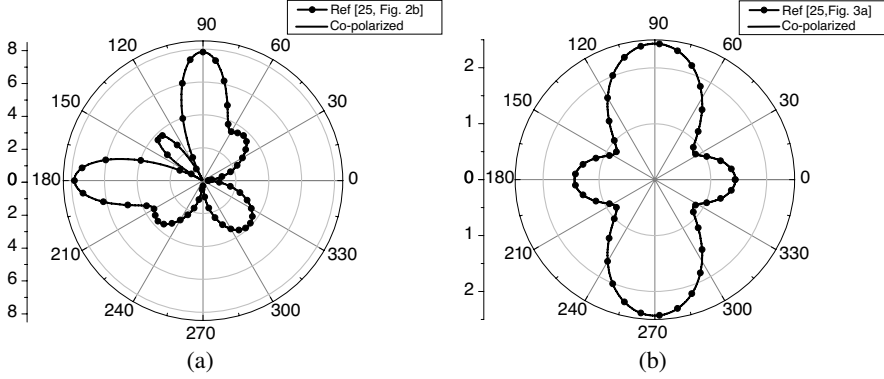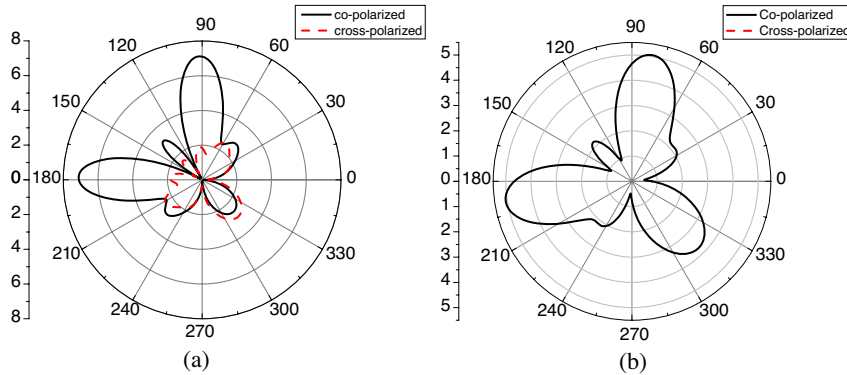
Figure 2. Far-zone scattered field patterns of a Dielectric coated PEMC cylinder. (a) $M\eta_1 = \pm\infty$, $k_0 a = 5$, $k_0 b = 5.1$, $\varepsilon_r = 5$, $\mu_r = 1$, $\phi_1 = -90°$, $\phi_2 = 0°$, $E_{01} = E_{02} = 1$. (b) $M\eta_1 = \pm\infty$, $k_0 a = 2$, $k_0 b = 2.4$, $\varepsilon_r = 5$, $\mu_r = 1$, $\phi_1 = 0°$, $\phi_2 = 180°$, $E_{01} = E_{02} = 1$.

