RADIATION OF A HORIZONTAL DIPOLE IN THE PRESENCE OF A THREE-LAYERED REGION AND MICROSTRIP ANTENNA

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Abstract—In this paper, we study in detail both the trapped surface wave and lateral wave excited by a dipole antenna parallel to the plane boundaries of a three-layered region in spherical coordinate. An approximate formula is obtained for the solution of the electric-type pole equation when a condition is satisfied by $\sqrt{k_1^2 - k_0^2}\, l \leq 0.6$. Similarly, an approximate formula is obtained for the solution of the magnetic-type pole equation when a condition is satisfied by $\sqrt{k_1^2 - k_0^2}\, l - \pi/2 \leq 1$. Furthermore, because of its useful applications in microstrip antenna, the radiation patterns of a patch antenna with specific current distributions are treated specifically. Analysis and computations are carried out in several typical cases. It is seen that, for the component $E_{0\Theta}$, the total field is determined primarily by the trapped surface wave of electric type, and, for the component $E_{0\Phi}$, the total field is determined primarily by the trapped surface wave of magnetic type.

1. INTRODUCTION

The problem of the electromagnetic field of a horizontal or vertical dipole in a multi-layered region has been visited widely by many investigators [1–28], especially including Sommerfeld, Wait, and King. In the past decade, several investigators have revisited the old problem and concluded that the trapped surface wave, which is determined by the sums of residues of the poles, can be excited efficiently by a dipole source in the presence of a three-layered region [29–36].

In a recent paper [37], the radiation from a vertical electric dipole in the presence of a three or four-layered region was investigated. It

is found that the far-field patterns on the air-dielectric boundary are
determined primarily the trapped surface, which is not considered in
chapter 8 of [13]. The new development in [37], naturally, rekindled
the interest in the further study on the radiation of a horizontal electric
dipole in the presence of a three-layered region. It is found that it is
more important to study the radiation from a horizontal dipole in the
presence of a three-layered region because of its useful applications in
microstrip antenna.

   In what follows, we will attempt to treat in detail both the
radiation of a horizontal dipole in the presence of a three-layered region
and the radiation patterns of a patch antenna with specific current
distributions.
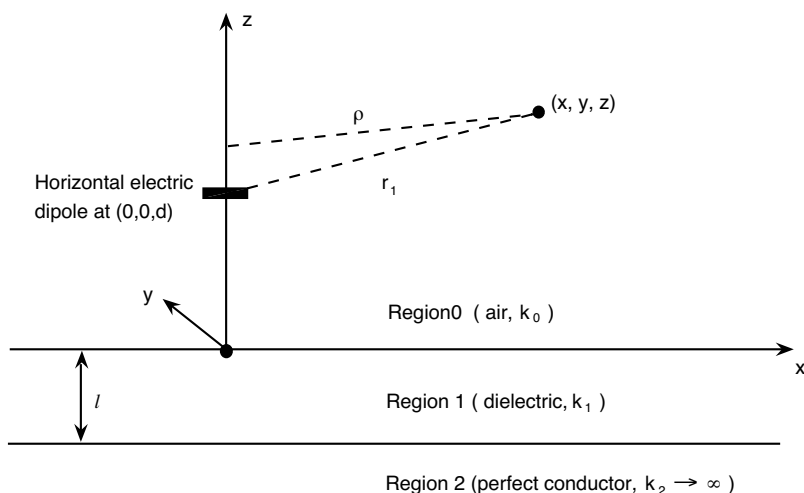
Figure 1. Unit horizontal electric dipole at $(0, 0, d)$ over the air-
dielectric boundary $z = 0$ in the presence of a three-layered region.

2. RADIATION OF A HORIZONTAL DIPOLE IN THE PRESENCE OF A THREE-LAYERED REGION

The relevant geometry and Cartesian coordinate system are shown in
Fig. 1, where a horizontal electric dipole in the $\hat{x}$ direction is located at
$(0, 0, d)$. Region 0 $(z \geq 0)$ is the space above the dielectric layer with
the air characterized by the permeability μ_0 and uniform permittivity
ϵ_0, Region 1 $(-l \leq z \leq 0)$ is the dielectric layer characterized by the
permeability μ_0, relative permittivity ϵ_{1r}, and Region 2 $(z \leq 0)$ is the
rest space occupied by a perfect conductor. Then, the wave numbers

in the three layers are

$$
\begin{aligned}
k_0 &= \omega\sqrt{\mu_0 \varepsilon_0} \\
k_1 &= \omega\sqrt{\mu_0 \varepsilon_0 \varepsilon_{1r}} \\
k_2 &\to \infty.
\end{aligned}
\tag{1}
$$

With the time dependence $e^{-i\omega t}$, the complete electromagnetic field generated by a horizontal dipole in the three-layered region in cylindrical coordinate system can be obtained in [32]. We write

$$
E_{0\rho}(\rho, \phi, z) = -\frac{\omega \mu_0 I dl}{4\pi k_0^2} \cos\phi
$$

$$
\cdot \left\{ -\left[\frac{2k_0}{r_1^2} + \frac{2i}{r_1^3} + \left(\frac{z-d}{r_1}\right)^2 \left(\frac{ik_0^2}{r_1} - \frac{3k_0}{r_1^2} - \frac{3i}{r_1^3}\right) \right] e^{ik_0 r_1} \right.
$$

$$
+ \left[\frac{2k_0}{r_2^2} + \frac{2i}{r_2^3} + \left(\frac{z+d}{r_2}\right)^2 \cdot \left(\frac{ik_0^2}{r_2} - \frac{3k_0}{r_2^2} - \frac{3i}{r_2^3}\right) \right] e^{ik_0 r_2} + \pi k_0^2
$$

$$
\cdot \sum_j \frac{\gamma_{0E}^* \gamma_{1E}^* \tan\gamma_{1E}^* l}{q'(\lambda_{jE}^*)} e^{i\gamma_{0E}^*(z+d)} \lambda_{jE}^* \left[H_0^{(1)}(\lambda_{jE}^*\rho) - H_2^{(1)}(\lambda_{jE}^*\rho) \right]
$$

$$
+ \pi k_0^2 \sum_j \frac{\tan\gamma_{1B}^* l}{p'(\lambda_{jB}^*)} \cdot e^{i\gamma_{0B}^*(z+d)} \lambda_{jB}^* \left[H_0^{(1)}(\lambda_{jB}^*\rho) + H_2^{(1)}(\lambda_{jB}^*\rho) \right]
$$

$$
+ 2e^{-i\frac{\pi}{4}} k_0^3 A \sqrt{\frac{2}{\pi k_0 \rho}} \cdot e^{ik_0 r_2} \cdot \left[-e^{i\frac{\pi}{4}} \sqrt{\frac{\pi}{2k_0\rho}} \left(\frac{z+d}{\rho} + iA \right) \right.
$$

$$
+ \frac{\pi}{2} A^2 \cdot \exp\left(-i\frac{k_0\rho}{2} \left(\frac{z+d}{\rho} - iA \right)^2 \right)
$$

$$
\cdot \text{erfc}\left(\sqrt{-i\frac{k_0\rho}{2} \left(\frac{z+d}{\rho} - iA \right)^2} \right) \right] - 2ik_0^3 \sqrt{\frac{1}{\pi k_0 \rho}} \cdot e^{ik_0 r_2}
$$

$$
\cdot \left(\frac{-i}{k_0\rho} \right) \cdot \left[\sqrt{\frac{\pi}{k_0\rho}} - \frac{\pi}{\sqrt{2}} e^{i\frac{\pi}{4}} T \exp\left(-i\frac{k_0\rho}{2} \left(\frac{z+d}{\rho} + iT \right)^2 \right) \right.
$$

$$
\left. \left. \cdot \text{erfc}\left(\sqrt{-\frac{ik_0\rho}{2} \left(\frac{z+d}{\rho} + iT \right)^2} \right) \right] \right\}
\tag{2}
$$

$$
E_{0\phi}(\rho, \phi, z) = -\frac{\omega \mu_0 I dl}{4\pi k_0^2} \sin\phi
$$

$$
\cdot \left\{ -\left(\frac{ik_0^2}{r_1} - \frac{k_0}{r_1^2} - \frac{i}{r_1^3} \right) e^{ik_0 r_1} + \left(\frac{ik_0^2}{r_2} - \frac{k_0}{r_2^3} - \frac{i}{r_2^3} \right) e^{ik_0 r_2} + \pi k_0^2 \right.
$$

$$\cdot \sum_j \frac{\gamma_{0E}^* \gamma_{1E}^* \tan \gamma_{1E}^* l}{q'(\lambda_{jE}^*)} e^{i\gamma_{0E}^*(z+d)} \lambda_{jE}^* \left[H_0^{(1)}(\lambda_{jE}^* \rho) + H_2^{(1)}(\lambda_{jE}^* \rho) \right]$$

$$+ \pi k_0^2 \sum_j \frac{\tan \gamma_{1B}^* l}{p'(\lambda_{jB}^*)} \cdot e^{i\gamma_{0B}^*(z+d)} \lambda_{jB}^* \left[H_0^{(1)}(\lambda_{jB}^* \rho) - H_2^{(1)}(\lambda_{jB}^* \rho) \right]$$

