## EXPERIMENTAL TECHNIQUE USING AN INTERFERENCE PATTERN FOR MEASURING DIRECTIONAL FLUCTUATIONS OF A LASER BEAM CREATED BY A STRONG THERMAL TURBULENCE

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Abstract—In the view of measuring directional fluctuations of a thin laser beam sent through a heated turbulent jet, an optical method using interference and diffraction with the out coming beam is proposed. The experimental set-up is described. A new technique for separating directional fluctuations of the laser beam is explained. From the measurement of the interference pattern perturbations, are deduced the Rms of the laser beam deflection angle, the spectrum of directional fluctuations of the laser beam, and the value of a scattering coefficient characterizing the heated turbulent jet. The measured spectrum reveals a -8/3 power law and the value obtained for that coefficient is nearly equal to that found in previous works. This agreement enables to conclude that the experimental technique used is efficient and satisfactory.

## 1. INTRODUCTION

The behaviour of optical wave propagating through inhomogeneous or random media has been the subject of many theoretical investigations [1–11]. It continues to be of a great interest because of their various technological applications, notably in the domains of engineering [12, 13], medical diagnostic [14–17], electromagnetic interactions with biologic systems [18, 19], remote sensing [20–22], radars [23–25], satellite communications [26], and defence laser weapons [27, 28].

We consider in this paper, a laser beam which propagates through a strong thermal turbulence created from a hot wind tunnel jet. It is well known that the laser beam propagating in those conditions is randomly perturbed; more precisely, the phase, intensity and direction of the laser wave undergo turbulent fluctuations due to random refractive index fluctuations of the medium, which are mainly caused by temperature fluctuations. We assume that the pressure variations in the hot jet spread at very high speed such that the pressure is nearly uniform for two neighbouring points in the jet. The validity of this hypothesis can be verified by founding that the direction of propagation of the laser beam through the turbulent jet remains nearly rectilinear when the jet is not heated.

Many experimental investigations [29–32] have been carried out in order to enable a best understanding of the laser beam behaviour when it propagates through a heated turbulent medium. In this paper, we have focused our interest on the study of directional fluctuations of a thin laser beam. The purpose of this work is to conceive an optical method based on the interference pattern perturbations for measuring those fluctuations. The paper is organized as follows: In Section 2, are described the experimental set-up and the unperturbed interference pattern. Perturbations of the light intensity curve in this pattern are studied in Section 3. In Section 4, we finally describe the measurement technique for separating directional fluctuations of the laser beam, present the results obtained, and make comparisons with previously published data.

#### 2. EXPERIMENTAL SET-UP

### 2.1. Characteristics of the Heated Turbulent Jet

A heated turbulent jet of air is issued from a rectangular nozzle of a wind tunnel already used in [7,8]. The ambient medium is at rest. The mean velocity and mean temperature along the direction of interest, that is, the direction parallel to the nozzle exhaust, are plotted in Figure 1(a). The mean velocity curve shows that the total width of the jet along this direction is X = 200 mm. The values of temperature Rms and refractive index Rms, along the same direction are plotted in Figure 1(b). The above measurements are carried out when the jet is fully developed and the results obtained are similar to those found in [7].

#### 2.2. Optical Set-up

The set-up is presented in Figure 2. The experimental device includes two ordinary lenses  $L_1$  and  $L_2$  (focal distances  $F_1 = F_2 = 100 \text{ mm}$ ), two objective lenses  $l_1$  and  $l_2$  (focal distances  $f_1 = 17 \text{ mm}$ ,  $f_2 = 4.7 \text{ mm}$ ),



Figure 1. Characteristics of the heated turbulent jet along the propagation direction of the laser beam. (a) Mean velocities and mean temperatures. (b) Rms of temperature fluctuations and Rms of refractive index fluctuations.

two Young holes  $P_1$  and  $P_2$ , a photomultiplier (PM), a photodiode, a thin laser beam with initial diameter 0.8 mm, created from a 5 mW continuous wave He-Ne laser (wavelength = 6328 Å). The optical system is built such that the image focus of the objective  $l_1$  and that of the lens  $L_1$  coincide respectively with the object focus of the lens  $L_1$  and that of the lens  $L_2$ . The Young holes are placed on the image



**Figure 2.** Experimental set up (1), (4) Drawing tables; (2) Spectrum analysis device; (3) Oscilloscope.

focus plane of the lens  $L_1$ .