$5, \mu_r = 1$ and two waves are considered to be incident on the configuration, with the angles of incidence $\phi_1 = -90°$, $\phi_2 = 0°$ and amplitudes $E_{01} = E_{02} = 1$. In Figure 2(b), we have taken $M\eta_1 = \pm\infty$, $k_0 a = 2, k_0 b = 2.4, \epsilon_r = 5, \mu_r = 1$ with the two waves incident with $\phi_1 = 0°$, $\phi_2 = 180°$ and $E_{01} = E_{02} = 1$. From these two plots it is obvious that the co-polarized component shows an excellent agreement with the previously published work while the cross-polarized component in these plots is zero as PEMC material becomes PEC for $M\eta_1 = \pm\infty$. In Figure 3(a), co- and cross-polarized scattered field patterns in response to two incident plane waves ($\phi_1 = -90°, \phi_2 = 0°, E_{01} = E_{02} = 1$), are presented with $M\eta_1 = 1$, $k_0 a = 5, k_0 b = 5.1, \epsilon_r = 5, \mu_r = 1$. Figure 3(b) shows the co- and cross-polarized scattered field patterns with the same configuration as in Figure 3(a) except that here we have taken $M\eta_1 = 0$. From Figure 3(b), it is obvious that the cross-polarized component is zero as PEMC behaves like PMC for $M\eta_1 = 0$. In Figure 4, the dielectric coating layer is changed by a metamaterial layer with $k_0 a = 5, k_0 b = 5.1, \epsilon_r = -5, \mu_r = -1$ and the same two plane waves are exciting the coated PEMC cylinder. Figure 4(a) shows the co- and cross-polarized scattered field patterns with $M\eta_1 = 1$, while Figure 4(b) shows the scattered co- and cross-polarized fields for $M\eta_1 = 0$. In Figure 5, a dielectric ($\epsilon_r = 9.8$, $\mu_r = 1$) coated PEMC cylinder is excited by three incident plane waves with ($\phi_1 = 0°, \phi_2 = 180°, \phi_3 = 135°, E_{01} = E_{02} = E_{03} = 1$). Figure 5(a) exhibits TM and TE scattered co-polarized field patterns in response to the
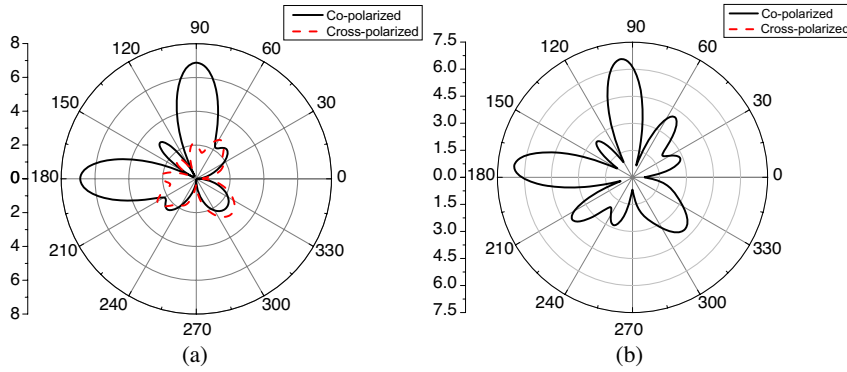
Figure 3. Far-zone scattered field form a PEMC cylinder coated with a dielectric material. (a) $M\eta_1 = 1$, $k_0a = 5$, $k_0b = 5.1$, $\varepsilon_r = 5$, $\mu_r = 1$, $\phi_1 = -90$, $\phi_2 = 0$. (b) $M\eta_1 = 0$, $k_0a = 5$, $k_0b = 5.1$, $\varepsilon_r = 5$, $\mu_r = 1$, $\phi_1 = -90$, $\phi_2 = 0$.
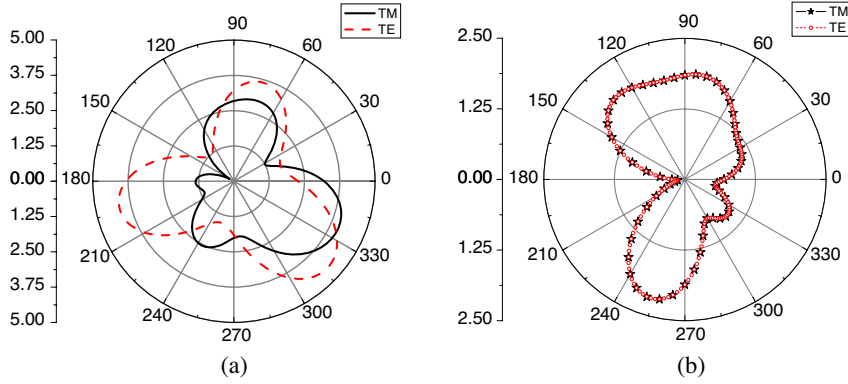