$$+ 2 e^{-i\frac{\pi}{4}} k_0^3 A \sqrt{\frac{2}{\pi k_0 \rho}} e^{ik_0 r_2} \left(\frac{-i}{k_0 \rho} \right) \left[-e^{i\frac{\pi}{4}} \sqrt{\frac{\pi}{2k_0 \rho}} \left(\frac{z+d}{\rho} + iA \right) \right.$$

$$+ \frac{\pi}{2} A^2 \exp\left(-i\frac{k_0 \rho}{2} \left(\frac{z+d}{\rho} - iA \right)^2 \right)$$

$$\left. \cdot \operatorname{erfc}\left(\sqrt{-i\frac{k_0 \rho}{2} \left(\frac{z+d}{\rho} + iA \right)^2} \right) \right] - 2 i k_0^3 \sqrt{\frac{1}{\pi k_0 \rho}} \cdot e^{ik_0 r_2}$$

$$\cdot \left[\sqrt{\frac{\pi}{k_0 \rho}} - \frac{\pi}{\sqrt{2}} \cdot e^{i\frac{\pi}{4}} \cdot T \exp\left(-i\frac{k_0 \rho}{2} \left(\frac{z+d}{\rho} + iT \right)^2 \right) \right.$$

$$\left. \left. \cdot \operatorname{erfc}\left(\sqrt{-i\frac{k_0 \rho}{2} \left(\frac{z+d}{\rho} + iT \right)^2} \right) \right] \right\} \tag{3}$$

$$E_{0z}(\rho, \phi, z) = \frac{i\omega \mu_0 I dl}{4\pi k_0^2} \cos \phi$$

$$\cdot \left\{ - \left(\frac{\rho}{r_1} \right) \left(\frac{z-d}{r_1} \right) \cdot \left(\frac{k_0^2}{r_1} + \frac{3 i k_0}{r_1^2} - \frac{3}{r_1^3} \right) e^{ik_0 r_1} \right.$$

$$+ \left(\frac{\rho}{r_2} \right) \cdot \left(\frac{z+d}{r_2} \right) \left(\frac{k_0^2}{r_2} + \frac{3 i k_0}{r_2^2} - \frac{3}{r_2^3} \right) e^{ik_0 r_2}$$

$$+ 2\pi k_0^2 \sum_j \frac{\gamma_{1E}^* \tan \gamma_{1E}^* l}{q'(\lambda_{jE}^*)} e^{i\gamma_{0E}^*(z+d)} \left(\lambda_{jE}^* \right)^2 H_1^{(1)}(\lambda_{jE}^* \rho)$$

$$+ 2 i k_0^3 A \cdot \sqrt{\frac{1}{\pi k_0 \rho}} \cdot e^{ik_0 r_2}$$

$$\cdot \left[\sqrt{\frac{\pi}{k_0 \rho}} + \frac{\pi}{\sqrt{2}} \cdot e^{i\frac{\pi}{4}} \cdot A \exp\left(-i\frac{k_0 \rho}{2} \left(\frac{z+d}{\rho} - iA \right)^2 \right) \right.$$

$$\left. \left. \operatorname{erfc}\left(\sqrt{-i\frac{k_0 \rho}{2} \left(\frac{z+d}{\rho} - iA \right)^2} \right) \right] \right\} \tag{4}$$

$$B_{0\rho}(\rho, \phi, z) = -\frac{\mu_0 I dl}{4\pi} \sin \phi \left\{ - \left(\frac{z-d}{r_1} \right) \left(\frac{i k_0}{r_1} - \frac{1}{r_1^2} \right) e^{ik_0 r_1} \right.$$

$$+ \left(\frac{z+d}{r_2} \right) \left(\frac{ik_0}{r_2} - \frac{1}{r_2^2} \right) e^{ik_0 r_2} + \pi k_0^2$$

$$\cdot \sum_j \frac{\gamma_{1E}^* \tan \gamma_{1E}^* l}{q'(\lambda_{jE}^*)} e^{i\gamma_{0E}^*(z+d)} \lambda_{jE}^* \left[H_0^{(1)}(\lambda_{jE}^* \rho) + H_2^{(1)}(\lambda_{jE}^* \rho) \right]$$

$$+ \pi \sum_j \frac{\gamma_{0B}^* \tan \gamma_{1B}^* l}{p'(\lambda_{jB}^*)} \lambda_{jB}^* e^{i\gamma_{0B}^*(z+d)} \left[H_0^{(1)}(\lambda_{jB}^* \rho) - H_2^{(1)}(\lambda_{jB}^* \rho) \right]$$

$$- 2k_0^2 A \sqrt{\frac{1}{\pi k_0 \rho}} e^{ik_0 r_2} \cdot \left(\frac{-i}{k_0 \rho} \right)$$

$$\cdot \left[\frac{\pi}{k_0 \rho} + \frac{\pi}{\sqrt{2}} e^{i\frac{\pi}{4}} A \cdot \exp\left(-i \frac{k_0 \rho}{2} \left(\frac{z+d}{\rho} - iA \right)^2 \right) \right.$$

$$\cdot \text{erfc}\left(\sqrt{-i \frac{k_0 \rho}{2} \left(\frac{z+d}{\rho} - iA \right)^2} \right) \right]$$

$$+ 2k_0^2 e^{i\frac{\pi}{4}} \sqrt{\frac{2}{\pi k_0 \rho}} e^{ik_0 r_2} \left\{ -\frac{\sqrt{2}}{2} e^{i\frac{\pi}{4}} \left(\frac{z+d}{2} - iT \right) \cdot \sqrt{\frac{\pi}{k_0 \rho}} \right.$$

$$+ \frac{\pi}{2} T^2 \exp\left(-i \frac{k_0 \rho}{2} \left(\frac{z+d}{\rho} + iT \right)^2 \right)$$

$$\left. \left. \cdot \text{erfc}\left(\sqrt{-i \frac{k_0 \rho}{2} \left(\frac{z+d}{\rho} + iT \right)^2} \right) \right] \right\} \tag{5}$$

$$B_{0\phi}(\rho, \phi, z) = -\frac{\mu_0 I dl}{4\pi} \cos \phi \cdot \left\{ -\left(\frac{z-d}{r_1} \right) \left(\frac{ik_0}{r_1} - \frac{1}{r_1^2} \right) e^{ik_0 r_1} \right.$$

$$+ \left(\frac{z+d}{r_2} \right) \left(\frac{ik_0}{r_2} - \frac{1}{r_2^2} \right) e^{ik_0 r_2} + \pi k_0^2$$

$$\cdot \sum_j \frac{\gamma_{1E}^* \tan \gamma_{1E}^* l}{q'(\lambda_{jE}^*)} e^{i\gamma_{0E}^*(z+d)} \lambda_{jE}^* \left[H_0^{(1)}(\lambda_{jE}^* \rho) - H_2^{(1)}(\lambda_{jE}^* \rho) \right]$$

$$+ \pi \sum_j \frac{\gamma_{0B}^* \tan \gamma_{1B}^* l}{P'(\lambda_{jB}^*)} e^{i\gamma_{0B}^*(z+d)} \lambda_{jB}^* \left[H_0^{(1)}(\lambda_{jB}^* \rho) + H_2^{(1)}(\lambda_{jB}^* \rho) \right]$$

$$- 2k_0^2 A \cdot \sqrt{\frac{1}{\pi k_0 \rho}} e^{ik_0 r_2} \cdot \left[\sqrt{\frac{\pi}{k_0 \rho}} + \frac{\pi}{\sqrt{2}} e^{i\frac{\pi}{4}} A \right.$$

$$\cdot \exp\left(-i \frac{k_0 \rho}{2} \left(\frac{z+d}{\rho} - iA \right)^2 \right)$$

$$\cdot \text{erfc}\left(\sqrt{-i\frac{k_0\rho}{2}\left(\frac{z+d}{\rho}-iA\right)^2}\right)\Bigg] + 2k_0^2 e^{i\frac{\pi}{4}}\sqrt{\frac{2}{\pi k_0\rho}}e^{ik_0r_2}$$

$$\cdot\left(\frac{-i}{k_0\rho}\right)\left[-\frac{\sqrt{2}}{2}e^{i\frac{\pi}{4}}\left(\frac{z+d}{2}-iT\right)\sqrt{\frac{\pi}{k_0\rho}}\right.$$

$$+\frac{\pi}{2}T^2\exp\left(-i\frac{k_0\rho}{2}\left(\frac{z+d}{\rho}+iT\right)^2\right)$$

$$\left.\left.\cdot\text{erfc}\left(\sqrt{-i\frac{k_0\rho}{2}\left(\frac{z+d}{\rho}+iT\right)^2}\right)\right]\right]\Bigg\} \tag{6}$$

$$B_{0z}(\rho,\phi,z) = \frac{i\mu_0 Idl}{4\pi}\sin\phi$$

$$\cdot\left\{-\left(\frac{\rho}{r_1}\right)\left(\frac{k_0}{r_1}+\frac{i}{r_1^2}\right)e^{ik_0r_1}+\left(\frac{\rho}{r_2}\right)\left(\frac{k_0}{r_2}+\frac{i}{r_2^2}\right)e^{ik_0r_2}\right.$$

$$+2\pi\sum_j\frac{\lambda_{jB}^{*\,2}\tan\gamma_{1B}^*l}{p'(\lambda_{jB}^*)}e^{i\gamma_{0B}^*(z+d)}H_1^{(1)}(\lambda_{jB}^*\rho)-2k_0^2\sqrt{\frac{1}{\pi k_0\rho}}$$

