Progress In Electromagnetics Research, PIER 84, 363–377, 2008

DETERMINISTIC BLIND BEAMFORMING FOR ELECTROMAGNETIC VECTOR SENSOR ARRAY

X. Zhang and D. Xu

Electronic Engineering Department
Nanjing University of Aeronautics & Astronautics
Nanjing 210016, China

Abstract—Deterministic blind beamforming algorithms try to separate superpositions of source signals impinging on electromagnetic vector sensor array by using deterministic properties of the signals. This paper links electromagnetic-vector-sensor-array beamforming problem to the parallel factor (PARAFAC) model, which is an analysis tool rooted in psychometrics and chemometrics. Exploiting this link, it derives a deterministic blind beamforming algorithm. The blind beamforming algorithm doesn't require DOA (direction of arrival) information and polarization information. The simulation results reveal that the performance of the blind beamforming algorithm for electromagnetic vector sensor array is close to nonblind MMSE method, and this algorithm works well in array error condition.

1. INTRODUCTION

Electromagnetic vector sensor arrays have some inherent advantages over traditional sensor arrays, since they have the capability of separating signals based on their polarization characteristics [1, 2], as well as spatial diversity. Electromagnetic vector sensor arrays are used widely in the communication, radio and navigation [3–6]. In the context of array signal processing, beamforming is concerned with the reconstruction of source signals from the outputs of a sensor array. Classically, beamforming [7–12] requires knowledge of a direction vector of the desired source. Maximum likelihood signal estimation method for electromagnetic vector sensor array *s* is proposed in [13]. Filtering performance of electromagnetic vector sensor array in completely polarized case is investigated in [14]. Acoustic vector sensor beamforming and direction estimation is investigated in [15]. The beamforming methods mentioned above are nonblind methods, since they require the knowledge of DOA [16] and polarization information. Blind beamforming tries to recover source signals without this information, relying instead on various structural properties, and blind beamforming is also regarded as blind source separation. Some blind beamforming algorithms use known space/time manifold structure (like ESPRIT [17]). More recently, new blind beamformers have been proposed that are not based on specific channel models, but instead exploit properties of the signals, e.g., finite alphabet, constant modulus, known pulse shape/spreading, or cyclostationarity [18–21]. Van der Veen [22] provides an excellent overview of algebraic methods for deterministic blind beamforming under sensor arrays. Sidiropoulos [23] has investigated identifiability issues in the context of deterministic blind beamforming for sensor array, and the viewpoint derives from the theory of low-rank decomposition of multiway arrays, known as Parallel factor (PARAFAC) analysis which was shown to be the core problem underlying deterministic blind beamforming. PARAFAC analysis has been first introduced as a data analysis tool in psychometrics, most of the research in the area is conducted in the context of chemometrics [24], spectrophotometric, chromatographic and flow injection analyses. Harshman [25] developed the PARAFAC model. At the same time, Caroll and Chang [26] introduced the canonical decomposition model, which is essentially identical to PARAFAC. In signal processing field, PARAFAC can be thought of as a generalization of ESPRIT and joint approximate diagonalization [27, 28].

Our work extends blind beamfoming for sensor array in [23] to blind beamforming for electromagnetic vector sensor array. This paper links the electromagnetic vector-sensor-array beamforming problem to the PARAFAC model and derives a deterministic blind beamforming whose performance is close to nonblind minimum mean-squared error (MMSE). The proposed algorithm supports small sample sizes, and even works well in array error condition. Most notably, it does not require knowledge of the DOA and polarization information. Instead, this algorithm relies on a fundamental result of Kruskal [29] regarding the uniqueness of low-rank three-way array decomposition. A prime advantage is that the blind beamformers are not dependent on channel properties or array calibration.

This paper is structured as follows. Section 2 develops data model. Section 3 discusses identifiability issues and deals with algorithmic issues. Section 4 presents simulation results, and Section 5 summarizes our conclusions.

2. THE DATA MODEL

Consider a uniform linear array consisting of M electromagnetic vector sensor, as shown in Fig. 1. Each electromagnetic vector sensor is triple-orthogonal dipoles. Each dipole in the array is a short dipole (In general, dipole length is shorter than half wavelength), so the output voltage from each dipole is proportional to the electric field component along that dipole. There are orthogonal short dipoles, the x-, y- and z-axis dipoles, parallel to the x, y and z axes, respectively. The lth dipole pair, $l = 1, 2, \ldots, M$, has its center on the y-axis at $y = (l-1)d$. The distance d between two adjacent dipole pairs is assumed to be a half wavelength to avoid angle ambiguity problems.

