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PLANAR SLAB OF CHIRAL NIHILITY METAMATERIAL BACKED BY FRACTIONAL DUAL/PEMC INTERFACE

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Abstract—Fields inside the chiral nihility slab which is backed by perfect electric conductor are determined. It is noted that both electric and magnetic fields exist inside the grounded chiral nihility slab when it is excited by a plane wave. Electric field inside the slab disappears for excitation due to an electric line source. Magnetic field inside the slab disappears when geometry changes to corresponding dual geometry. Dual geometry means chiral nihility slab backed by perfect magnetic conductor and excited by a magnetic line source. Using fractional curl operator, fields are determined for fractional order geometries which may be regarded as intermediate step between the two geometries which are related through principle of duality. Discussion is extended for chiral nihility slab which is backed by perfect electromagnetic conductor (PEMC).

1. INTRODUCTION

Chiral nihility is a special kind of chiral medium, for which the real part of permittivity and permeability are simultaneously zero or refractive index become zero at certain frequency known as nihility frequency [1–3]. In chiral nihility, the two eigenwaves are still circularly polarized but one of them is a backward wave. For backward waves phase velocity is antiparallel to the corresponding Poynting vector. Phenomena of negative refraction occurs when a plane wave enters from vacuum to chiral nihility medium. That is, when a plane wave obliquely hits the interface due to vacuum and chiral nihility, one refracted eigenwave propagates on one side of normal at certain angle while other eigenwave propagates at same angle on other side of the normal to the interface. Another interesting phenomena of negative reflection of both eigenwaves occurs at the interface between chiral nihility and perfect electric conductor plane [2]. Due to these two phenomenon, both electric field and power flow disappear in particular regions of planer waveguide, composed of chiral nihility slabs backed by perfect electric conductors, when it is excited by an electric line source [3]. On the other hand, magnetic field and power flow disappears in particular regions of planer waveguide, composed of chiral nihility slabs backed by perfect magnetic conductors, when it is excited by a magnetic line source.

Our interest is to study behavior of fields inside and outside chiral nihility slab which is backed by perfect electric conductor. Another geometry which is dual to the first geometry has also been considered. Geometry containing chiral nihility slab backed by PEC and excited by an electric line source and geometry containing chiral nihility slab backed by PMC and excited by a magnetic line source are dual of each other. For each geometry uniform plane wave or line source has been considered as a source of excitation. Difference in behavior of fields inside grounded chiral nihility slab due to line source excitation and plane wave excitation is noted.

Discussion is further extended to two general geometries, first deals with chiral nihility slab backed by fractional dual interface while other deals with chiral nihility slab backed by perfect electromagnetic conductor (PEMC). PEC and PMC become special cases of each general geometry. Field corresponding to fractional or intermediate geometries between the two dual geometries are studied. Fractional geometries have been obtained using fractional curl operator [4].

2. GROUNDED CHIRAL NIHILITY SLAB

Consider a slab of chiral nihility metamaterial. The slab is of infinite length and is backed by perfect electric conductor (PEC). Front face of the chiral nihility slab is located at $z = d_1$ while perfect electric conductor is located at location $z = d_2$, where $d_2 > d_1$. The chiral nihility slab backed by PEC has been termed as grounded chiral nihility slab.

A linearly polarized uniform plane wave, with time dependency time harmonic $\exp(-j\omega t)$, is obliquely incident on the grounded chiral nihility slab. The electric and magnetic fields inside and outside the grounded chiral nihility slab may be written in terms of unknown coefficients as [5, 6]

$$\begin{aligned}\mathbf{E}_0 = \exp(ik_y y)\big[&\hat{\mathbf{x}}\exp(ik_{0z}z) + A^-\mathbf{N}_R^-\exp(-ik_{0z}z)\\ &+B^-\mathbf{N}_L^-\exp(-ik_{0z}z)\big], \quad z<d_1\end{aligned} \tag{1}$$

$$\mathbf{E}_1 = \exp(ik_y y)\left[E^+\mathbf{M}_R^+\exp(ik_z^+ z) + F^+\mathbf{M}_L^+\exp(ik_z^- z)\right.$$
$$\left.+E^-\mathbf{M}_R^-\exp(-ik_z^+ z) + F^-\mathbf{M}_L^-\exp(-ik_z^- z)\right], \quad d_1 < z < d_2 \quad (2)$$