$$\cdot\left[\sqrt{\frac{\pi}{k_0\rho}}-\frac{\pi}{\sqrt{2}}e^{i\frac{\pi}{4}}\cdot T\cdot\exp\left(-i\frac{k_0\rho}{2}\left(\frac{z+d}{\rho}+iT\right)^2\right)\right.$$

$$\left.\left.\cdot\text{erfc}\left(\sqrt{-i\frac{k_0\rho}{2}\left(\frac{z+d}{\rho}+iT\right)^2}\right)\right]e^{ik_0r_2}\right\} \tag{7}$$

where

$$r_1 = \sqrt{\rho^2+(z-d)^2}, \qquad r_2 = \sqrt{\rho^2+(z+d)^2} \tag{8}$$

$$q'(\lambda_{jE}^*) = -\frac{k_1^2\lambda_{jE}^*}{\gamma_{0E}^*}+i\frac{k_0^2\lambda_{jE}^*}{\gamma_{1E}^*}\left(\tan\gamma_{1E}^*l+\gamma_{1E}^*l\sec^2\gamma_{1E}^*l\right) \tag{9}$$

$$p'(\lambda_{jB}^*) = -\frac{\lambda_{jB}^*}{\gamma_{1B}^*}+i\lambda_{jB}^*\left(\frac{\tan\gamma_{1B}^*l}{\gamma_{0B}^*}+\frac{\gamma_{0B}^*l}{\gamma_{1B}^*}\sec^2\gamma_{1B}^*l\right) \tag{10}$$

$$A = \frac{k_0}{k_1}\sqrt{k_1^2-k_0^2}\tan\left(\sqrt{k_1^2-k_0^2}l\right) \tag{11}$$

$$T = \frac{\sqrt{k_1^2-k_0^2}}{k_0\tan(\sqrt{k_1^2-k_0^2}l)} \tag{12}$$

$$\text{erfc}\left(\sqrt{-ip^*}\right) = \sqrt{2}e^{-i\frac{\pi}{4}}F(p^*) \tag{13}$$

$$p_1^* = \frac{k_0\rho}{2}\left(\frac{z+d}{\rho}+iT\right)^2 \tag{14}$$

$$p_2^* = \frac{k_0\rho}{2}\left(\frac{z+d}{\rho}-iA\right)^2. \tag{15}$$

It is noted that λ_{jE}^* and λ_{jB}^* are the j-th poles of the integrals of electric-type(TM) wave and magnetic-type (TE) wave, respectively. Correspondingly,

$$\gamma_{mE}^* = \sqrt{k_m^2-(\lambda_{jE}^*)^2}; \quad m=0,1 \tag{16}$$

$$\gamma_{mB}^* = \sqrt{k_m^2-(\lambda_{jB}^*)^2}; \quad m=0,1. \tag{17}$$

Here the subscript m refers Regions 0 and 1 and j refers to a number of poles.

Subjecting a condition $z+d \ll \rho$, it is known that $r_1 \sim r_0 - d\cos\Theta$, $r_2 \sim r_0 + d\cos\Theta$ in phases and $r_1 \sim r_2 \sim r_0$ in amplitudes. Taking into account the relations between cylindrical coordinate and spherical coordinate, and $\rho = r_0\sin\Theta$, $z = r_0\cos\Theta$, we have

$$E_{0r}(r_0,\Theta,\Phi) = E_{0\rho}(\rho,\phi,z)\sin\Theta + E_{0z}(\rho,\phi,z)\cos\Theta \tag{18}$$
$$E_{0\Theta}(r_0,\Theta,\Phi) = E_{0\rho}(\rho,\phi,z)\cos\Theta + E_{0z}(\rho,\phi,z)\cos\Theta \tag{19}$$
$$E_{0\Phi}(r_0,\Theta,\Phi) = E_{0\phi}(\rho,\phi,z) \tag{20}$$
$$B_{0r}(r_0,\Theta,\Phi) = B_{0\rho}(\rho,\phi,z)\sin\Theta + B_{0z}(\rho,\phi,z)\cos\Theta \tag{21}$$
$$B_{0\Theta}(r_0,\Theta,\Phi) = B_{0\rho}(\rho,\phi,z)\cos\Theta - B_{0z}(\rho,\phi,z)\sin\Theta \tag{22}$$
$$B_{0\Phi}(r_0,\Theta,\Phi) = B_{0\phi}(\rho,\phi,z). \tag{23}$$

When the far-field conditions $|p_1^*| \geq 4$ and $|p_2^*| \geq 4$ are satisfied, the Fresnel-integral terms are written in the forms

$$k_0^2 \cdot iT\sqrt{\frac{\pi}{k_0\rho}} \cdot e^{-ip_1^*} \cdot F(p_1^*) = \frac{ik_0\rho}{\rho^2} \cdot \frac{iT}{iT+(z+d)/\rho}$$
$$+\frac{iT}{\rho^2}\left[\frac{1}{iT+(z+d)/\rho}\right]^3 \tag{24}$$

$$k_0^2 \cdot (-iA)\sqrt{\frac{\pi}{k_0\rho}} \cdot e^{-ip_2^*} \cdot F(p_2^*) = \frac{ik_0\rho}{\rho^2} \cdot \frac{-iA}{-iA+(z+d)/\rho}$$
$$+\frac{-iA}{\rho^2}\left[\frac{1}{-iA+(z+d)/\rho}\right]^3 \tag{25}$$

In practical applications the antenna is usually placed on the surface of the earth, either as a based-insulated dipole or a monopole

base-driven against a buried ground system. In this case $d/r_0 \sim 0$, $k_0 d \sim 0$, substitutions (2)–(7) into (18)–(23) lead to the following formulas.

$$
\begin{aligned}
E_{0r} &= -\frac{\omega\mu_0 Idl\cos\Phi}{2\pi k_0^2}e^{ik_0 r_0}\cdot\left\{-\frac{k_0+ik_0^2 r_0 A^2\sin\Theta}{r_0^2\sin\Theta}\right.\\
&\quad +\frac{iTk_0 r_0(\cos\Theta+iT\sin\Theta)^2+T\sin\Theta}{r_0^3(\cos\Theta+iT\sin\Theta)^3}\\
&\quad \left.+\frac{\begin{pmatrix}k_0^2 A^2 r_0(-iA\sin\Theta+\cos\Theta)^2(i\cos\Theta+A\sin\Theta)\\+A^2 k_0\sin\Theta\cos\Theta-ik_0 A^3\sin^2\Theta\end{pmatrix}}{r_0^2(-iA\sin\Theta+\cos\Theta)^3}\right\}\\
&\quad -\frac{\omega\mu_0 Idl\cos\Phi}{4}\cdot\left\{\sum_j\frac{\gamma_{0E}^*\gamma_{1E}^*\sin\Theta\tan(\gamma_{1E}^* l)}{q'(\lambda_{jE}^*)}e^{i\gamma_{0E}^* r_0\cos\Theta}\right.\\
&\quad \cdot\lambda_{jE}^*\cdot\left[H_0^{(1)}(\lambda_{jE}^* r_0\sin\Theta)-H_2^{(1)}(\lambda_{jE}^* r_0\sin\Theta)\right]\\
&\quad -\sum_j\frac{2i\cos\Theta\gamma_{1E}^*\tan(\gamma_{1E}^* l)}{q'(\lambda_{jE}^*)}e^{i\gamma_{0E}^* r_0\cos\Theta}\lambda_{jE}^{*\,2}H_1^{(1)}(\lambda_{jE}^* r_0\sin\Theta)\\
&\quad +\sum_j\frac{\tan(\gamma_{1B}^* l)\sin\Theta}{p'(\lambda_{jB}^*)}e^{i\gamma_{0B}^* r_0\cos\Theta}\\
&\quad \left.\cdot\lambda_{jB}^*\left[H_0^{(1)}(\lambda_{jB}^* r_0\sin\Theta)+H_2^{(1)}(\lambda_{jB}^* r_0\sin\Theta)\right]\right\}
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
E_{0\Theta} &= -\frac{\omega\mu_0 Idl\cos\Phi}{2\pi k_0^2}e^{ik_0 r_0}\\
&\quad \cdot\left\{-\frac{k_0^2 Ar_0+k_0\cos\Theta+ik_0^2 A^2 r_0\sin\Theta\cos\Theta}{r_0^2\sin^2\Theta}\right.\\
&\quad +\frac{\begin{pmatrix}k_0^2 r_0 A^2(\cos\Theta-iA\sin\Theta)^2(A\cos\Theta-i\sin\Theta)\\-k_0 A^2\sin^2\Theta-iA^3 k_0\sin\Theta\cos\Theta\end{pmatrix}}{r_0^2(\cos\Theta-iA\sin\Theta)^3}\\
&\quad \left.+\frac{iTk_0 r_0\cos\Theta(\cos\Theta+iT\sin\Theta)^2+T\sin\Theta\cos\Theta}{r_0^3\sin\Theta(\cos\Theta+iT\sin\Theta)^3}\right\}\\
&\quad -\frac{\omega\mu_0 Idl\cos\Phi}{4}\left\{\sum_j\frac{\gamma_{0E}^*\gamma_{1E}^*\tan(\gamma_{1E}^* l)}{q'(\lambda_{jE}^*)}e^{\gamma_{0E}^* r_0\cos\Theta}\right.
\end{aligned}
$$