Figure 4. Far-zone scattered field by a PEMC cylinder coated with a Metamaterial layer. (a) $M\eta_1 = 1$, $k_0a = 5$, $k_0b = 5.1$, $\varepsilon_r = -5$, $\mu_r = -1$, $\phi_1 = -90$, $\phi_2 = 0$. (b) $M\eta_1 = 0$, $k_0a = 5$, $k_0b = 5.1$, $\varepsilon_r = -5$, $\mu_r = -1$, $\phi_1 = -90$, $\phi_2 = 0$.

three incident plane waves with $M\eta_1 = 1$, while Figure 5(b) shows the TM and TE cross-polarized field patterns from the same configuration. Figure 6 shows the behavior of the metamaterial ($\epsilon_r = -9.8$, $\mu_r = -1$) coated PEMC cylinder in response to the three incident plane waves ($\phi_1 = 0°, \phi_2 = 180°, \phi_3 = 135°, E_{01} = E_{02} = E_{03} = 1$) for both the TM and TE cases with $M\eta_1 = 1$. Figure 6(a) shows the co-polarized scattered filed patterns while Figure 6(b) presents the cross-polarized

scattered filed patterns.

Figure 7 and Figure 8, show the dependence of the co- and cross-polarized components of the diffracted far-field patterns on the thickness of the coating layer. In these two figures the radius of the inner cylinder, is taken as $k_a = 1$, while the thickness of the coating layer is varied between $k_b = 1$ and $k_0 b = 4$. Figure 7(a) presents
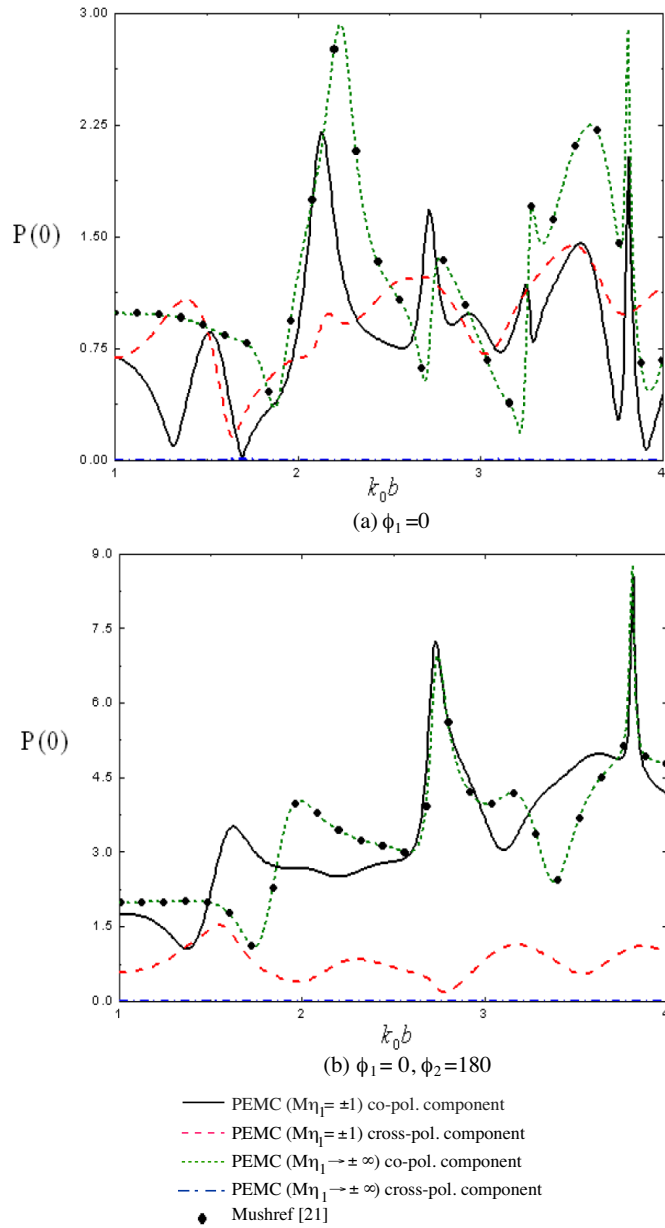