$$\mathbf{H}_0 = \exp(ik_y y)\left[\frac{1}{k_0\eta_0}\left\{\hat{\mathbf{y}}k_{0z}\exp(ik_{0z}z) - \hat{\mathbf{z}}k_y\exp(ik_{0z}z)\right\}\right.$$
$$\left.-\frac{i}{\eta_0}\left\{A^-\mathbf{N}_R^-\exp(-ik_{0z}z) - B^-\mathbf{N}_L^-\exp(-ik_{0z}z)\right\}\right], \quad z < d_1 \quad (3)$$

$$\mathbf{H}_1 = \exp(ik_y y)\frac{-i}{\eta}\left[E^+\mathbf{M}_R^+\exp(ik_z^+ z) - F^+\mathbf{M}_L^+\exp(ik_z^- z)\right.$$
$$\left.+E^-\mathbf{M}_R^-\exp(-ik_z^+ z) - F^-\mathbf{M}_L^-\exp(-ik_z^- z)\right], \quad d_1 < z < d_2 \quad (4)$$

where

$$\mathbf{N}_R^\pm = \hat{x} \pm \frac{ik_{0z}}{k_0}\hat{y} - \frac{ik_y}{k_0}\hat{z} \quad (5)$$

$$\mathbf{N}_L^\pm = \hat{x} \mp \frac{ik_{0z}}{k_0}\hat{y} + \frac{ik_y}{k_0}\hat{z} \quad (6)$$

$$\mathbf{M}_R^\pm = \hat{x} \pm \frac{ik_z^+}{k^+}\hat{y} - \frac{ik_y}{k^+}\hat{z} = \hat{x} \pm \frac{ik_z^\pm}{k^\pm}\hat{y} - \frac{ik_y}{k^\pm}\hat{z} \quad (7)$$

$$\mathbf{M}_L^\pm = \hat{x} \mp \frac{ik_z^+}{k^+}\hat{y} + \frac{ik_y}{k^+}\hat{z} = \hat{x} \mp \frac{ik_z^\pm}{k^\pm}\hat{y} + \frac{ik_y}{k^\pm}\hat{z} \quad (8)$$

Superscript $\pm$ in Equations (5)–(8) represents the eigenwaves propagating in the $\pm z$ direction. The subscript R and L refer to the RCP and LCP eigenwaves satisfying the dispersion relations as

$$k_y^2 + (k_z^\pm)^2 = (k^\pm)^2$$

where $k^\pm = \pm\omega\kappa$ at the nihility frequency. In above equations, $k_0 = \omega\sqrt{\mu_0\epsilon_0}$, $\eta_0 = \sqrt{\mu_0/\epsilon_0}$, and $\eta = \sqrt{\mu/\epsilon}$. k_{0z} and k_y satisfy the dispersion relation

$$k_y^2 + k_{0z}^2 = k_0^2$$

It may be noted that relation $k_z^+ = -k_z^-$ holds for all modes propagating inside the slab.

Unknown coefficients in field expressions (1)–(4) may be obtained using the boundary conditions. At $z = d_2$, tangential components of electric field $\mathbf{E}_1$ must be zero. Application of boundary condition to x and y components of electric field at $z = d_2$ and imposing restriction $k_z^+ = -k_z^-$ yields

$$E^\pm = -F^\mp \quad (9)$$

Tangential components of electric and magnetic fields across the dielectric interface located at $z = d_1$ must be continues. Continuity of x-components and y-components of electric field yields

$$E^- = \frac{\exp\left(ik_{0z}d_1 + ik_z^+ d_1\right)}{2R_f} \tag{10}$$

$$E^+ = -\frac{\exp\left(ik_{0z}d_1 - ik_z^+ d_1\right)}{2R_f} \tag{11}$$

where

$$R_f = \frac{k_0\eta_0 k_z^+}{k^+\eta k_{0z}}$$

Substitution of unknowns coefficients in expressions (2) and (4) yields electric and magnetic fields inside the slab. On the other, if we excite the grounded slab by a electric line source, it can be shown that electric field inside the slab disappears. That is

$$\int_{-\infty}^{\infty} \mathbf{E}_1(k_y)dk_y = 0 \tag{12}$$

Using duality principle, it can be shown that chiral nihility slab backed by PMC does not contain magnetic field at nihility frequency when it is excited by a magnetic line source. That is

$$\int_{-\infty}^{\infty} \mathbf{H}_1(k_y)dk_y = 0$$

In both geometries, for line source excitation, there is no power inside the grounded chiral nihility slab.