$$\cdot \lambda_{jE}^* \cos\Theta \left[H_0^{(1)}(\lambda_{jE}^* r_0 \sin\Theta) - H_2^{(1)}(\lambda_{jE}^* r_0 \sin\Theta) \right]$$

$$+ 2i \sin\Theta \sum_j \frac{\gamma_{1E}^* \tan(\gamma_{1E}^* l)}{q'(\lambda_{jE}^*)} e^{\gamma_{0E}^* r_0 \cos\Theta} \lambda_{jE}^{*2} H_1^{(1)}(\lambda_{jE}^* r_0 \sin\Theta)$$

$$+ \cos\Theta \sum_j \frac{\tan(\gamma_{1B}^* l)}{p'(\lambda_{jB}^*)} e^{i\gamma_{0B}^* r_0 \cos\Theta}$$

$$\left. \cdot \lambda_{jB}^* \left[H_0^{(1)}(\lambda_{jB}^* r_0 \sin\Theta) + H_2^{(1)}(\lambda_{jB}^* r_0 \sin\Theta) \right] \right\} \tag{27}$$

$$
\begin{aligned}
E_{0\Phi} =\ & -\frac{\omega\mu_0 Idl \sin\Phi}{2\pi k_0^2} e^{ik_0 r_0} \left\{ \frac{iAk_0 \cos\Theta - k_0 A^2 \sin\Theta - ik_0^2 r_0 \sin^2\Theta}{r_0^2 \sin^3\Theta} \right. \\
& - \frac{ik_0 A^3 r_0(-iA\sin\Theta + \cos\Theta)^2 + A^3 \sin\Theta}{r_0^3 \sin\Theta(-iA\sin\Theta + \cos\Theta)^3} \\
& \left. + \frac{-k_0^2 T r_0(iT\sin\Theta + \cos\Theta)^2 + iT k_0 \sin\Theta}{r_0^2(iT\sin\Theta + \cos\Theta)^3} \right\} \\
& - \frac{\omega\mu_0 Idl \sin\Phi}{4} \left\{ \sum_j \frac{\gamma_{0E}^* \gamma_{1E}^* \tan(\gamma_{1E}^* l)}{q'(\lambda_{jE}^*)} e^{i\gamma_{0E}^* r_0 \cos\Theta} \right. \\
& \cdot \lambda_{jE}^* \left[H_0^{(1)}(\lambda_{jE}^* r_0 \sin\Theta) + H_2^{(1)}(\lambda_{jE}^* r_0 \sin\Theta) \right] \\
& + \sum_j \frac{\tan(\gamma_{1B}^* l)}{p'(\lambda_{jB}^*)} e^{\gamma_{0B}^* r_0 \cos\Theta} \cdot \lambda_{jB}^* \\
& \left. \cdot \left[H_0^{(1)}(\lambda_{jB}^* r_0 \sin\Theta) - H_2^{(1)}(\lambda_{jB}^* r_0 \sin\Theta) \right] \right\} \tag{28}
\end{aligned}
$$

$$
\begin{aligned}
B_{0r} =\ & -\frac{\mu_0 Idl \sin\Phi}{2\pi} e^{ik_0 r_0} \left\{ \frac{iA - k_0 r_0 \left(T\sin\Theta - i\cos\Theta + \frac{ir_0 \sin\Theta \cos\Theta}{2} \right)}{r_0^2 \sin\Theta} \right. \\
& + \frac{\left(\begin{array}{c} k_0 T r_0(iT\sin\Theta + \cos\Theta)^2(\cos\Theta + iT\sin\Theta) \\ + T\sin\Theta(T\sin\Theta - i\cos\Theta) \end{array} \right)}{r_0^2(iT\sin\Theta + \cos\Theta)^3} \\
& \left. \cdot \frac{iA^2 \sin\Theta - k_0 A^2 r_0(-iA\sin\Theta + \cos\Theta)^2}{k_0 r_0^3(-iA\sin\Theta + \cos\Theta)^3} \right\} \\
& - \frac{\mu_0 Idl \sin\Phi}{4} \left\{ \sum_j \frac{k_0^2 \gamma_{1E}^* \sin\Theta \tan(\gamma_{1E}^* l)}{q'(\lambda_{jE}^*)} e^{i\gamma_{0E}^* r_0 \cos\Theta} \right.
\end{aligned}
$$

$$
\cdot \lambda_{jE}^* \left[H_0^{(1)}(\lambda_{jE}^* r_0 \sin\Theta) + H_2^{(1)}(\lambda_{jE}^* r_0 \sin\Theta) \right]
$$

$$
+ \sum_j \frac{\gamma_{0B}^* \sin\Theta \tan(\gamma_{1B}^* l)}{p'(\lambda_{jB}^*)} e^{i\gamma_{0B}^* r_0 \cos\Theta}
$$

$$
\cdot \lambda_{jB}^* \left[H_0^{(1)}(\lambda_{jB}^* r_0 \sin\Theta) - H_2^{(1)}(\lambda_{jB}^* r_0 \sin\Theta) \right]
$$

$$
\left. -2i\cos\Theta \sum_j \frac{\lambda_{jB}^{*\,2} \tan(\gamma_{1B}^* l)}{p'(\lambda_{jB}^*)} e^{i\gamma_{0B}^* r_0 \cos\Theta} \cdot H_1^{(1)}(\lambda_{jB}^* r_0 \sin\Theta) \right\} \quad (29)
$$

$$
B_{0\Theta} = -\frac{\mu_0 I dl \sin\Phi}{2\pi} e^{ik_0 r_0}
$$

$$
\cdot \left\{ \frac{iA\cos\Theta - k_0 r_0 \sin\Theta \left(T\cos\Theta + i\sin\Theta + \frac{ir_0 \cos^2\Theta}{2} \right)}{r_0^2 \sin^2\Theta} \right.
$$

$$
+ \frac{\begin{pmatrix} k_0 T r_0 (iT\sin\Theta + \cos\Theta)^2 (iT\cos\Theta - \sin\Theta) \\ + T\sin\Theta(T\cos\Theta + i\sin\Theta) \end{pmatrix}}{r_0^2 (iT\sin\Theta + \cos\Theta)^3}
$$

$$
\left. + \frac{iA^2 \sin\Theta \cos\Theta - k_0 A^2 r_0 \cos\Theta(-iA\sin\Theta + \cos\Theta)^2}{k_0 r_0^3 \sin\Theta(-iA\sin\Theta + \cos\Theta)^3} \right\}
$$

$$
- \frac{\mu_0 I dl \sin\Phi}{4} \left\{ \sum_j \frac{k_0^2 \gamma_{1E}^* \cos\Theta \tan(\gamma_{1E}^* l)}{q'(\lambda_{jE}^*)} e^{i\gamma_{0E}^* r_0 \cos\Theta} \right.
$$

$$
\cdot \lambda_{jE}^* \left[H_0^{(1)}(\lambda_{jE}^* r_0 \sin\Theta) + H_2^{(1)}(\lambda_{jE}^* r_0 \sin\Theta) \right]
$$

$$
+ \sum_j \frac{\gamma_{0B}^* \cos\Theta \tan(\gamma_{1B}^* l)}{p'(\lambda_{jB}^*)} e^{i\gamma_{0B}^* r_0 \cos\Theta}
$$

$$
\cdot \lambda_{jB}^* \left[H_0^{(1)}(\lambda_{jB}^* r_0 \sin\Theta) - H_2^{(1)}(\lambda_{jB}^* r_0 \sin\Theta) \right]
$$

$$
\left. + \sum_j \frac{2i\lambda_{jB}^{*\,2} \sin\Theta \tan(\gamma_{1B}^* l)}{p'(\lambda_{jB}^*)} e^{i\gamma_{0B}^* r_0 \cos\Theta} H_1^{(1)}(\lambda_{jB}^* r_0 \sin\Theta) \right\} \quad (30)
$$

$$
B_{0\Phi} = -\frac{\mu_0 I dl \cos\Phi}{2\pi} e^{ik_0 r_0} \left\{ -\frac{k_0 A r_0 \sin\Theta + \frac{r_0 \cos\Theta}{2} - iT}{r_0^2 \sin^2\Theta} \right.
$$

$$
- \frac{ik_0 A^2 r_0(-iA\sin\Theta + \cos\Theta)^2 + A^2 \sin\Theta}{r_0^2(-iA\sin\Theta + \cos\Theta)^3}
$$

$$+\frac{k_0 T^2 r_0 (iT\sin\Theta + \cos\Theta)^2 - iT^2\sin\Theta}{k_0 r_0^3 \sin\Theta(iT\sin\Theta + \cos\Theta)^3} e^{ik_0 d\cos\Theta}\Bigg\}$$

$$-\frac{\mu_0 I dl \cos\Phi}{4}\Bigg\{ k_0^2 \sum_j \frac{\gamma_{1E}^* \tan(\gamma_{1E}^* l)}{q'(\lambda_{jE}^*)} e^{i\gamma_{0E}^* r_0 \cos\Theta} \cdot \lambda_{jE}^*$$

$$\cdot\left[H_0^{(1)}(\lambda_{jE}^* r_0 \sin\Theta) - H_2^{(1)}(\lambda_{jE}^* r_0 \sin\Theta) \right]$$

$$+\sum_j \frac{\gamma_{0B}^* \tan(\gamma_{1B}^* l)}{p'(\lambda_{jB}^*)} \cdot \lambda_{jB}^* \cdot e^{i\gamma_{0B}^* r_0 \cos\Theta}$$

$$\cdot\left[H_0^{(1)}(\lambda_{jB}^* r_0 \sin\Theta) + H_2^{(1)}(\lambda_{jB}^* r_0 \sin\Theta) \right]\Bigg\}. \qquad (31)$$