The laser beam is sent parallel to the nozzle exhaust at a distance  $L = 200 \,\mathrm{mm}$  from the nozzle aperture. After having traversed the hot turbulent jet, it is expanded by a first afocal system (lenses  $L_1$ ,  $l_1$ ) which enables to illuminate the two Young holes, situated on a vertical screen, symmetrically to the optical axis of the system. This axis coincides with the direction of the unperturbed laser beam, that is, the direction of the laser beam before entering into the jet. A second nearly afocal system  $(L_2, l_2)$  enables to project the obtained interference pattern on the plane (E') where the diaphragm of a photo detector is placed. Unlike the optical set-up previously described in [7], the experimental device used in this paper includes an acquisition system of data, connected to a detector for measurements of directional fluctuations of the laser beam, by means of a suitable treatment of the light signal transmitted. All studied frequencies are situated within the linear response range of the photo detector and it has a linear response up to a frequency of 1 GHz. The digitized data are stored on magnetic disk for subsequent analysis.

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For improving the sensitivity of the photomultiplier and reducing its electronic noise, the electrical tension between its cathode (C) and anode (A) is chosen to be equal to 1500 V; the electrical resistance and capacitor connecting to C and A are respectively equal to  $30 \times 10^5 \Omega$ and  $10^5 \text{ pF}$ .

### 2.3. The Unperturbed Interference Pattern

If A denotes the constant amplitude of the plane laser wave in the plane (E) of the Young holes, 2a the diameter of each hole, d the distance between the centres of the holes,  $\lambda$  the laser radiation wavelength, it is shown [33] that the amplitude of the light wave in the interference pattern situated at the  $(x_2, y_2)$  image focus plane of the lens  $L_2$ , is given by the relation:

$$F'(x_2, y_2) = 4\pi A a^2 \left(\frac{J_1(z_2)}{z_2}\right) \cos\left(\frac{\pi d}{\lambda F_2} x_2\right)$$
(1)

where  $z_2 = 2\pi \frac{a\sqrt{x_2^2 + y_2^2}}{\lambda F_2}$  and  $J_1$  is the Bessel function of 1st order. Knowing that the second system is afocal and that the (x, y)

Knowing that the second system is afocal and that the (x, y) observation plane is situated at a distance D (D = 250 cm) from the objective  $l_2$ , the distribution law F(x, y) of light amplitude in the (x, y) plane is given by the relation (1) if  $x_2$  and  $y_2$  are replaced by  $x_2 = f_2 x/D$  and  $y_2 = f_2 y/D$ . So, the distribution law of the intensity of light in the interference pattern observed at the plane of the photo detector diaphragm can be deduced:

$$I(x, y) = 16\pi^2 A^2 a^4 \left(\frac{J_1(z)}{z}\right)^2 \cos^2\left(\frac{\pi f_2 d}{\lambda F_2 D}x\right)$$
(2)

with  $z = 2\pi \frac{af_2\sqrt{x^2+y^2}}{\lambda F_2 D}$ .

We have focused our interest on the central diffraction spot because this spot is sufficient for determining all geometrical and optical properties of the interference pattern. The width of this spot being  $2X_0$  where  $X_0$  is the first zero of the Bessel function  $J_1$ , it contains n brilliant fringes if  $X_0/x_0 = n$ , where  $x_0$  is the first zero of the function  $\cos^2$ . This leads to the relation:  $a/d = 3.832/n\pi$ . It is well known that the fringes are all the more large and illuminated as both distances d and a are small. The smallest radius we have been able to build for the Young holes is a = 1 mm and the distance between the holes is d = 3.75 mm. So, the experimental value of the ratio a/dis equal to 0.266. This value must be compared to the theoretical ratio 0.244 which corresponds to five brilliant fringes contained in the central diffraction spot and having same width = 9 mm.

In the absence of the turbulent jet and at the ambient temperature, we have obtained five brilliant vertical fringes within the central diffraction spot, having same width nearly equal to 9 mm, as it is shown in Figure 3(a). This curve is plotted by performing a continuous transversal exploration of the interference pattern plane by means of the photomultiplier; it must be compared to the predicted curve (Figure 3(b)) corresponding to experimental data and obtained from Equation (2). Both curves present the same characteristics but the uncertainty latitude of the results given by the photomultiplier cannot enable to highlight the two brilliant extreme fringes because their light intensity is too small. The minimum and maximum of the transmitted electrical tension are  $V_{\rm min} = 0.05 \,\mathrm{mV}$  and  $V_{\rm max} = 1.60 \,\mathrm{mV}$ ; the visibility coefficient  $\gamma$  of the fringes is then equal to:

$$\gamma = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}} = 0.94$$
(3)