Fractionalization of a given ordinary operators may be used to explore the intermediate geometries between the two given geometries. The two given geometries must be connected through the given ordinary operator. Frac-tional curl operator has been used to study various problems [7–16]. A linear operator may be fractionalized using recipe given in [4], which dictates that fractionalization of an operator means fractionalization of its eigenvalues. Our interest is to note the behavior of electric and magnetic fields inside the chiral nihility slab for different values of the order of curl operator. In other words, our interest is to see how fields in a geometry changes to fields in the dual geometry.

3. CHIRAL NIHILITY SLAB BACKED BY FRACTIONAL DUAL INTERFACE

Using the concept of fractional curl operator $(\nabla\times)^\alpha$ [4], we can write Maxwell equations for time harmonic fields as

$$(\hat{\mathbf{k}}_i\times)\mathbf{E}_{0fd} = (\eta_0\mathbf{H}_{0fd}) \tag{13}$$
$$(\hat{\mathbf{k}}_i\times)(\eta_0\mathbf{H}_{0fd}) = -\mathbf{E}_{0fd} \tag{14}$$
$$(\hat{\mathbf{k}}_i^{\pm}\times)\mathbf{E}_{1fd} = (\eta\mathbf{H}_{1fd}) \tag{15}$$
$$(\hat{\mathbf{k}}_i^{\pm}\times)(\eta\mathbf{H}_{1fd}) = -\mathbf{E}_{1fd} \tag{16}$$

Subscript fd stands for fractional dual. Fractional dual fields for region inside and outside the slab may be obtained as

$$\begin{aligned}
\mathbf{E}_{0fd} &= (\hat{\mathbf{k}}_i\times)^\alpha\eta_0\mathbf{H}_0\\
\eta_0\mathbf{H}_{0fd} &= (\hat{\mathbf{k}}_i\times)^\alpha\mathbf{E}_0\\
\mathbf{E}_{1fd} &= (\hat{\mathbf{k}}_i^{\pm}\times)^\alpha\eta\mathbf{H}_1\\
\eta\mathbf{H}_{fd} &= (\hat{\mathbf{k}}_i^{\pm}\times)^\alpha\mathbf{E}_1
\end{aligned}$$

and these fields must satisfy the Maxwell equations.

In order to deal with above equations, eigenvalues and eigenvector of operators $(\hat{\mathbf{k}}_i\times)$ and $(\hat{\mathbf{k}}_i^{\pm}\times)$ are required. So first we calculate the eigenvalues and eigenvectors of these cross product operators.

Eigenvalues and eigenvectors of operator $\hat{\mathbf{k}}_1\times = \left(\frac{k_y\hat{y}+k_{0z}\hat{z}}{k_0}\right)\times$

$$\begin{aligned}
\mathbf{A}_{11} &= \frac{1}{\sqrt{2}}\left[\hat{x}+i\frac{k_{0z}}{k_0}\hat{y}-i\frac{k_y}{k_0}\hat{z}\right] = \mathbf{N}_R^+, && a_{11} = -i\\
\mathbf{A}_{12} &= \frac{1}{\sqrt{2}}\left[\hat{x}-i\frac{k_{0z}}{k_0}\hat{y}+i\frac{k_y}{k_0}\hat{z}\right] = \mathbf{N}_L^+, && a_{12} = +i\\
\mathbf{A}_{13} &= i\frac{k_y}{k_0}\hat{y}+i\frac{k_{0z}}{k_0}\hat{z}, && a_{13} = 0
\end{aligned}$$

Eigenvalues and eigenvectors of operator $\hat{\mathbf{k}}_2\times = \left(\frac{k_y\hat{y}-k_{0z}\hat{z}}{k_0}\right)\times$ are

$$\begin{aligned}
\mathbf{A}_{21} &= \frac{1}{\sqrt{2}}\left[\hat{x}+i\frac{k_{0z}}{k_0}\hat{y}+i\frac{k_y}{k_0}\hat{z}\right] = \mathbf{N}_R^-, && a_{21} = +i\\
\mathbf{A}_{22} &= \frac{1}{\sqrt{2}}\left[\hat{x}-i\frac{k_{0z}}{k_0}\hat{y}-i\frac{k_y}{k_0}\hat{z}\right] = \mathbf{N}_L^-, && a_{22} = -i\\
\mathbf{A}_{23} &= i\frac{k_y}{k_0}\hat{y}-i\frac{k_{0z}}{k_0}\hat{z}, && a_{23} = 0
\end{aligned}$$