3. ANALYTICAL TECHNIQUES FOR DETERMINING THE POLES λ_E AND λ_M

In this section we will analyze the pole equations for both electric-type (TM) and magnetic-type (TE) waves. The pole equation for electric-type (TM) wave is expressed as follows [32]:

$$q(\lambda) = k_1^2 \gamma_0 - ik_0^2 \gamma_1 \tan\gamma_1 l = 0 \qquad (32)$$

It follows that

$$\frac{d\lambda}{dl} = \frac{k_0^2 (k_1^2 - \lambda^2)\sec^2\left(\sqrt{k_1^2 - \lambda^2}l\right)}{\frac{k_1\lambda}{\sqrt{\lambda^2 - k_0^2}} + k_0^2 \tan\left(\sqrt{k_1^2 - \lambda^2}l\right) + k_0^2 l\lambda\sec^2\left(\sqrt{k_1^2 - \lambda^2}l\right)}. \qquad (33)$$

When $\tan\sqrt{k_1^2 - \lambda^2}l = \frac{k_1^2\sqrt{\lambda^2 - k_0^2}}{k_0^2\sqrt{k_1^2 - \lambda^2}}$ is substituted into (33), and considering that k_1 is a real number, it is seen that all the terms of the above equation in the right are positive because of the poles λ_{jE} are between k_0 and k_1. So that $\sqrt{k_1^2 - \lambda^2}l$ is always positive, and it means that $\frac{d\lambda}{dl} > 0$. Then, it is known that λ_{jE} increase as l in the interval $n\pi \le \sqrt{k_1^2 - k_0^2}l \le (n+1)\pi$. When a condition is satisfied by $\sqrt{k_1^2 - k_0^2}l < 0.6$, we write

$$\lambda_E = k_0 + \frac{\left[-k_1^2\sqrt{2k_0} + \sqrt{2k_1^4 k_0 + 8k_0^5(k_1^2 - k_0^2)l^2}\right]^2}{16k_0^6 l^2}. \qquad (34)$$

Similarly, the pole equation for magnetic-type (TE) wave is expressed in the following form [32].

$$p(\lambda) = \gamma_1 - i\gamma_0 \tan\gamma_1 l = 0. \tag{35}$$

Obviously, (35) can be re-written as

$$\sqrt{k_1^2 - \lambda^2} \tan\left(\sqrt{k_1^2 - k_0^2} l - \frac{\pi}{2}\right) - \sqrt{\lambda^2 - k_0^2} = 0. \tag{36}$$

Then,

$$\frac{d\lambda}{dl} = \frac{\sqrt{k_1^2 - \lambda^2}\sqrt{\lambda^2 - k_0^2}\sec^2\left(\sqrt{k_1^2 - \lambda^2} l\right)}{\frac{\lambda}{\sqrt{k_1^2 - \lambda^2}} + \frac{\lambda l \sqrt{\lambda^2 - k_0^2}\sec^2\left(\sqrt{k_1^2 - \lambda^2} l\right)}{\sqrt{k_1^2 - \lambda^2}} - \frac{\lambda\tan\left(\sqrt{k_1^2 - \lambda^2} l\right)}{\sqrt{\lambda^2 - k_0^2}}}. \tag{37}$$

Substituting $\tan\gamma_1 l = -\frac{\sqrt{k_1^2 - \lambda^2}}{\sqrt{\lambda^2 - k_0^2}}$ into (37), and considering that k_1 is a real number, it is easily verified that $\frac{d\lambda}{dl} > 0$ in the interval $k_0 < \lambda < k_1$. When a condition is satisfied by $\sqrt{k_1^2 - k_0^2} l - \pi/2 \ll 1$, we have

$$\tan\left(\sqrt{k_1^2 - k_0^2} l - \frac{\pi}{2}\right) \approx \sqrt{k_1^2 - k_0^2} l - \frac{\pi}{2} \tag{38}$$

From (37), after algebraic manipulation, we write

$$\lambda_B = k_0 + \frac{\left(\sqrt{2k_0} - \sqrt{2k_0 - 4A_b B_b}\right)^2}{4B_b^2} \tag{39}$$

where

$$A_b = \sqrt{k_1^2 - k_0^2}\left(\sqrt{k_1^2 - k_0^2} l - \frac{\pi}{2}\right) \tag{40}$$

$$B_b = \frac{\pi}{2} \cdot \frac{k_0}{\sqrt{k_1^2 - k_0^2}} - 2k_0 l. \tag{41}$$

As shown in Fig. 2, it is seen that the approximate values of λ_E are agreement with the corresponding accurate values under the condition $\sqrt{k_1^2 - k_0^2} l < 0.6$ for electric-type (TM) wave. Similarly, from Fig. 3, it is seen that the approximate values of λ_M are agreement with the corresponding accurate values under the condition $\sqrt{k_1^2 - k_0^2} l - \pi/2 \ll 1$ for magnetic-type (TE) wave.
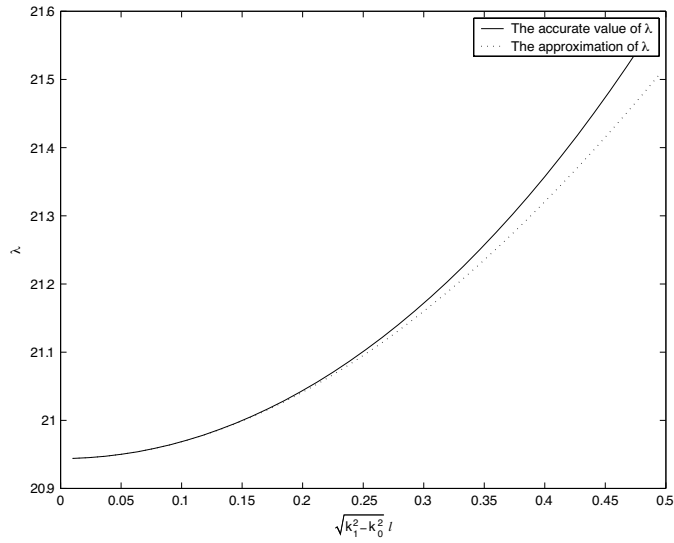
Figure 2. The values of λ vs. the thickness l of the dielectric layer for the electric-type (TM) wave with $f = 1\,\text{GHz}$ and $\epsilon_{1r} = 2.65$.
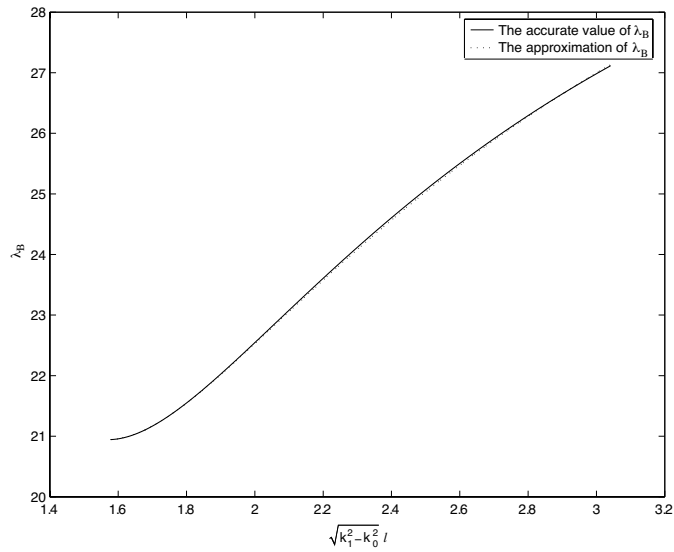
Figure 3. The values of λ vs. the thickness l of the dielectric layer for the magnetic-type (TE) wave with $f = 1\,\text{GHz}$ and $\epsilon_{1r} = 2.65$.
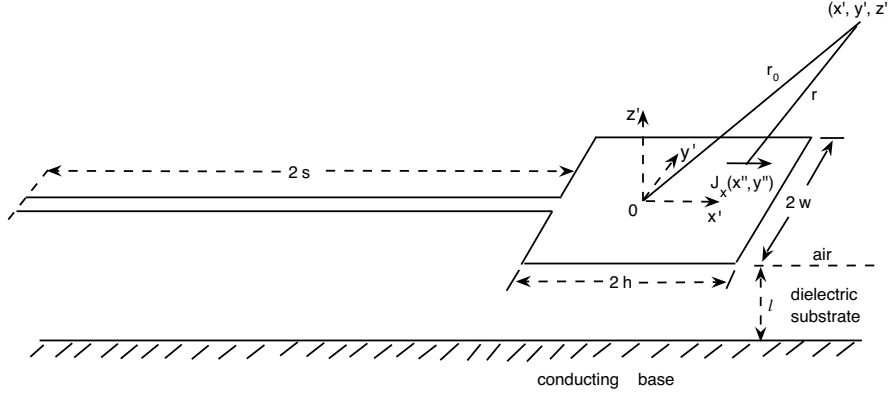
Figure 4. Microstrip patch antenna end-driven by microstrip transmission line.