## 3. ABOUT THE INTERFERENCE PATTERN PERTURBATIONS CREATED BY THE HEATED TURBULENT JET

After having obtained the unperturbed interference pattern described in the previous section, we put the wind tunnel running and wait for the fully developed turbulence. The laser beam then traverses the hot turbulent jet before reaching the optical system. The refractive index fluctuations in the jet caused by temperature fluctuations create random fluctuations of the laser beam direction. Those index fluctuations also create phase fluctuations in the laser beam wave front and then, induce intensity fluctuations of light wave by diffraction process.

Hence, the interference pattern is perturbed by two kinds of fluctuations. First, the laser beam intensity fluctuations which are caused by amplitude fluctuations and phase fluctuations, and which randomly modify the light intensity curve in the interference pattern. Second, the laser beam wandering with respect to the optical axis causes a global random displacement of fringes along random directions. Assuming that those perturbations remain equally distributed around the optical axis, the unperturbed interference pattern can be regarded as the mean state of the interference pattern around which perturbations occur. Let us study each kind of perturbation.



Figure 3. Distribution law of light intensity in the unperturbed interference pattern (a) Measurements (b) Predicted curve using experimental data.

### 3.1. Relationship between the Interference Pattern Perturbations and the Laser Beam Characteristics

At the outlet of the heated turbulent jet, the laser beam intensity is defined to be proportional to  $|A_{urb}|^2 = A_{turb}A^*_{turb}$  where  $A_{turb}$  denotes the turbulent complex amplitude of the laser beam. This intensity takes into account the diffraction process occurring in the laser beam wave front and due to phase fluctuations  $\varphi = 2\pi\mu X/\lambda$ , where  $\mu$  is the refractive index fluctuations, and X the total width jet  $\approx 200 \text{ mm}$ . So,  $I_{turb}$  can be written as follows:

$$I_{turb} = I_0 + \delta I_{turb}(i_\alpha, i_\varphi) \tag{4}$$

where  $I_0$  is the light intensity of the unperturbed incident laser beam, and  $\delta I_{turb}$  denotes light intensity fluctuations of the laser beam measured with respect to  $I_0$  at the outlet of the jet. The quantity  $\delta I_{turb}$  depends on fluctuations  $i_{\alpha}$  and  $i_{\varphi}$  respectively due to amplitude fluctuations  $\alpha$  and phase fluctuations  $\varphi$ .

In addition, the laser beam assumed to remain parallel, undergoes a random deflection angle  $\varepsilon$  at the outlet of the jet which induces at the plane of the Young holes, a random angle  $\beta$  between the optical axis and the laser beam such that:

$$\beta = \frac{f_1}{F_1}\varepsilon\tag{5}$$

Let us suppose that  $\beta$  is defined in the vertical plane. This creates a phase difference  $\psi = 2\pi\beta d/\lambda$  between the light rays entering into the holes. By applying the Fourier transform of the distribution law of light amplitude in the plane of the holes, and by using the considerations already explained in Section 2.3, one obtains the distribution law of light amplitude in the (x, y) image focus plane of objective  $l_2$ , that is:

$$F_p(x,y) = 2A_{turb}S\left(\frac{J_1(z)}{z}\right)\left(\exp\left(-j\frac{\pi f_2 d}{\lambda F_2 D}x\right) + \exp(-j\psi)\exp\left(j\frac{\pi f_2 d}{\lambda F_2 D}x\right)\right)$$
(6)

where S is the hole surface =  $a^2\pi$ , and  $z = 2\pi \frac{af_2\sqrt{x^2+y^2}}{\lambda F_2 D}$ .

By using the amplitude distribution law given in Equation (6) and by setting  $I_{turb} = 16|A_{turb}|^2 S^2$ , the distribution law of light intensity  $I_p(x, y)$  in the perturbed interference pattern can be written as follows.

$$I_p(x,y) = I_{turb} \left(\frac{J_1(z)}{z}\right)^2 \cos^2\left(\frac{\pi f_2 d}{\lambda F_2 D} x - \frac{\psi}{2}\right) \tag{7}$$