Figure 5. Far-zone scattered field from a dielectric coated PEMC cylinder, $M\eta_1 = 1$, $k_0 a = 1$, $k_0 b = 2$, $\varepsilon_r = 9.8$, $\mu_r = 1$, $\phi_1 = 0°$, $\phi_2 = 180°$, $\phi_3 = 135°$. (a) Co-polarized components. (b) Cross-polarized components.
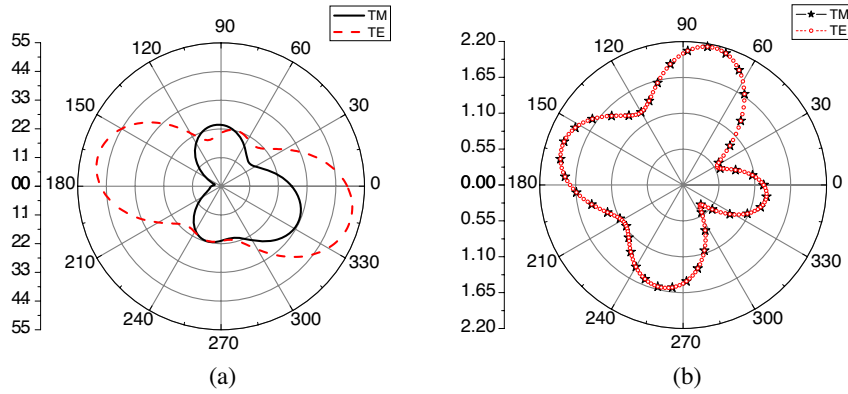

Figure 6. Far-zone scattered field from a Metamaterial coated PEMC cylinder, $M\eta_1 = 1$, $k_0 a = 1$, $k_0 b = 2$, $\varepsilon_r = -9.8$, $\mu_r = -1$, $\phi_1 = 0°$, $\phi_2 = 180°$, $\phi_3 = 135°$. (a) Co-polarized components. (b) Cross-polarized components.

(a) $\phi_1 = 0$

(b) $\phi_1 = 0$, $\phi_2 = 180$

—— PEMC ($M\eta_1 = \pm 1$) co-pol. component

- - - PEMC ($M\eta_1 = \pm 1$) cross-pol. component

········ PEMC ($M\eta_1 \rightarrow \pm \infty$) co-pol. component

—·—· PEMC ($M\eta_1 \rightarrow \pm \infty$) cross-pol. component

◆   Mushref [21]