Similarly eigenvalues and eigenvectors of operator $\hat{\mathbf{k}}_1^+ \times = \left(\frac{k_y\hat{y}+k_z^+\hat{z}}{k_0}\right)\times$

$$\begin{aligned}
\mathbf{A}_{11}^+ &= \frac{1}{\sqrt{2}}\left[\hat{x}+i\frac{k_z^+}{k^+}\hat{y}-i\frac{k_y}{k^+}\hat{z}\right]=\mathbf{M}_R^+, && a_{11}^+=-i\\
\mathbf{A}_{12}^+ &= \frac{1}{\sqrt{2}}\left[\hat{x}-i\frac{k_z^+}{k^+}\hat{y}+i\frac{k_y}{k^+}\hat{z}\right]=\mathbf{M}_L^-, && a_{12}^+=+i\\
\mathbf{A}_{13}^+ &= i\frac{k_y}{k^+}\hat{y}+i\frac{k_z^+}{k^+}\hat{z}, && a_{13}^+=0
\end{aligned}$$

Eigenvalues and eigenvector of operator $\hat{\mathbf{k}}_2^+ \times = \left(\frac{k_y\hat{y}-k_z^+\hat{z}}{k_0}\right)\times$ are

$$\begin{aligned}
\mathbf{A}_{21}^+ &= \frac{1}{\sqrt{2}}\left[\hat{x}+i\frac{k_z^+}{k^+}\hat{y}+i\frac{k_y}{k^+}\hat{z}\right]=\mathbf{M}_L^-, && a_{21}^+=i\\
\mathbf{A}_{22}^+ &= \frac{1}{\sqrt{2}}\left[\hat{x}-i\frac{k_z^+}{k^+}\hat{y}-i\frac{k_y}{k^+}\hat{z}\right]=\mathbf{M}_R^+, && a_{22}^+=-i\\
\mathbf{A}_{23}^+ &= i\frac{k_y}{k^+}\hat{y}-i\frac{k_z^+}{k^+}\hat{z}, && a_{23}^+=0
\end{aligned}$$

Fractional dual fields are obtained by fractionalizing the eigenvalues of corresponding linear operator as given below

$$\begin{aligned}
\mathbf{E}_{0fd} &= \exp(ik_yy)\left[(-i)^\alpha\frac{1}{\sqrt{2}}\mathbf{A}_{11}+(+i)^\alpha\frac{1}{\sqrt{2}}\mathbf{A}_{12}\right]\exp\left(ik_{0z}z\right)\\
&\quad+\left[(-i)^\alpha A^-\mathbf{N}_R^-+(i)^\alpha B^-\mathbf{N}_L^-\right]\exp(ik_yy-ik_{0z}z)\\
\mathbf{E}_{1fd} &= \exp(ik_yy)\left[(-i)^\alpha E^+\mathbf{M}_R^+\exp(ik_z^+z)+(i)^\alpha F^+\mathbf{M}_L^+\exp(ik_z^-z)\right.\\
&\quad\left.+(-i)^\alpha E^-\mathbf{M}_R^-\exp(-ik_z^+z)+(i)^\alpha F^-\mathbf{M}_L^-\exp(-ik_z^-z)\right]\\
\eta_0\mathbf{H}_{0fd} &= \exp(ik_yy)\left[(-i)^{\alpha+1}\frac{1}{\sqrt{2}}\mathbf{A}_{11}+(+i)^{\alpha+1}\frac{1}{\sqrt{2}}\mathbf{A}_{12}\right]\exp\left(ik_{0z}z\right)\\
&\quad\left[+(-i)^{\alpha+1}A^-\mathbf{N}_R^-+(i)^{\alpha+1}B^-\mathbf{N}_L^-\right]\exp(ik_yy-ik_{0z}z)\\
\eta\mathbf{H}_{1fd} &= \exp(ik_yy)\left[(-i)^{\alpha+1}E^+\mathbf{M}_R^+\exp(ik_z^+z)+(i)^{\alpha+1}F^+\mathbf{M}_L^+\exp(ik_z^-z)\right.\\
&\quad\left.+(-i)^{\alpha+1}E^-\mathbf{M}_R^-\exp(-ik_z^+z)+(i)^{\alpha+1}F^-\mathbf{M}_L^-\exp(-ik_z^-z)\right]
\end{aligned}$$