From Equation (7), one concludes that the perturbed interference pattern can be regarded as an interference pattern created by two Young holes, in which the maximum of light is a random distribution  $(I_p)_{\text{max}}$  and the brilliant central fringe undergoes a random displacement  $\delta$  such that:

$$(I_p)_{\max} = I_{turb} \left(\frac{J_1(z)}{z}\right)^2 = (I_0 + \delta I_{turb}) \left(\frac{J_1(z)}{z}\right)^2$$
 (8a)

$$\delta = \psi \left( \frac{\lambda F_2 D}{2\pi f_2 d} \right) = \left( \frac{f_1}{F_1} \right) \left( \frac{F_2}{f_2} \right) D\varepsilon$$
(8b)

In addition, Equation (7) shows that the intensity of light  $I_p$  in the perturbed interference pattern observed by the detector can be regarded as the superposition of three quantities, written as follows:

$$I_p = I + \delta I_{intensity} + \delta I_{direction} \tag{9}$$

where I is the intensity of light in the unperturbed interference pattern given in Equation (2);  $\delta I_{intensity}$  and  $\delta I_{direction}$  are the two kinds of interference pattern perturbations respectively caused by intensity fluctuations and directional fluctuations of the laser beam, and given by:

$$\delta I_{intensity} = \delta I_{turb}(i_{\alpha}, i_{\varphi}) \left(\frac{J_1(z)}{z}\right)^2 \cos^2\left(\frac{\pi f_2 d}{\lambda F_2 D} x - \frac{\pi f_1 d}{\lambda F_1}\varepsilon\right)$$
(10a)

$$\delta I_{direction} = I_0 \left(\frac{J_1(z)}{z}\right)^2 \left(\cos^2\left(\frac{\pi f_2 d}{\lambda F_2 D} x - \frac{\pi f_1 d}{\lambda F_1} \varepsilon\right) - \cos^2\left(\frac{\pi f_2 d}{\lambda F_2 D} x\right)\right)$$
(10b)

# **3.2.** Interference Pattern Perturbations Due to Laser Beam Intensity Fluctuations

The positions of the points corresponding to the minimum or maximum of light intensity in the interference pattern are deduced from Equation (2) and are given by:

$$x_{k} = \left(\frac{2k+1}{2}\right) \frac{\lambda F_{2}D}{f_{2}d}, \quad x'_{k} = k \frac{\lambda F_{2}D}{f_{2}d}$$
(11)  

$$k = 0, 1, 2, \dots$$

Relations (11) prove that these positions depend only on the characteristics of the optical system  $(\lambda, F_2, f_2, D, d)$ ; so, they do not change in spite of interference pattern perturbations due to laser beam intensity fluctuations.



Figure 4. Amplitude scheme of the light intensity perturbations considered at three points (A, B, C) of the brilliant central fringe and caused by fluctuations of the laser beam intensity ((a) unperturbed curve). These perturbations occur between two similar curves  $I = I_{\max}(x)$  (curve (b)) and  $I = I_{\min}(x)$  (curve (c)), and the condition  $\Delta V_A > \Delta V_B > \Delta V_C$  is then satisfied.

Let us focus our interest on the brilliant central fringe situated in the central diffraction spot of the interference pattern. In Figure 4, we have illustrated the predicted configuration of the perturbed light intensity curve. In this figure, we show that perturbations of the interference pattern due to laser beam intensity fluctuations act between two curves ( $I = I_{\max}(x)$  and  $I = I_{\min}(x)$ ) which are similar and correspond to the maximal and minimal values of light intensity. By studying three different positions A, B, and C of the detector on the central fringe (see Figure 4), we can conclude that, if the detector is placed at the position corresponding to the maximum of the intensity curve (position A), the fluctuations of the received light intensity are stronger than those measured at any point of central fringe.

# **3.3.** Interference Pattern Perturbations Due to Fluctuations of the Laser Beam Direction

The interference pattern perturbations caused by the fluctuations of the laser beam direction exhibit an opposite behaviour, compared to those created by the laser beam intensity fluctuations. For given



Figure 5. Amplitude scheme of the light intensity perturbations considered at three points (A, B, C) of the brilliant central fringe and caused by the same displacement  $\Delta x$  of fringes. These perturbations are created by directional fluctuations of the laser beam and satisfy the condition  $\Delta V_A < \Delta V_C < \Delta V_B$ 

directions along the x axis having fluctuations of equal Rms, it is shown (Figure 5) that those perturbations are very small and can be neglected, if the detector is placed at point A; they are very strong at B and cannot be neglected at C. We have verified that the above results remain unchanged for displacements taken along the y axis. For this, we have placed the axis joining the holes in the horizontal direction in order to obtain horizontal fringes, that is, to create displacements of the fringes along the y axis.