Figure 7. Co and cross-polarized components of the diffracted field patterns (TM case), versus $k_0 b$ at $\phi = 0$ for $k_0 a = 1$, $\varepsilon_r = 5$, $\mu_r = 1$.
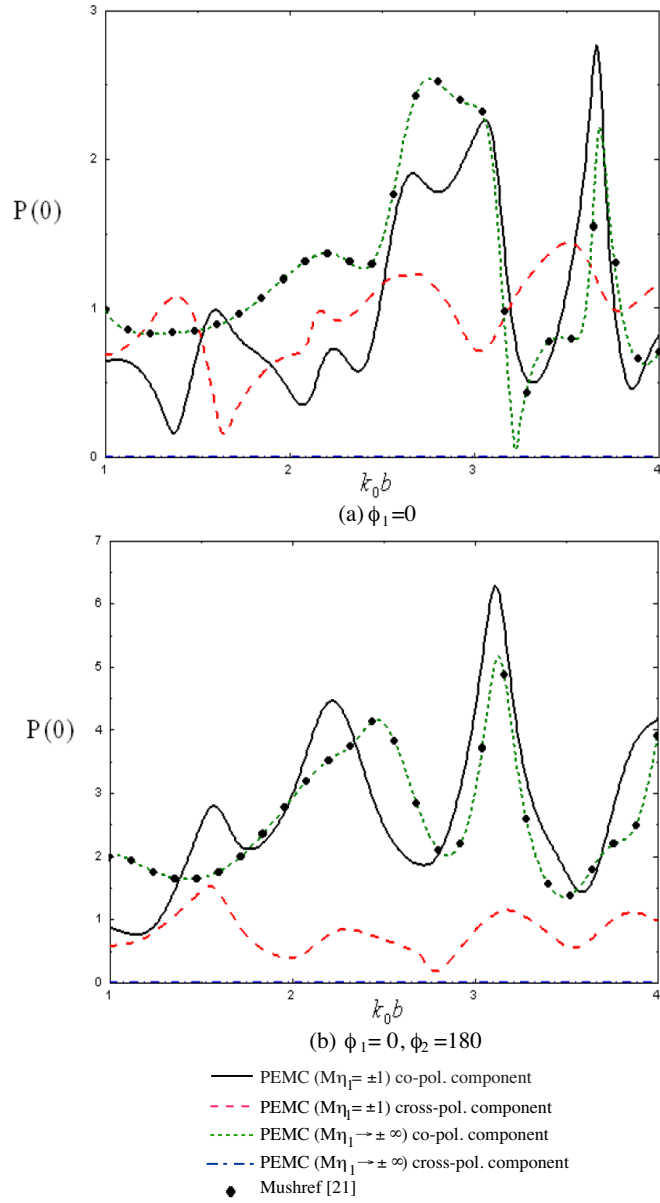
Figure 8. Co and cross-polarized components of the diffracted field patterns (TM case), versus $k_0 b$ at $\phi = 0$ for $k_0 a = 1$, $\varepsilon_r = 5$, $\mu_r = 5$.