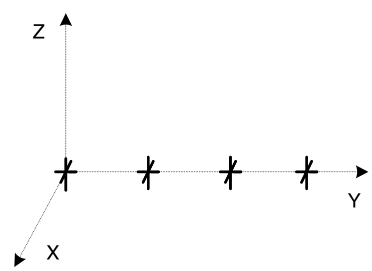

Figure 1. The structure of electromagnetic vector sensor array.

2.1. The Received Signal Model for Electromagnetic Vector Sensor

We begin by considering the polarization of an incoming signal. Suppose there is an antenna at the origin of a spherical coordinate system. Assume that a signal $b(t)$ is arriving from direction θ, φ, where φ is the elevation angle and θ is the azimuth angle. Let this signal be a transverse electromagnetic (TEM) wave, and consider the polarization ellipse produced by the electric field in a fixed transverse plane. Polarization parameters are γ, η. We characterize the antenna in terms of its response to linearly polarized signals in the x, and y directions. Let E_x, E_y and E_z be the complex voltage induced at the antenna output terminals by an incoming electromagnetic signal with a unit electric field polarized entirely in the x, y and z directions, respectively. The total output voltage and magnetic from this antenna in response to the electromagnetic signal is

$$y_o(t) = \begin{bmatrix} E_x \\ E_y \\ E_z \\ H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\varphi & -\sin\varphi \\ \cos\theta\sin\varphi & \cos\varphi \\ -\sin\theta & 0 \\ \sin\varphi & \cos\theta\cos\varphi \\ -\cos\varphi & \cos\theta\sin\varphi \\ 0 & -\sin\theta \end{bmatrix} \begin{bmatrix} \sin\gamma e^{j\eta} \\ \cos\gamma \end{bmatrix} b(t) = sb(t) \quad (1)$$

where $s = \begin{bmatrix} \cos\theta\cos\varphi & -\sin\varphi \\ \cos\theta\sin\varphi & \cos\varphi \\ -\sin\theta & 0 \\ \sin\varphi & \cos\theta\cos\varphi \\ -\cos\varphi & \cos\theta\sin\varphi \\ 0 & -\sin\theta \end{bmatrix} \begin{bmatrix} \sin\gamma e^{j\eta} \\ \cos\gamma \end{bmatrix}$ is the polarization vector, and it relates to polarization and DOA information.

2.2. The Received Signal Model for Electromagnetic Vector Sensor Array

Assume that a signal $b(t)$ arrives at the uniform linear array with M electromagnetic vector sensor. The received signal of the electromagnetic vector sensor array is shown as follows.

$$y(t) = \left[s^T, qs^T, \dots, q^{M-1}s^T\right]^T b(t) = (a \otimes s)b(t) \quad (2)$$

where $\otimes$ is Kronecker product. s is the polarization vector. $a = [1, q, \dots, q^{M-1}]^T$ is the direction vector, $q = e^{-j2\pi d \sin\theta/\lambda}$.

When K sources impinge the electromagnetic vector sensor array, the received signal at the output of the electromagnetic vector sensor array is

$$X = [a_1 \otimes s_1, a_2 \otimes s_2, \dots, a_K \otimes s_K]B^T = [A \circ S]B^T \quad (3)$$

where a_i and s_i are the direction vector and polarization vector of the ith source, respectively. $B = [b_1^T, b_2^T, \dots, b_K^T] \in \mathbb{R}^{N\times K}$ is the source matrix, where b_i the transmit signal of the ith source. $A \circ S$ is Khatri-Rao product. $A = [a_1, a_2, \dots, a_K] \in \mathbb{C}^{M\times K}$ is the direction matrix. $S = [s_1, s_2, \dots, s_K] \in \mathbb{C}^{6\times K}$ is the polarization matrix. Eq. (3) can be denoted as

$$X = \begin{bmatrix} X_{..1} \\ X_{..2} \\ \vdots \\ X_{..M} \end{bmatrix} = \begin{bmatrix} SD_1(A) \\ SD_2(A) \\ \vdots \\ SD_M(A) \end{bmatrix} B^T \quad (4)$$