### 4. MEASUREMENTS

## 4.1. Measurement of Interference Pattern Perturbations Caused by Total Fluctuations of the Laser Beam (Intensity + Direction)

Measurements of the Rms of light intensity fluctuations have been carried out along the axis perpendicular to fringes and are plotted in Figure 6, for the brilliant central fringe of the central diffraction spot. In the same figure, one can make comparisons between those measurements and the mean values of light intensity. It can be concluded that those Rms are very small when the light intensity is small, and then, they can be neglected at both borders of the central



**Figure 6.** Rms of total fluctuations (amplitude + phase + direction) measured in the brilliant central fringe of the interference pattern and compared to the mean light intensity (a) Mean light intensity (b) Rms of total fluctuations.

brilliant fringe. They increase at the central zone of that fringe and reach their maximal value at point B corresponding to the half value of the maximum of light intensity.

# 4.2. Measurements of the Spectrum of Directional Fluctuations of the Laser Beam

For measuring the spectrum of directional fluctuations of the laser beam, we use the perturbed interference pattern, by placing the detector at point B, according to considerations detailed in Section 3. We have taken many precautions in order to ensure that there is no additional cause for the perturbations of the electrical tension transmitted by the detector. The fluctuations of light intensity in the perturbed interference pattern are normalized and defined as:

$$i(x,y) = \frac{I_p(x,y) - I(x,y)}{I(x,y)} = \frac{\delta I_{intensity} + \delta I_{direction}}{I}$$
(12)

The quantity i is also equal to the fluctuations of the logarithm of I.

The measured temporal spectra of the normalized directional fluctuations of the laser beam can be deduced from the spectra of i, and are presented in Figure 7, for two values of the mean velocity of



Figure 7. Spectrum of the normalized directional fluctuations of the laser beam for two values of the mean jet velocity, the photo detector being placed at point B in the interference pattern: (a)  $\overline{U} = 8 \text{ m/s}$ ; (b)  $\overline{U} = 12.5 \text{ m/s}$ . Increasing the mean velocity causes a displacement of the spectrum curve towards high frequencies.

the turbulent jet. ( $\overline{U} = 8 \text{ m/s}$  (a),  $\overline{U} = 12.5 \text{ m/s}$  (b)). Figure 7 reveals a well defined range for high frequencies described by the -8/3 power law.

# 4.3. RMS Measurement of the Displacements of the Brilliant Central Fringe

The distribution functions of light intensity plotted for the brilliant central fringe in Figures 3 and 6 reveal the existence of a "linear variation zone" situated on both sides of the inflexion point B, such that within that zone, the light intensity variations  $\Delta I$  are proportional to displacement variations  $\Delta x$  along the Ox axis. This hypothesis of linearity holds if  $|\Delta x| < x_0/2$ ,  $x_0$  being the first zero of the function  $\cos^2$  defined in Equation (2), that is, the half width of the fringes given by:  $x_0 = \lambda F_2 D/2f_2 d$ . Knowing that the perturbations of the displacement of the brilliant central fringe remain very small, we expect that the above condition is verified.

Consequently, for measuring a random displacement  $\delta_x$  of the

brilliant central fringe which causes a small variation  $\delta I$  of light intensity, we place the detector at B. Also, using relation (2), we can write the following equation:

$$\frac{\delta I}{\delta_x} = \left(\frac{dI}{dx}\right)_B = \left(\frac{dI}{dx}\right), \quad \text{for} \quad x = \frac{x_0}{2} \tag{13}$$

Assuming that the small quantity  $\delta I$  can be regarded as a light intensity fluctuation  $i_x$  defined around the light intensity curve of the unperturbed interference pattern, relation (6) leads to the following result:

$$\delta_x = -\left(\frac{\lambda F_2 D}{\pi f_2 d}\right) \frac{i_x}{I_m},\tag{14}$$

with  $I_m = 16\pi^2 a^4 A^2 \left(\frac{J_1(z)}{z}\right)_{z=0} = \text{maximum value of light intensity.}$ The real displacement  $\delta$  of the brilliant central fringe occurring