(a) $\phi_1 = 0$

(b) $\phi_1 = 0$, $\phi_2 = 180$

——— PEMC ($M\eta_1 = \pm 1$) co-pol. component
– – – PEMC ($M\eta_1 = \pm 1$) cross-pol. component
········ PEMC ($M\eta_1 \rightarrow \pm \infty$) co-pol. component
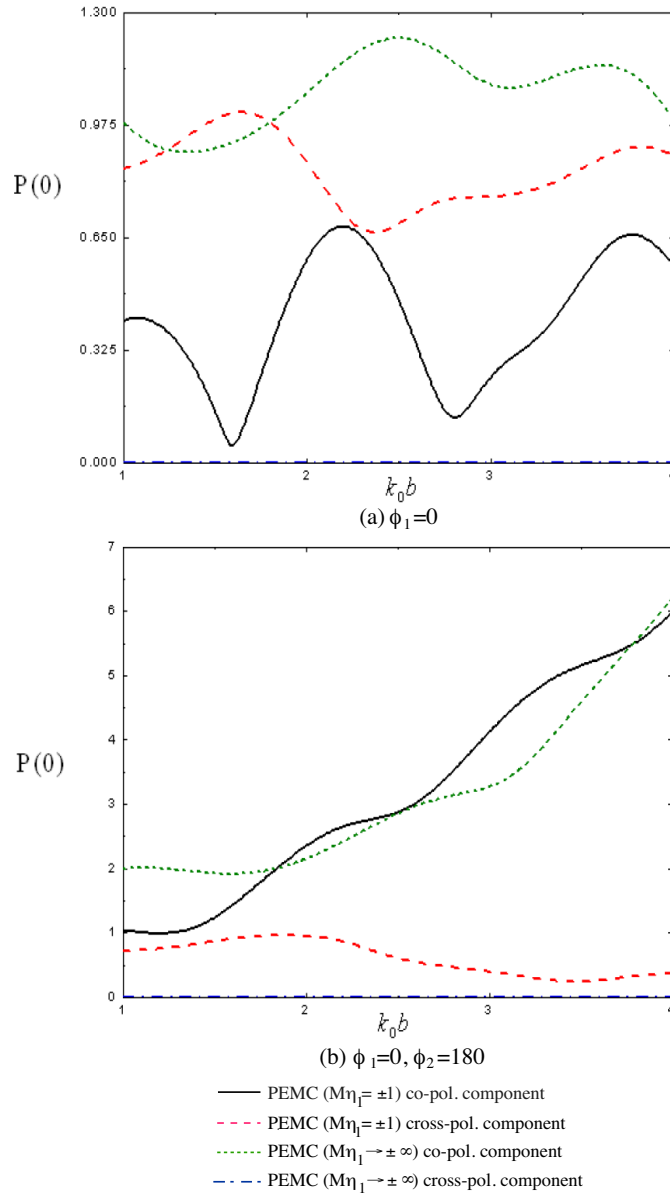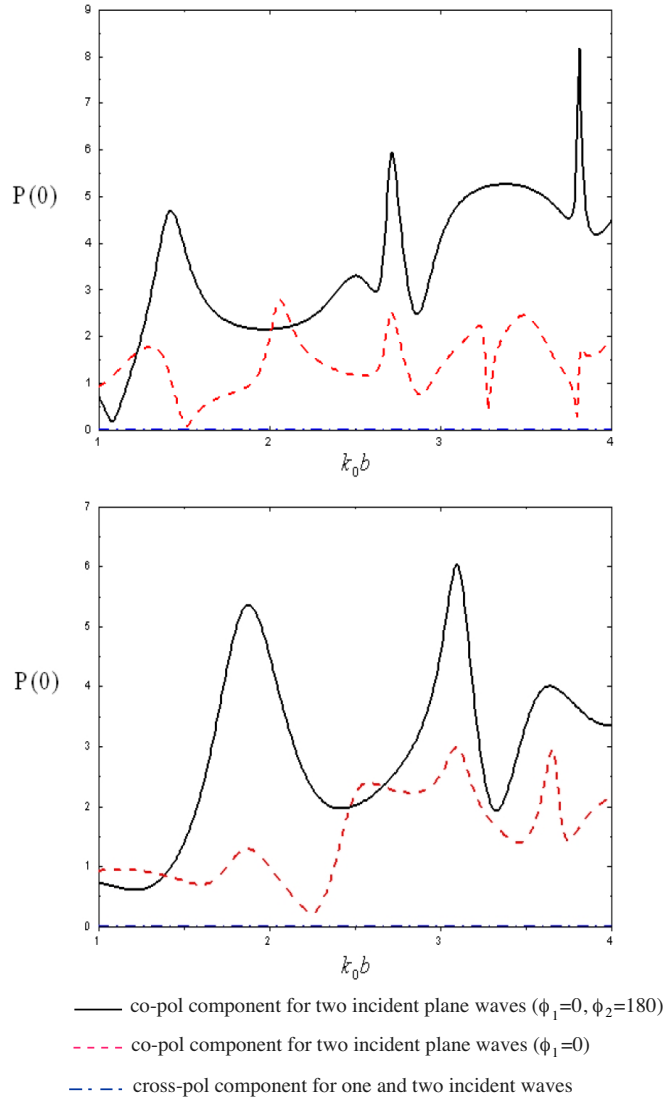– · – · PEMC ($M\eta_1 \rightarrow \pm \infty$) cross-pol. component

Figure 9. Co and cross-polarized components of the diffracted field patterns (TE case), versus $k_0 b$ at $\phi = 0$ for $k_0 a = 1$, $\varepsilon_r = 2.2$, $\mu_r = 1$.