4. MICROSTRIP ANTENNA

Microstrip consist of strip transmission lines and antenna on the surface of a thin dielectric layer coating a highly conducting base, which can be regarded as a perfect conductor. The basic elements of them are horizontal electric dipoles. In this section, we will treat a rectangular patch antenna end-driven by transmission lines, as illustrated in Fig. 4. The current-density distribution of a patch antenna with a width $2w$ and length $2h$ has the following form.

$$J_x(x'', y'') = \frac{I_x(0)}{2w} \cos(k_L x''); \qquad \begin{cases} -h \leq x'' \leq h \\ -w \leq y'' \leq w \end{cases} \qquad (42)$$

where $k_L = k_0\sqrt{\epsilon_{1r\ eff}}$ is the wave number that would characterize the patch as a segment of a microstrip transmission line for which $\epsilon_{1r\ eff}$ is the relative effective permittivity at the operating frequency, $I_x(0)$ is the total current traversing the center line $x'' = 0$ of the patch. In the present study we assume that the transverse distribution of the $\hat{x}'$-directed current is uniform. Actually, there is a $\hat{y}'$-directed current with large peaks at the edges $|y''| = w$ and a minimum at the center $y'' = 0$. Since the impedance of the patch is very large when the driving-point current is small compared to $I_x(0)$, the assumed current (42) should be a good approximation. The approximate formulas for the characteristic impedance Z_c of the microstrip transmission line and the relative effective permittivity $\epsilon_{1r\ eff}$ were given in (3.20)–(3.22) by Hoffman [38]. The relative permittivity $\epsilon_{1r\ eff}$ is approximated as

follows:

$$\epsilon_{1r\ eff} = \frac{\epsilon_{1r} + 1}{2} + \frac{\epsilon_{1r} - 1}{2} \left(1 + \frac{5l}{w}\right)^{-1/2}. \tag{43}$$

In the far-field zone, $r \sim r_0 = (x'^2 + y'^2 + z'^2)$ is adequate in amplitudes. In the phases, it is necessary to use the more accurate formula.

$$r \sim \left(r_0^2 - 2x'x'' - 2y'y''\right)^{1/2} = r_0 \left(1 - 2\frac{x'x''}{r_0^2} - 2\frac{y'y''}{r_0^2}\right)^{1/2}. \tag{44}$$

Following the similar steps addressed in Sec. 15.12 of [12] or Sec. 9.7 of [13], and use is made of the above approximations, the field factor for the patch antenna is written in the following form.

$$P(\Theta, \Phi) = \frac{I_x(0)}{2w} J(x'')J(y'')$$

$$= I_x(0) \left[\frac{2k_L \sin(k_L h)\cos(k_0 h \sin\Theta\cos\Phi)}{k_L^2 - k_0^2 \cos^2\Phi\sin^2\Theta} \right]$$

$$\cdot \left[\frac{\sin(k_0 w \sin\Phi\sin\Theta)}{k_0 w \sin\Phi\sin\Theta}\right]. \tag{45}$$

If h is chosen as $k_L h = \pi/2$, the field factor (45) reduces to (15.12.14) in the book [12] or (9.159) in [13]. Multiplying (27), (28), (30), and (31) by the field factor in (45), the far-field components of the patch antenna are expressed as follows:

$$[E_{0\Theta}^r(r_0, \Theta, \Phi)]_p = B_{0\Theta}(r_0, \Theta, \Phi) \cdot P(\Theta, \Phi) \tag{46}$$

$$[B_{0\Theta}^r(r_0, \Theta, \Phi)]_p = B_{0\Phi}(r_0, \Theta, \Phi) \cdot P(\Theta, \Phi) \tag{47}$$

$$[E_{0\Phi}^r(r_0, \Theta, \Phi)]_p = B_{0\Theta}(r_0, \Theta, \Phi) \cdot P(\Theta, \Phi) \tag{48}$$

$$[B_{0\Phi}^r(r_0, \Theta, \Phi)]_p = B_{0\Phi}(r_0, \Theta, \Phi) \cdot P(\Theta, \Phi). \tag{49}$$

It is noted that these are valid in the far-field zone and not valid in the intermediate zone.

5. COMPUTATIONS AND DISCUSSIONS

Considering the electric field at the high frequency of interest, $f = 1\,\text{GHz}$, and the far-field conditions $|p_1^*| \geq 4$ and $|p_2^*| \geq 4$, with

$\epsilon_{1r} = 2.56$ and $l = 0.04\,\text{m}$ or $l = 0.1\,\text{m}$, the radial distance ranges over $r_0 = 7\,\text{m}$. As shown in Figs. 5 and 6, the total field, both the trapped surface waves of electric type (TM) and magnetic type (TE), and the lateral wave are calculated from (27) for $E_{0\Theta}^r(r_0, \Theta, \Phi)$ and (28) for $E_{0\Phi}^r(r_0, \Theta, \Phi)$, respectively. Graphs of the amplitudes of the components $E_{0\Theta}^r(r_0, \Theta, \Phi)$ and $E_{0\Phi}^r(r_0, \Theta, \Phi)$ for a unit horizontal dipole vs. the angular degrees of Θ are computed and shown in Figs. 7 and 8, respectively. It is noted that the DRL waves include the direct wave, ideal reflected wave, and lateral wave. From Figs. 5–8, it is seen that, for the component $E_{0\Theta}$, the total field is determined primarily by the trapped surface wave of electric type, and, for the component $E_{0\Phi}$, the total field is determined primarily by the trapped surface wave of magnetic type when both the dipole point and observation point are located on the air-dielectric bound.
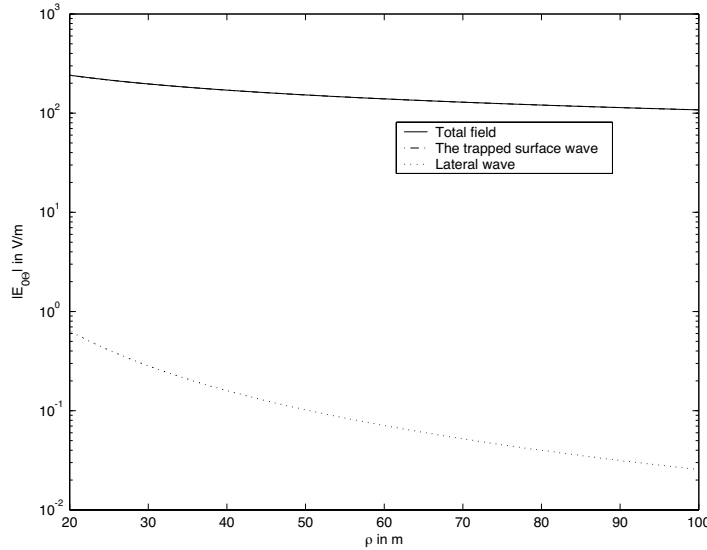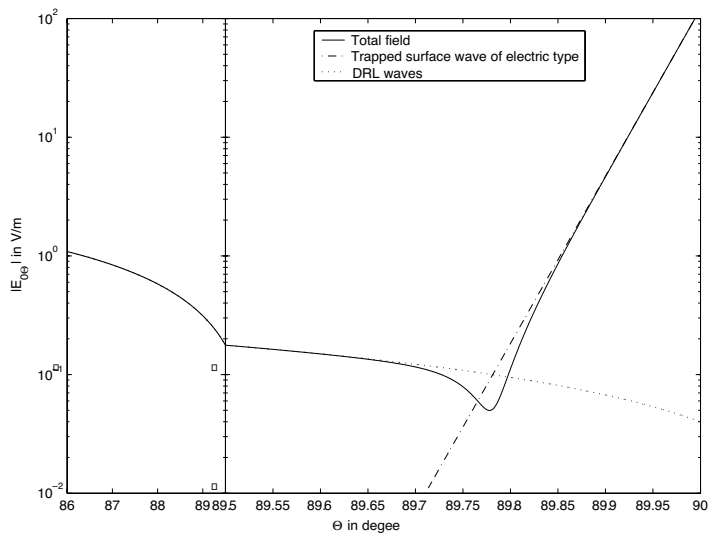

Figure 5. Electric field component $E_{0\Theta}$ in V/m with $f = 1$ GHz, $\epsilon_{1r} = 2.65$, $l = 0.1\,\text{m}$, $d = z = 0\,\text{m}$, and $\Phi = 0$.