The real displacement  $\delta$  of the brilliant central fringe occurring along any direction, we must take into account the other displacement component  $\delta_y$  defined along the y axis, such that:  $\delta^2 = \delta_x^2 + \delta_y^2$ . For this, we have to turn the axis joining the Young holes such that it becomes horizontal; then, the new fringes obtained are horizontal and relation (14) remains valid for the displacement  $\delta_y$  corresponding to the light intensity fluctuation  $i_y$ . So, we obtain the following formula which enables to deduce the variance of the displacement of the brilliant central fringe from the variance measurements of the electrical tension fluctuations  $v_x = V_x - V_0$  and  $v_y = V_y - V_0$ ;  $V_x$  and  $V_y$  are the transmitted turbulent electrical signal, and  $V_0 = V_m/2$  is the value corresponding to the unperturbed interference pattern.

$$\overline{\delta^2} = \frac{1}{4} \left( \frac{\lambda F_2 D}{\pi f_2 d} \right)^2 \frac{\left( \overline{v_x^2} + \overline{v_y^2} \right)}{V_0^2} \tag{15}$$

We have measured the following Rms:  $\sqrt{v_x^2} = 1.65 \times 10^{-1} \,\mathrm{mV}$  and  $\sqrt{v_y^2} = 1.65 \times 10^{-1} \,\mathrm{mV}$ . This leads to the following result for the Rms of the displacement of the brilliant central fringe:  $\sqrt{\delta^2} = 0.41 \,\mathrm{mm}$ .

# 4.4. Comparison of the Obtained Results with the Published Data

In the previous work carried out under the same experimental conditions, Ngo Nyobe and Pemha [8] have measured a coefficient related to the light diffusion of the heated turbulent jet and named

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diffusion coefficient  $D_{\mu}$ . This coefficient is defined as:

$$D_{\mu} = -\frac{\overline{\mu^2}}{2} \int_0^\infty \Delta R_{\mu}(r, 0, 0) dr$$
 (16)

 $R_{\mu}$  being the correlation coefficient of the refractive index fluctuations  $\mu$  and  $\Delta$  the Laplacian operator.

The value they have obtained is  $D_{\mu} = 2.27 \times 10^{-9} \,\mathrm{m}^{-1}$ . From the calculated Rms of the displacement of the brilliant central fringe found in this work, we can derive another value of that coefficient. So, a comparison can be made between both values.

In fact, it is proved [1] that the variance of the deflection angle  $\varepsilon$  of the laser beam in the turbulent medium is given by the equation:

$$\overline{\varepsilon^2} = 4D_\mu \sigma / (\bar{n})^2 \tag{17}$$

where  $\overline{n}$  is the mean value of the refractive index of the jet and  $\sigma$ is the distance traversed by the laser beam in the turbulent medium. Equation (17) holds if the geometrical optics approximation is valid; this is rigorously proved in [7]. The random global displacement  $\delta$  of the fringes being caused by the random deflection angle of the laser beam at the outlet of the jet, the relation between both quantities is given in Section 3.1 (see Equation (8b)). Hence, using Equations (8b) and (17), the diffusion coefficient  $D_{\mu}$  can be determined, as a function of the quantities  $\overline{\delta^2}$ ,  $\sigma$ , and  $\overline{n}$ , that is:

$$D_{\mu} = \frac{\left(\overline{n}\right)^2}{4\sigma D^2} \left(\frac{F_1}{f_1} \frac{f_2}{F_2}\right)^2 \overline{\delta^2}$$
(18)

By taking the approximated value  $\overline{n} \approx 1$  for the hot air in the turbulent jet, and by assuming that the whole path traversed by the laser beam in the jet is equal to the total width X of the jet (X = 200 mm), we have calculated the diffusion coefficient from Equation (18). The value we have obtained is:  $D_{\mu} = 2.44 \times 10^{-9} \,\mathrm{m}^{-1}$ . This result is nearly equal to the previous one obtained in [8].

#### 5. CONCLUSION

This paper shows the possibility of determining the directional fluctuations of a thin laser beam, when it propagates through a heated turbulent jet, by using an optical technique. The experimental method is based on the measurements of the interference-diffraction pattern perturbations of the out coming laser beam and the optical device is connected to an acquisition system of data for statistical investigations. The spectrum of fluctuations of the laser beam direction measured from the fluctuations of light intensity in the perturbed interference pattern reveals a -8/3 power law. The Rms values of the directional perturbations of the interference pattern have been measured. Those results were then used for calculating a scattering coefficient, well defined in previous works as the diffusion coefficient of the jet. The value obtained for this coefficient is nearly equal to that found in previous papers.

This agreement enables to conclude that the experimental technique used is efficient and satisfactory.

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