Figure 10. Co and cross-polarized components of the diffracted field patterns (TM case), versus $k_0 b$ at $\phi = 0$ for $k_0 a = 1$, $\varepsilon_r = 5$, $\mu_r = 1$, $M\eta_1 = 0$.
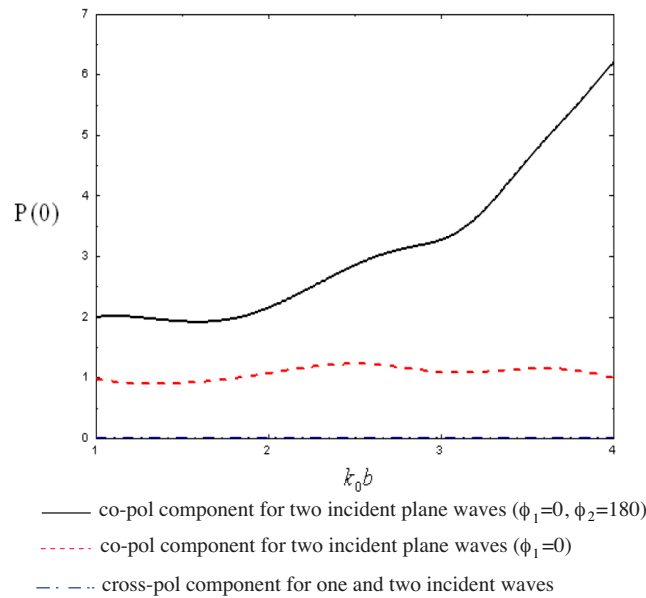
the variations in the co- and cross-polarized field components, for a single incident plane wave, versus $k_0 b$, when $\epsilon_r = 5$ and $\mu_r = 1$, with $\phi_1 = 0°$. Figure 7(b) shows the variations of the field components

for two incident plane waves, with $(\phi_1 = 0°, \phi_2 = 180°$. Figure 8(a) and Figure 8(b) show the co- and cross-polarized components of the diffracted far-field with the variation in the coating thickness for one and two incident plane waves respectively. Here in Figure 8, coating layer has the parameters as $\epsilon_r = 1$ and $\mu_r = 5$. From Figures 7 and 8, we can see that the co-polarized components of the coated PEMC cylinder with $M\eta_1 = \pm\infty$ shows a good agreement with that of a coated PEC cylinder, discussed by Mushref [22]. And cross-polarized component vanishes in this case.

Figures 9(a) and 9(b), show the co- and cross-polarized components of the diffracted far-field in the TE case, for one and two incident plane waves, respectively. Here in Figure 9, we have chosen the coating layer with constitutive parameters $\epsilon_r = 2.2$ and $\mu_r = 1$ and the admittance parameter is taken as $M\eta_1 = \pm1$ and $M\eta_1 = \pm\infty$.

Figures 10 and 11, show the co- and cross-polarized components of the diffracted far-field from a PEMC cylinder with $M\eta_1 = 0$ (PMC case), for TM and TE case respectively. Each plot in these two figures contains co- and cross-polarized components for one and two incident plane waves. From these these plots, it is obvious that the cross-

—— co-pol component for two incident plane waves (ϕ_1=0, ϕ_2=180)

----- co-pol component for two incident plane waves (ϕ_1=0)

—·—·· cross-pol component for one and two incident waves

Figure 11. Co and cross-polarized components of the diffracted field patterns (TE case), versus $k_0 b$ at $\phi = 0$ for $k_0 a = 1$, $\varepsilon_r = 2.2$, $\mu_r = 1$, $M\eta_1 = 0$.

polarized component for both the single and double incident plane waves vanishes when we choose $M\eta_1 = 0$.

4. CONCLUSIONS

It has been observed that for $M\eta_1 \to \infty$, geometry behaves like a coated PEC cylinder, while it shows the characteristics of a coated PMC cylinder at $M\eta_1 = 0$. It is seen that the cross-polarized component of the diffracted far-field, exhibits a similar behavior for both types of polarizations for a particular type of the coating layer. But it is different for two different types of the coating. And the behavior of the co-polarized component of the diffracted far-field, for a coated PEMC cylinder varies with the type of polarization and type of the coating layer.

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