where $D_m(.)$ is understood as an operator that extracts the mth row of its matrix argument and constructs a diagonal matrix out of it. $X_{..m}$ is

$$X_{..m} = SD_m(A)B^T, \quad m = 1, 2, \ldots, M \tag{5}$$

where $X_{..m}$ is the mth slice along spatial direction. In the presence of noise, the received signal model becomes

$$\tilde{X}_{..m} = X_{..m} + V_{..m} = SD_m(A)B^T + V_{..m}, \quad m = 1, 2, \ldots, M \tag{6}$$

where $V_{..m}$, the $6 \times N$ matrix, is the received noise corresponding to the mth slice.

The signal in (5) is also denoted through rearrangements

$$\begin{gathered} x_{m,n,p} = \sum_{f=1}^{K} a_{m,f} s_{n,f} b_{p,f}, \\ m = 1, \ldots, M; \quad n = 1, \ldots, N; \quad p = 1, 2, \ldots, 6 \end{gathered} \tag{7}$$

where $a_{m,f}$ stands for the (m, f) element of A matrix, and similarly for the others. The signal in (7) is well known as the trilinear mode, trilinear decomposition or PARAFAC analysis. Eq. (6) can be interpreted as slicing the 3-D data in a series of slices (2-D data) along the spatial direction. The symmetry of the trilinear model in (7) allows two more matrix system rearrangements, which can be interpreted as slicing the three-way data along different directions. In particular

$$Y_{..p} = BD_p(S)A^T, \quad p = 1, 2, \ldots, 6 \tag{8}$$

$$Z_{..n} = AD_n(B)S^T, \quad n = 1, 2, \ldots, N \tag{9}$$

where $Y_{..p}$ is the pth slice in polarization direction. $Z_{..n}$ is the nth slice in the temporal direction.

3. DETERMINISTIC BLIND BEAMFORMING FOR ELECTROMAGNETIC VECTOR SENSOR ARRAY

3.1. Trilinear Decomposition

TALS (Trilinear Alternating Least Square) algorithm is the common data detection method for trilinear model [29]. TALS algorithm is discussed in detail as follows.

According to (4), least squares fitting is

$$\min_{A,S,B} \left\| \begin{bmatrix} \tilde{X}_{..1} \\ \tilde{X}_{..2} \\ \vdots \\ \tilde{X}_{..M} \end{bmatrix} - \begin{bmatrix} SD_1(A) \\ SD_2(A) \\ \vdots \\ SD_M(A) \end{bmatrix} B^T \right\|_F \tag{10}$$

where $\|\|_F$ stands for the Frobenius norm. $\tilde{X}_{..m}$, $m = 1, 2, \ldots, M$, are the noisy slices. Least squares update for the source matrix B is

$$\hat{B}^T = \begin{bmatrix} \hat{S}D_1\left(\hat{A}\right) \\ \hat{S}D_2\left(\hat{A}\right) \\ \vdots \\ \hat{S}D_M\left(\hat{A}\right) \end{bmatrix}^+ \begin{bmatrix} \tilde{X}_{..1} \\ \tilde{X}_{..2} \\ \vdots \\ \tilde{X}_{..M} \end{bmatrix} \tag{11}$$

where $[.]^+$ stands for pseudo-inverse. $\hat{A}$ and $\hat{S}$ denote previously obtained estimates of A and S.

Similarly, from the second way of slicing the 3-D data: $Y_{..p} = BD_p(S)A^T$, $p = 1, 2, \ldots, 6$. According to symmetry of the trilinear model, the costing function in (10) is rewritten as follows

$$\min_{A,S,B} \left\| \begin{bmatrix} \tilde{Y}_{..1} \\ \tilde{Y}_{..2} \\ \vdots \\ \tilde{Y}_{..6} \end{bmatrix} - \begin{bmatrix} BD_1(S) \\ BD_2(S) \\ \vdots \\ BD_6(S) \end{bmatrix} A^T \right\|_F \tag{12}$$