Expressing k_z^- as $(-k_z^+)$, above equations yields the following

$$\begin{aligned}
\mathbf{E}_{0fd} &= \left[\cos\left(\frac{\alpha\pi}{2}\right)\hat{x}+\frac{k_{0z}}{k_0}\sin\left(\frac{\alpha\pi}{2}\right)\hat{y}\right.\\
&\quad\left.-\frac{k_y}{k_0}\sin\left(\frac{\alpha\pi}{2}\right)\hat{z}\right]\exp(ik_yy+ik_{0z}z)
\end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{2}\left[\hat{x}2\cos\left(\frac{\alpha\pi}{2}\right)-\hat{y}2\frac{k_{0z}}{k_0}\sin\left(\frac{\alpha\pi}{2}\right)\right.\\
&\left.-\hat{z}2\frac{k_y}{k_0}\sin\left(\frac{\alpha\pi}{2}\right)\right]\exp(ik_y y-ik_{0z}(z-2d_1)) \qquad (17)\\
\mathbf{E}_{1fd} \;=\;& E^+\left[-2i\sin\left(\frac{\alpha\pi}{2}\right)\hat{x}-2\frac{k_z^+}{k^+}\sin\left(\frac{\alpha\pi}{2}\right)\hat{y}\right.\\
&\left.-2i\frac{k_y}{k^+}\cos\left(\frac{\alpha\pi}{2}\right)\hat{z}\right]\exp(ik_y y+ik_z^+ z)\\
&+E^-\left[-2i\sin\left(\frac{\alpha\pi}{2}\right)\hat{x}-2\frac{k_z^+}{k^+}\sin\left(\frac{\alpha\pi}{2}\right)\hat{y}\right.\\
&\left.-2i\frac{k_y}{k^+}\cos\left(\frac{\alpha\pi}{2}\right)\hat{z}\right]\exp(ik_y y-ik_z^+ z) \qquad (18)\\
\eta_0\mathbf{H}_{0fd} \;=\;& \left[\cos\left(\frac{(\alpha+1)\pi}{2}\right)\hat{x}+\frac{k_{0z}}{k_0}\sin\left(\frac{(\alpha+1)\pi}{2}\right)\hat{y}\right.\\
&\left.-\frac{k_y}{k_0}\sin\left(\frac{(\alpha+1)\pi}{2}\right)\hat{z}\right]\exp(ik_y y+ik_{0z}z)
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{2}\left[\hat{x}2\cos\left(\frac{(\alpha+1)\pi}{2}\right)-\hat{y}2\frac{k_{0z}}{k_0}\sin\left(\frac{(\alpha+1)\pi}{2}\right)\right.\\
&\left.-\hat{z}2\frac{k_y}{k_0}\sin\left(\frac{(\alpha+1)\pi}{2}\right)\right]\exp(ik_y y-ik_{0z}(z-2d_1)) \qquad (19)\\
\eta_0\mathbf{H}_{1fd} \;=\;& E^+\left[-2i\sin\left(\frac{(\alpha+1)\pi}{2}\right)\hat{x}-2\frac{k_z^+}{k^+}\sin\left(\frac{(\alpha+1)\pi}{2}\right)\hat{y}\right.\\
&\left.-2i\frac{k_y}{k^+}\cos\left(\frac{(\alpha+1)\pi}{2}\right)\hat{z}\right]\exp(ik_y y+ik_z^+ z)\\
&+E^-\left[-2i\sin\left(\frac{(\alpha+1)\pi}{2}\right)\hat{x}-2\frac{k_z^+}{k^+}\sin\left(\frac{(\alpha+1)\pi}{2}\right)\hat{y}\right.\\
&\left.-2i\frac{k_y}{k^+}\cos\left(\frac{(\alpha+1)\pi}{2}\right)\hat{z}\right]\exp(ik_y y-ik_z^+ z) \qquad (20)
\end{aligned}
$$

Changing values of α between zero and one, we can find behavior of fields inside intermediate geometries. $\alpha = 0$ and $\alpha = 1$ reproduces the PEC and PMC cases respectively. In next section Chiral nihility slab backed by PEMC characterized by admittance parameter M is considered. $M\eta \to \pm\infty$ and $M\eta \to 0$ reproduce PEC and PMC cases respectively.