In Figs. 9 and 10, graphs of the amplitudes of the components $[E_{0\Theta}^r(r_0, \Theta, \Phi)]_p$ as calculated from (46) and $[E_{0\Phi}^r(r_0, \Theta, \Phi)]_p$ from (47) for the patch antenna vs. the angular degrees of Φ are shown with $f = 1\,\text{GHz}$, $\epsilon_{1r} = 2$, $r_0 = 10\,\text{m}$, $k_L h = \pi/2$, $w = h$, $l = 0.1\,\text{m}$, $d = 0\,\text{m}$, $I_0 = 1\,\text{A}$, and $\Theta = \pi/2$. It is found that the amplitude of the total wave is approximately equal to that of the trapped surface wave under the conditions of $d = 0$ and $\Theta = \pi/2$. This also demonstrates that total
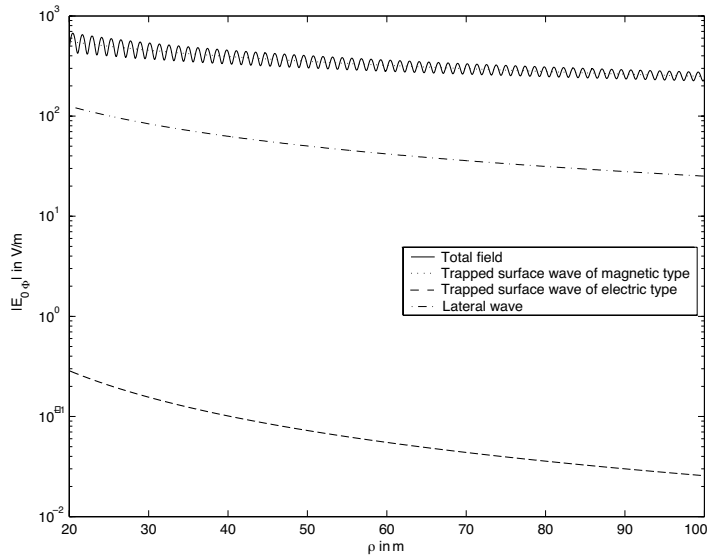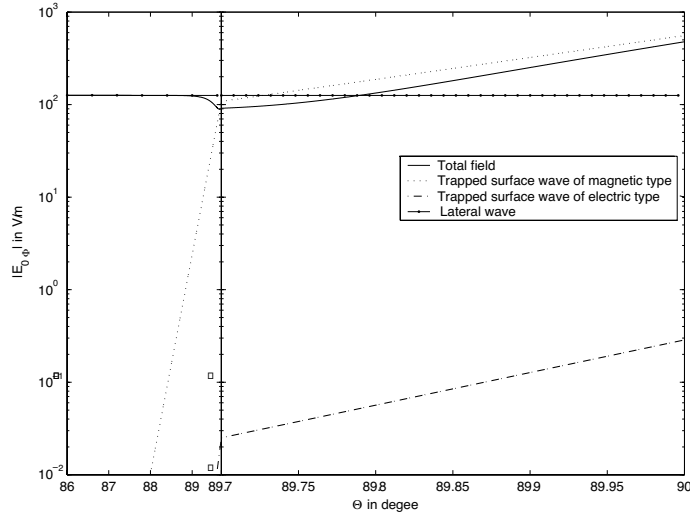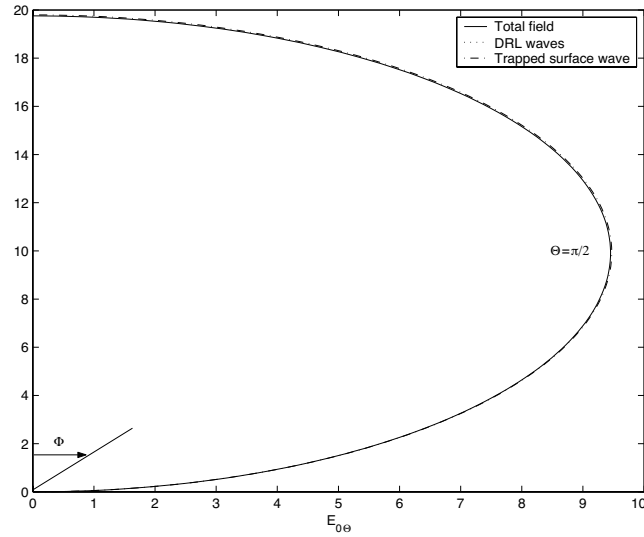
Figure 6. Electric field component $E_{0\Phi}$ in V/m with $f = 1$ GHz, $\epsilon_{1r} = 2.65$, $l = 0.1$ m, $d = z = 0$ m, and $\Phi = \pi/2$.
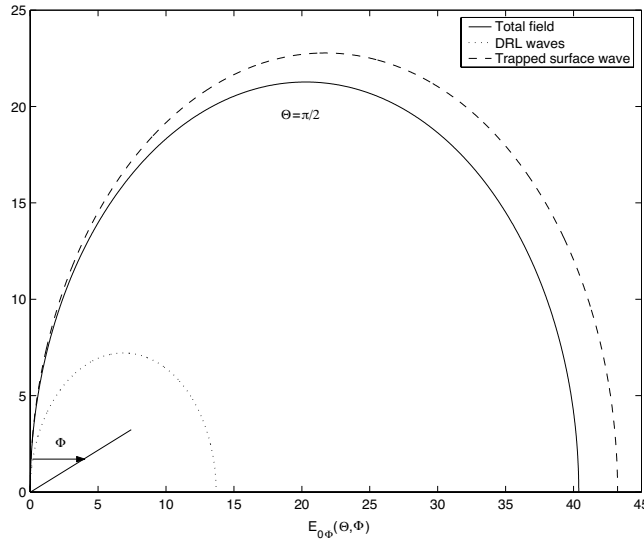

Figure 7. Electric field component $E_{0\Theta}$ in V/m vs. Θ with $f = 1$ GHz, $\epsilon_{1r} = 2.65$, $r_0 = 80$ m, $l = 0.1$ m, $d = 0$ m, and $\Phi = 0$.

Figure 8. Electric field component $E_{0\Phi}$ in V/m vs. Θ with $f = 1\,\mathrm{GHz}$, $\epsilon_{1r} = 2.65$, $r_0 = 20\,\mathrm{m}$, $l = 0.1\,\mathrm{m}$, $d = 0\,\mathrm{m}$, and $\Phi = \pi/2$.

Figure 9. Electric field component $[E_{0\Theta}^r(r_0, \Theta, \Phi)]_p$ radiated by a patch antenna vs. Φ with $f = 1\,\mathrm{GHz}$, $\epsilon_{1r} = 2$, $r_0 = 10\,\mathrm{m}$, $k_L h = \pi/2$, $w = h$, $l = 0.1\,\mathrm{m}$, $d = 0\,\mathrm{m}$, $I_0 = 1\,\mathrm{A}$, and $\Theta = \pi/2$.

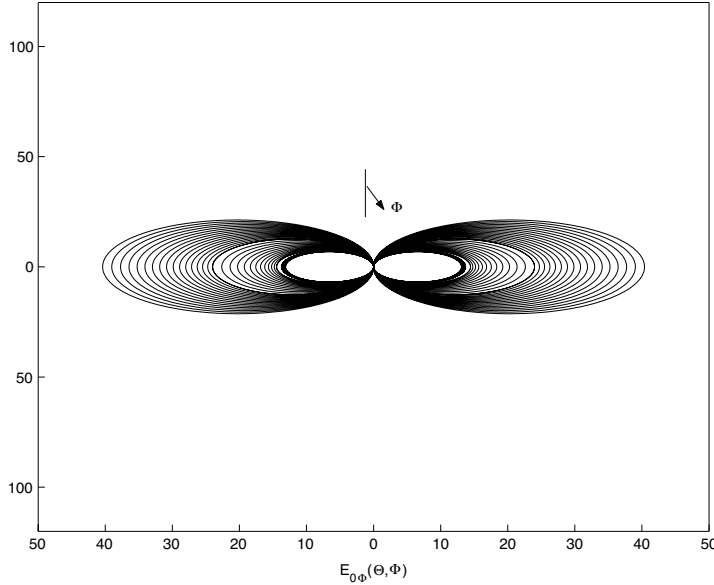**Figure 10.** $[E_{0\Phi}^r(r_0, \Theta, \Phi)]_p$ radiated by a patch antenna with $f = 1\,\mathrm{GHz}$, $\epsilon_{1r} = 2$, $r_0 = 10\,\mathrm{m}$, $k_L h = \pi/2$, $w = h$, $l = 0.1\,\mathrm{m}$, $d = 0\,\mathrm{m}$, $I_0 = 1\,\mathrm{A}$, and $\Theta = \pi/2$.
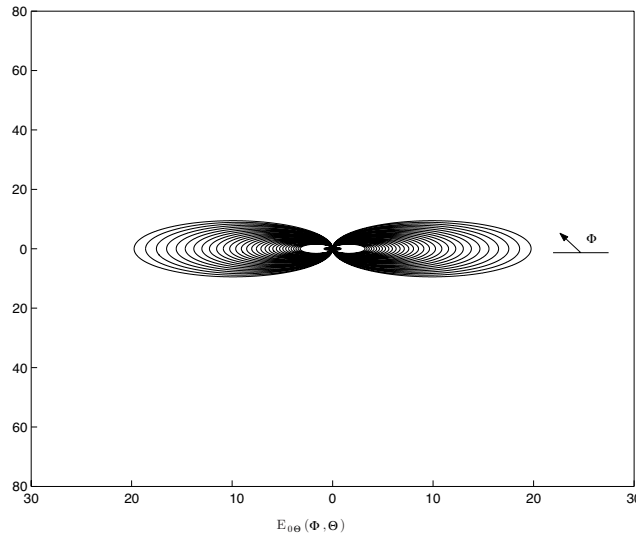
Figure 11. Three-dimensional diagram of the component $[E_{0\Theta}^r(r_0, \Theta, \Phi)]_p$ radiated by a patch antenna with $f = 1\,\mathrm{GHz}$, $\epsilon_{1r} = 2$, $r_0 = 10\,\mathrm{m}$, $k_L h = \pi/2$, $w = h$, $l = 0.1\,\mathrm{m}$, $d = 0\,\mathrm{m}$, and $I_0 = 1\,\mathrm{A}$.