And the conditional LS update for matrix A is

$$\hat{A}^T = \begin{bmatrix} \hat{B}D_1\left(\hat{S}\right) \\ \hat{B}D_2\left(\hat{S}\right) \\ \vdots \\ \hat{B}D_6\left(\hat{S}\right) \end{bmatrix}^+ \begin{bmatrix} \tilde{Y}_{..1} \\ \tilde{Y}_{..2} \\ \vdots \\ \tilde{Y}_{..6} \end{bmatrix} \tag{13}$$

Finally, from the third way of slices: $Z_{..n} = AD_n(B)S^T$, $n =$

$1, 2, \ldots, N$. And then LS update for matrix S is

$$\hat{S}^T = \begin{bmatrix} \hat{A}D_1\left(\hat{B}\right) \\ \hat{A}D_2\left(\hat{B}\right) \\ \vdots \\ \hat{A}D_N\left(\hat{B}\right) \end{bmatrix}^+ \begin{bmatrix} \tilde{Z}_{..1} \\ \tilde{Z}_{..2} \\ \vdots \\ \tilde{Z}_{..N} \end{bmatrix} \tag{14}$$

According to (11), (13) and (14), matrices B, A and S are updated with conditioned least squares, respectively. The matrix update will stop until convergence.

TALS is optimal when noise is additive i.i.d. Gaussian [30]. TALS algorithm has several advantages: it is easy to implement, guarantee to converge and simple to extend to higher order data. Sometime TALS algorithm can be stuck in local minima [30, 31]. In this paper, we use the COMFAC algorithm [32] for trilinear decomposition. COMFAC algorithm is essentially a fast implementation of TALS, and can speeds up the LS fitting.

Deterministic blind beamforming algorithm is proposed in this paper. This blind beamforming uses trilinear decomposition to get the source matrix, and we make decision for the estimated source matrix to implement the beamforming.

3.2. Identifiablity

Definition **1** [29]: Consider a matrix $A \in \mathbb{C}^{I\times J}$. If $\text{rank}(A) = r$, then A contains a collection of r linearly independent columns. Moreover, if every $l \leq J$ columns of A are linearly independent, but this does not hold for every $l + 1$ columns, then A has k-rank $k_A = l$. Note that $k_A \leq \text{rank}(A), \forall A$.

Theorem **1** [23]: $X_{..m} = SD_m(A)B^T$, $m = 1, 2, \ldots, M$, where $A \in \mathbb{C}^{M\times K}$, $S \in \mathbb{C}^{6\times K}$, $B \in \mathbb{R}^{N\times K}$, considering that A is a matrix with Vandermonde characteristic. If

$$k_S + \min(M + k_B, 2K) \geq 2K + 2 \tag{15}$$

then A, B and S are unique up to permutation and scaling of columns, that is to say, any other triple $\bar{A}, \bar{B}, \bar{S}$ that construct $X_{..m}$ $(m = 1, 2, \ldots, M)$ is related to A, B and S via

$$\bar{A} = A\Pi\Delta_1, \quad \bar{B} = B\Pi\Delta_2, \quad \bar{S} = S\Pi\Delta_3 \tag{16}$$

where Π is a permutation matrix, and $\Delta_1, \Delta_2, \Delta_3$ are diagonal scaling matrices satisfying $\Delta_1\Delta_2\Delta_3 = I$ and I is an identity matrix.

Scale ambiguity and permutation ambiguity are inherent to the separation problem. This is not a major concern. Permutation ambiguity can be resolved by resorting to a priori or embedded information. The scale ambiguity can be resolved using automatic gain control, normalization or embedded information.

In our present context, for source-wise independent source signals, $k_B = \min(N, K)$; for source-wise independent polarization, $k_S = \min(6, K)$, and therefore, (15) becomes

$$\min(6, K) + \min(M + \min(N, K), 2K) \geq 2K + 2 \qquad (17)$$

If matrix A in *Theorem***1** is not a vandermonde matrix, according to [23], the identifiable condition is

$$\min(6, K) + \min(N, K) + \min(M, K) \geq 2K + 2 \qquad (18)$$

4. SIMULATION RESULTS

According to (6), we define SNR

$$SNR = 10 \log_{10} \frac{\sum_{m=1}^{M} \left\| SD_m(A)B^T \right\|_F^2}{\sum_{m=1}^{M} \left\| V_{..m} \right\|_F^2} \, dB \qquad (19)$$

Uniform linear array containing 8 electromagnetic vector sensors is used in the simulation. Assume that each source only has one path to electromagnetic vector sensor array. We assume Binary Phase Shift Keying (BPSK) modulated signal and additive gauss white noise.