4. CHIRAL NIHILITY SLAB BACKED BY PEMC

Here it assumed that slab of chiral nihility metamaterial is backed by perfect electromagnetic conductor (PEMC). PEMC is generalization of PEC and PMC and has been introduced by Lindell and Sihvola [17]. Chiral material and PEMC has been studied by many authors [18–27]. Fields given in Equations (1)–(4) can be assumed in regions inside and outside the slab. Unknown coefficients in field expressions (1)–(4) may be obtained using the related boundary conditions. At $z = d_2$, tangential components of field quantity $(M\mathbf{E}_1 + \mathbf{H}_1)$ must be zero [17]. Application of boundary condition to x and y components of electric field yields

$$E^{\pm} = -\left(\frac{M\eta + i}{M\eta - i}\right) F^{\mp} \tag{21}$$

Tangential components of electric and magnetic fields across the dielectric interface located at $z = d_1$ must be continues. Continuity of x-components and y-components of electric field yields

$$\begin{aligned}&\exp(ik_{0z}d_1) + (A^- + B^-)\exp(-ik_{0z}d_1)\\ =\ &\left(\frac{2i}{M\eta + i}\right)\left[E^+\exp(ik_z^+ d_1) + E^-\exp(-ik_z^+ d_1)\right]\end{aligned}$$

$$\begin{aligned}&(-A^- + B^-)\exp(-ik_{0z}d_1)\\ =\ &\frac{k_z^+ k_0}{k^+ k_{0z}}\left(\frac{2i}{M\eta + i}\right)\left[E^+\exp(ik_z^+ d_1) - E^-\exp(-ik_z^+ d_1)\right]\end{aligned}$$

Continuity of x-components and y-components of magnetic field yields

$$\begin{aligned}&(A^- - B^-)\exp(-ik_{0z}d_1)\\ =\ &\frac{\eta_0}{\eta}\left(\frac{2M\eta}{M\eta + i}\right)\left[E^+\exp(ik_z^+ d_1) + E^-\exp(-ik_z^+ d_1)\right]\end{aligned}$$

$$\begin{aligned}&\exp(ik_{0z}d_1) - (A^- + B^-)\exp(-ik_{0z}d_1)\\ =\ &\left(\frac{2M\eta}{M\eta + i}\right) R_f\left[E^+\exp(ik_z^+ d_1) - E^-\exp(-ik_z^+ d_1)\right]\end{aligned}$$

Solving above four equations simultaneously yields the unknown coefficients

$$E^- = -E^+\left(\frac{\eta^2 2iR_f + \eta_0^2 2M\eta}{-\eta^2 2iR_f + \eta_0^2 2M\eta}\right)\exp(2ik_z^+ d_1) \tag{22}$$

$$E^{+} = \frac{2\exp(ik_{0z}d_1 - ik_z^{+}d_1)}{P - QL} \tag{23}$$

where

$$P = \frac{2i + 2M\eta R_f}{M\eta + i}$$
$$Q = \left(\frac{\eta^2 2iR_f + \eta_0^2 2M\eta}{-\eta^2 2iR_f + \eta_0^2 2M\eta}\right)$$
$$L = \frac{2i - 2M\eta R_f}{M\eta + i}$$

It may be noted that under the limit $M\eta \to \pm\infty$, results derived in this section reduces to results derived in previous section for chiral nihility slab backed by PEC.

5. CONCLUSIONS

Both electric and magnetic fields exist inside the grounded chiral nihility slab when it is excited by a plane wave. Electric field inside the grounded chiral nihility slab disappears for excitation due to an electric line source. Magnetic field inside the slab disappears when geometry changes to corresponding dual geometry. Using fractional curl operator, fields are determined for geometries which may be regarded as intermediate step between the two dual geometries. Neither electric fields nor magnetic field disappears for fractional geometries either for plane wave or line source excitation. Using concept of fractional geometries, one can select appropriate geometry required regarding distribution of field and power inside the nihility slab.

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