We present Monte Carlo simulations that are to assess the bit error rate (BER) performance of the proposed blind beamforming algorithm. The number of Monte Carlo trials is 1000. The blind beamforming algorithm doesn't require DOA information and polarization information. We compare our blind beamforming algorithm with the nonblind minimum mean-squared error (MMSE) receiver. MMSE receiver offers a performance bound against which blind algorithms are often measured [33]. MMSE receiver assumes perfect knowledge of DOA, SNR and polarization information.

Note that N is the number of snapshots. We use MATLAB software and computer with Pentium 4, CPU 3.2 GHz, memory 504 MB as simulation environment. For 1000 Monte Carlo trials with $N = 100$, SNR $= 0$ and 2 sources, PARAFAC method requires 178s, and MMSE method requires 28s. The PARAFAC method requires more processing time than MMSE method. The performances of these algorithms under different N are shown in Fig. 2–Fig. 5. For the simulation, the number of the sources is 3.

Figure 2 presents large sample results for $N = 400$, From Fig. 2, we find that the blind beamforming algorithm is very close to nonblind MMSE method.

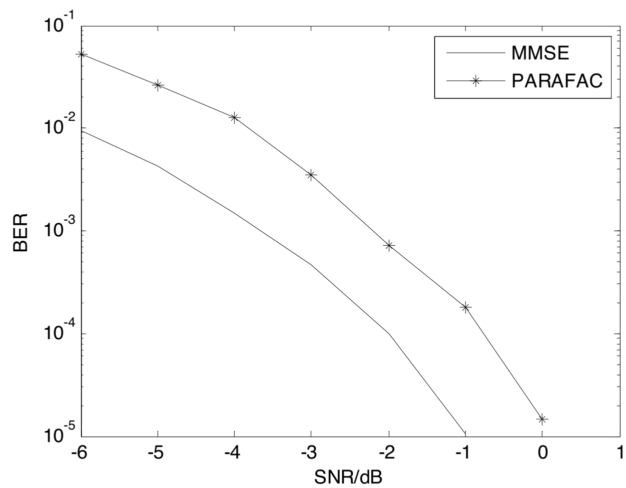

Figure 2. Algorithm performance comparison with $N = 400$.

Figure 3 and Fig. 4 depict results for $N = 200$ and 100, respectively. From Fig. 2 to Fig. 4 we find that the gap between the blind beamforming algorithm and (nonblind) MMSE increases as N decreases.

Figure 5 shows small sample results for $N = 30$. It is clear that the blind beamforming algorithm performs well even for small sample sizes.

The performance of the blind beamforming algorithm under different source number is investigated in the simulation. The source number is set 2, 3 and 4. The number of snapshots is 100. Fig. 6 shows the blind beamforming performance under different source numbers. From Fig. 6, we find that the blind beamforming performance of the proposed algorithm degrades with the increasing of the source number.

The blind beamforming algorithm performance in the array error condition is investigated. In this simulation, array error vector is

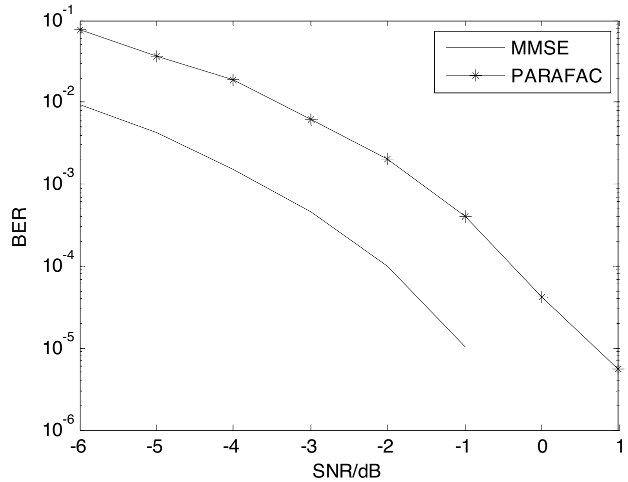

Figure 3. Algorithm performance comparison with $N = 200$.

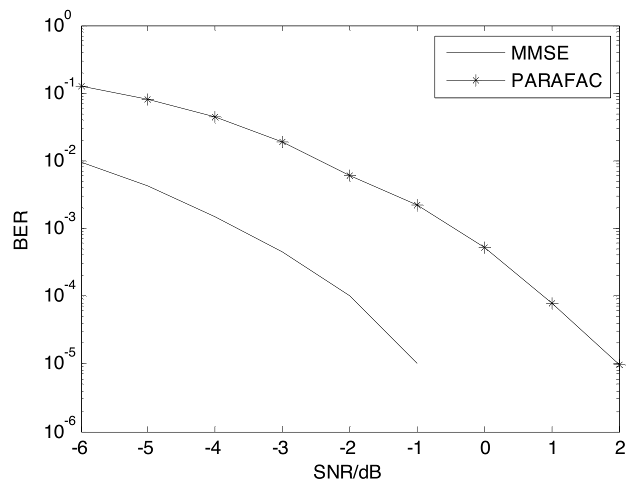

Figure 4. Algorithm performance comparison with $N = 100$.

the array gain and phase error vector. The array error vector $g = [1, 0.6071 - 0.4953i, 0.2083 + 0.7059i, 0.3497 - 0.7167i, 0.693 + 0.2916i, 0.4343 + 0.6883i, 0.330 + 0.6894i, 0.6678 - 0.5133i]$. Assume that array response vector for DOA $= \theta$ is $a(\theta)$, and then the array response vector with array error is $\text{diag}(g)a(\theta)$. The direction matrix with array error is not vandermonde matrix, and then the identifiable

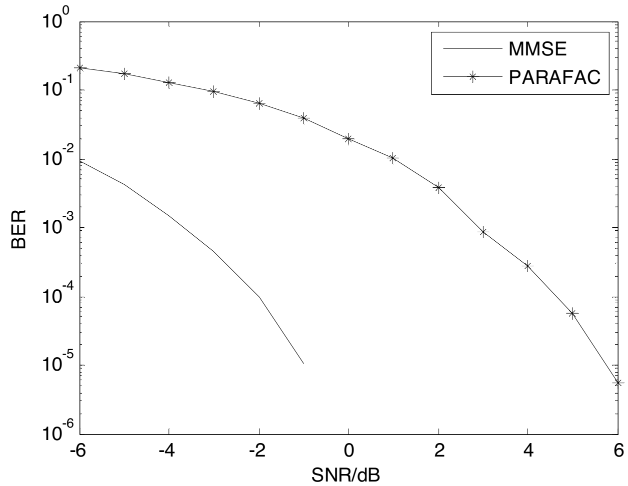

Figure 5. Algorithm performance comparison with $N = 30$.

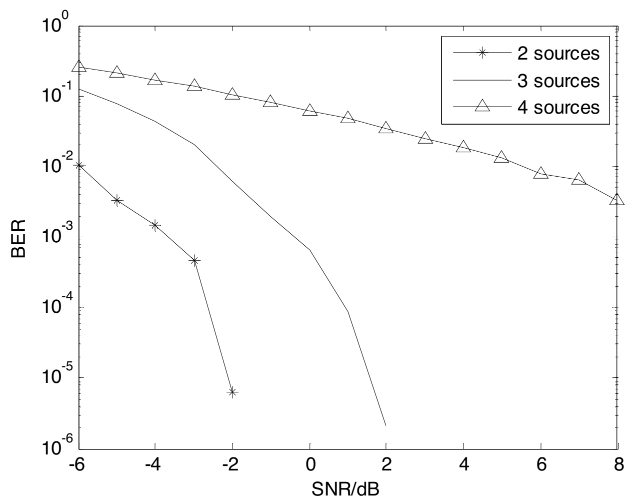

Figure 6. Algorithm performance comparison with different sources.

condition is shown in (18). The performance of the blind beamforming algorithm in array error condition is shown in Fig. 7. Fig. 7 shows that the blind beamforming algorithm has the better performance in the array error condition. The blind beamforming algorithm has robust characteristics to array error. When there exists mutual coupling between the elements, the direction matrix A is not a Vandermonde

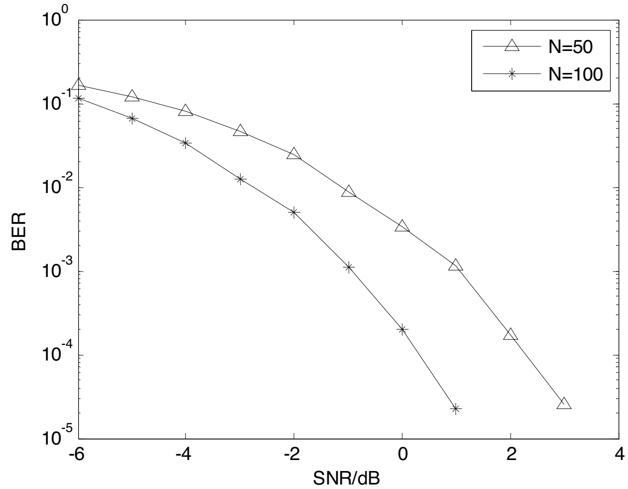

Figure 7. Algorithm performance with array error.

matrix. If satisfying the identifiable condition in (18), our proposed algorithm still works well in this condition.

5. CONCLUSIONS

This paper has developed a link between PARAFAC analysis and blind beamforming for electromagnetic vector sensor arrays. Relying on the uniqueness of low-rank three-way array decomposition and trilinear decomposition, a deterministic blind beamforming algorithm has been proposed. The algorithm doesn't require DOA information and polarization information. The simulation results reveal that the performance of the blind beamforming algorithm for electromagnetic vector sensor array is close to nonblind MMSE method, and the algorithm works well in array error condition and supports small sample sizes.

ACKNOWLEDGMENT

This work is supported by China NSF Grants (60801052) and Jiangsu NSF Grants (BK2007192). The authors wish to thank the anonymous reviewers for their valuable suggestions on improving this paper.

REFERENCES

1. Costanzo, S. and G. Dimassa, "Direct far-field computation from bi-polar near-field samples," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 9, 1137–1148, 2006.
2. Bao, X. L. and M. J. Ammann, "Comparison of several novel annular-ring microstrip patch antennas for circular polarization," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 11, 1427–1438, 2006.
3. Zhang, M.-T., Y.-B. Chen, Y.-C. Jiao, et al., "Dual circularly polarized antenna of compact structure for radar application," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 14, 1895–1902, 2006.
4. Panagopoulos, A. D. and G. E. Chatzaraki, "Outage performance of single/dual polarized fixed wireless access links in heavy rain climatic regions," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 3, 283–297, 2007.
5. Ng, J. W. P. and A. Manikas, "Polarisation-sensitive array in blind MIMO CDMA system," *Electronics Letters*, Vol. 41, No. 17, 49–50, August 18, 2005.
6. Ioannis, K. and G. B. Keith, "Base station polarization-sensitive adaptive antenna for mobile radio," *Third Annual International Conference on Universal Personal Communications*, 230–235, 1994.
7. Dessouky, M. I., H. A. Sharshar, and Y. A. Albagory, "A novel tapered beamforming window for uniform concentric circular arrays," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 14, 2077–2089, 2006.
8. Guo, B., Y. Wang, J. Li, et al., "Microwave imaging via adaptive beamforming methods for breast cancer detection," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 1, 53–63, 2006.
9. Jeong, Y.-S. and J.-H. Lee, "Estimation of time delay using conventional beamforming-based algorithm for UWB systems," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 15, 2413–2420, 2007.
10. Dessouky, M. I., H. A. Sharshar, and Y. A. Albagory, "Improving the cellular coverage from a high altitude platform by novel tapered beamforming technique," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 13, 1721–1731, 2007.
11. Babayigit, B., A. Akdagli, and K. Guney, "A clonal selection algorithm for null synthesizing of linear antenna arrays by

amplitude control," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 8, 1007–1020, 2006.

12. Naghshvarian-Jahromi, M., "Novel Ku band fan beam reflector back array antenna," *Progress In Electromagnetics Research Letters*, Vol. 3, 95–103, 2008.
13. Weiss, A. J. and B. Friedlander, "Maximum likelihood signal estimation for polarization sensitive arrays," *IEEE Transactions on Antennas and Propagation*, Vol. 41, No. 7, 918–925, July 1993.
14. Xu, Z., X. Wang, S. Xiao, et al., "Filtering performance of polarization sensitive array: Completely polarized case," *Acta Electronica Sinica*, Vol. 32, No. 8, 1310–1313, 2004.
15. Hawkes, M. and A. Nehorai, "Acoustic vector-sensor beamforming and capon direction estimation," *IEEE Transactions on Signal Processing*, Vol. 46, No. 9, 2291–2304, September 1998.
16. Landesa, L., I. T. Castro, J. M. Taboada, and F. Obelleiro, "Bias of the maximum likelihood doa estimation from inaccurate knowledge of the antenna array response," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 9, 1205–1217, 2007.
17. Roy, R. and T. Kailath, "ESPRIT — Estimation of signal parameters via rotational invariance techniques," *IEEE Trans. Acoust., Speech, Signal Process.*, Vol. 37, 984–995, July 1989.
18. Agee, B. G., S. V. Schell, and W. A. Gardner, "Spectral self-coherence restoral: A new approach to blind adaptive signal extraction using antenna arrays," *Proc. IEEE*, Vol. 78, 753–767, April 1990.
19. Anand, K., G. Mathew, and V. U. Reddy, "Blind separation of multiple co-channel BPSK signals arriving at an antenna array," *IEEE Signal Processing Lett.*, Vol. 2, 176–178, September 1995.
20. Shynk, J. J. and R. P. Gooch, "The constant modulus array for cochannel signal copy and direction finding," *IEEE Transactions on Signal Processing*, Vol. 44, 652–660, March 1996.
21. Talwar, S., M. Viberg, and A. Paulraj, "Blind separation of synchronous co-channel digital signals using an antenna array — Part I: Algorithms," *IEEE Transactions on Signal Processing*, Vol. 44, 1184–1197, May 1996.
22. Van der Veen, A.-J., "Algebraic methods for deterministic blind beamforming," *Proc. IEEE*, Vol. 86, 1987–2008, October 1998.
23. Sidiropoulos, N. D. and X. Liu, "Identifiability results for blind beamforming in incoherent multipath with small delay spread," *IEEE Transactions on Signal Processing*, Vol. 49, No. 1, 228–236, January 2001.

24. Smilde, A., R. Bro, and P. Geladi, *Multi-way Analysis: Applications in the Chemical Sciences*, John Wiley and Sons, Chichester, U.K., 2004.
25. Harshman, R. A., "Foundations of the PARAFAC procedure: Model and conditions for an 'explanatory' multi-mode factor analysis," *UCLA Working Papers Phonetics*, Vol. 16, 1–84, December 1970.
26. Carroll, J. D. and J. Chang, "Analysis of individual differences in multidimensional scaling via an N-way generalization of 'Eckart-Young' decomposition," *Psychometrika*, Vol. 35, No. 3, 283–319, 1970.
27. De Lathauwer, L., B. De Moor, and J. Vandewalle, "Computation of the canonical decomposition by means of a simultaneous generalized Schur decompositition," *SIAM J. Matrix Anal. Appl.*, Vol. 26, No. 2, 295–327, 2004.
28. De Lathauwer, L., "A link between the canonical decomposition in multilinear algebra and simultaneous matrix diagonalization," *SIAM J. Matrix Anal. Appl.*, Vol. 28, No. 3, 642–666, 2006.
29. Kruskal, J. B., "Three-way arrays: Rank and uniqueness of trilinear decompositions, with application to arithmetic complexity and statistics," *Linear Algebra Applicat.*, Vol. 18, 95–138, 1977.
30. Vorobyov, S. A., Y. Rong, N. D. Sidiropoulos, and A. B. Gershman, "Robust iterative fitting of multilinear models," *IEEE Transactions on Signal Processing*, Vol. 53, No. 8, 2678–2689, 2005.
31. Tomasi, G. and R. Bro, "A comparison of algorithms for fitting the PARAFAC model," *Computational Statistics & Data Analysis*, Vol. 50, 1700–1734, 2006.
32. Bro, R., N. D. Sidiropoulos, and G. B. Giannakis, "A fast least squares algorithm for separating trilinear mixtures," *Int. Workshop Independent Component and Blind Signal Separation Anal.*, Aussois, France, January 11–15, 1999.
33. Tsatsanis, M. K. and Z. Xu, "Performance analysis of minimum variance CDMA receivers," *IEEE Transactions on Signal Processing*, Vol. 46, 3014–3022, November 1998.