Figure 12. Three-dimensional diagram of $[E_{0\Phi}^r(r_0, \Theta, \Phi)]_p$ of antenna with $f = 1\,\text{GHz}$, $\epsilon_{1r} = 2$, $r_0 = 10\,\text{m}$, $k_L h = \pi/2$, $w = h$, $l = 0.1\,\text{m}$, $d = 0\,\text{m}$, and $I_0 = 1\,\text{A}$.

field is determined primarily by the trapped surface wave when both the dipole point and observation point are located on the air-dielectric boundary.

In Figs. 11 and 12, three-dimensional diagrams of the two components $[E_{0\Theta}^r(r_0, \Theta, \Phi)]_p$ and $[E_{0\Phi}^r(r_0, \Theta, \Phi)]_p$ for a patch antenna are shown, respectively. It is noted that the inner curves in Figs. 11 and 12 is for $\Theta = 86°$ and the outer curves for $\Theta = 90°$. For the component $[E_{0\Theta}^r(r_0, \Theta, \Phi)]_p$, the radiation is enlarged greatly in the region near $\Theta = \pi/2$ and $\Phi = 0$, or π. For the component $[E_{0\Phi}^r(r_0, \Theta, \Phi)]_p$, the radiation are enlarged greatly in the region near $\Theta = \pi/2$ and $\Phi = \pi/2$.

The radiation from a patch antenna can be determined in the same manner for other types of load, such as resonant lines with impedance load and matched line with travelling-wave currents.

6. CONCLUSIONS

The far-field radiation of a horizontal dipole in the presence of a three-layered region have been investigated for its possible applications in microstrip antenna. In the analysis and computations on the radiation

of a horizontal dipole and microstrip antenna, which are addressed in chapter 15 of [12] and chapter 9 of [13], the trapped surface wave is not considered. In this paper we consider both the trapped surface wave and lateral wave in detail. The approximate formulas have been obtained for determining the poles of the electric-type waves under the condition of $\sqrt{k_1^2 - k_0^2}\, l \leq 0.6$ and that of the magnetic-type waves under the condition of $\sqrt{k_1^2 - k_0^2}\, l - \pi/2 \leq 1$. In addition, the radiation patterns of a patch antenna with specific current distributions are analyzed. Computations show that, for the component $E_{0\Theta}$, the total field is determined primarily by the trapped surface wave of electric type, and, for the component $E_{0\Phi}$, the total field is determined primarily by the trapped surface wave of magnetic type.

REFERENCES

1. Sommerfeld, A., "Propagation of waves in wireless telegraphy," *Ann. Phys.*, Vol. 28, 665–736, Mar. 1909.

2. Sommerfeld, A., "Propagation of waves in wireless telegraphy," *Ann. Phys.*, Vol. 81, 1135–1153, Dec. 1926.

3. Wait, J. R., "Radiation from a vertical electric dipole over a stratified ground," *IRE Trans. Antennas Propagat.*, Vol. 1, 9–12, 1953; Vol. 2, 144–146, 1954.

4. Wait, J. R., "Radiation from a vertical electric dipole over a curved stratified ground ground," *Journal of Res. Nat. Bur. Standards*, Vol. 56, 232–239, 1956.

5. Wait, J. R., "Excitation of surface waves on conducting dielectric clad and corrugated surfaces," *Journal of Res. Nat. Bur. Standards*, Vol. 59, No. 6, 365–377, 1957.

6. Wait, J. R., "Electromagnetic surface waves," *Advances in Radio Research*, 157–219, Academic Press, New York, 1964.

7. Wait, J. R., *Electromagnetic Waves in Stratified Media*, 2nd edition, Pergamon, New York, 1970.

8. Wait, J. R., "Radiation from a vertical electric dipole located over laterally anisotropic groundplane," *Electronics Letters*, Vol. 26, No. 1, 74–76, 1990.

9. Wait, J. R., "Electromagnetic fields of a vertical electric dipole over a laterally anisotropic surface," *IEEE Trans. Antennas and Propagat.*, Vol. 38, No. 10, 1719–1723, 1990.

10. Mahmoud, S. F., "Remarks on 'The electromagnetic field of a vertical electric dipole over the earth or sea'," *IEEE Trans. Antennas Propagat.*, Vol. 46, No. 12, 1745–1946, 1999.

11. Collin, R. E., "Some observations about the near zone electric field of a hertzian dipole above a lossy earth," *IEEE Trans. Antennas Propagat.*, Vol. 52, No. 11, 3133–3137, 2004.

12. King, R. W. P., M. Owens, and T. T. Wu, *Lateral Electromagnetic Waves: Theory and Applications to Communications, Geophysical Exploration, and Remoting Sensing*, Springer-Verlag, 1992.

13. King, R. W. P., G. J. Fikioris, and R. B. Mack, *Cylindrical Antennas and Array*, University Press, Cambridge, 2002.

14. King, R. W. P. and S. S. Sandler, "The electromagnetic field of a vertical electric dipole over the earth or sea," *IEEE Trans. Antennas Propagat.*, Vol. 42, No. 3, 382–389, 1994.

15. King, R. W. P., "The electromagnetic field of a horizontal electric dipole in the presence of a three-layered region," *J. Appl. Phys.*, Vol. 69, No. 12, 7987–7995, 1991

16. King, R. W. P., "The electromagnetic field of a horizontal electric dipole in the presence of a three-layered region: Supplement," *J. Appl. Phys.*, Vol. 74, No. 8, 4845–4548, 1993.

17. King, R. W. P. and S. S. Sandler, "The electromagnetic field of a vertical electric dipole in the presence of a three-layered region," *Radio Sci.*, Vol. 29, No. 1, 97–113, 1994.

18. Wait, J. R., "Comment on 'The electromagnetic field of a vertical electric dipole in the presence of a three-layered region' by Ronold, W. P. King and S. S. Sandler," *Radio Sci.*, Vol. 33, No. 2, 251–253, 1998.

19. King, R. W. P. and S. S. Sandler, "Reply," *Radio Sci.*, Vol. 33, No. 2, 255–256, 1998.

20. Collin, R. E., "Hertzian dipole radiation over a lossy earth or sea: Some early and late 20th century controversies," *IEEE Antennas and Propagation Magazine*, Vol. 46, No. 2, 64–79, 2004.

21. Koledintseva, M. Y., V. V. Bodrov, I. V. Sourkova, M. M. Sabirov, and V. I. Sourkov, "Unified spectral technique application for study of radiator behavior near planar layered composites," *Progress In Electromagnetics Research*, PIER 66, 317–357, 2006.

22. Xu, X.-B. and Y. Huang, "An efficient analysis of vertical dipole antennas above a lossy half-space," *Progress In Electromagnetics Research*, PIER 74, 353–377, 2007.

23. Abo-Seida, O. M., "Far-field due to a vertical magnetic dipole in sea," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 6, 707–715, 2006.

24. Zheng, Q.-R., Y.-Q. Fu, and N.-C. Yuan, "Characteristics of planar PBG structures with a cover layer," *Journal of*

Electromagnetic Waves and Applications, Vol. 20, No. 11, 1439–1453, 2006.

25. Chen, H. T., J.-X. Luo, and D.-K. Zhang, "An analytic formula of the current distribution for the VLF horizontal wire antenna above lossy half-space," *Progress In Electromagnetics Research Letters*, Vol. 1, 149–158, 2008.

26. Sharma, A. and G. Singh, "Design of single Pin shorted three-dielectric-layered substrates rectangular patch microstrip antenna for communication systems," *Progress In Electromagnetics Research Letters*, Vol. 2, 157–165, 2008.

27. Attiya, A. M., "Dyadic Green's function of an elementary point source above a periodically-defected-grounded dielectric slab," *Progress In Electromagnetics Research B*, Vol. 4, 127–145, 2008.

28. Zhang, L., Y. H. Zhang, W. Hu, W. Yu, and G.-Q. Zhu, "Automatic incorporation of surface wave poles in discrete complex image method," *Progress In Electromagnetics Research*, PIER 80, 161–178, 2008.

29. Zhang, H. Q. and W. Y. Pan, "Electromagnetic field of a vertical electric dipole on a perfect conductor coated with a dielectric layer," *Radio Science*, Vol. 37, No. 4, 13-1–13-7, 2002.

30. Zhang, H.-Q., K. Li, and W.-Y. Pan, "The electromagnetic field of a vertical dipole on the dielectric-coated imperfect conductor," *Journal of Electromagnetic Waves and Applications*, Vol. 18, No. 10, 1305–1320, 2004.

31. Zhang, H.-Q., W.-Y. Pan, K. Li, and K.-X. Shen, "Electromagnetic field for a horizontal electric dipole buried inside a dielectric layer coated high Lossy half space," *Progress In Electromagnetics Research*, PIER 50, 163–186, 2005.

32. Li, K. and Y. Lu, "Electromagnetic field generated by a horizontal electric dipole near the surface of a planar perfect conductor coated with a uniaxial layer," *IEEE Trans. Antennas and Propagat.*, Vol. 53, No. 10, 3191–3200, 2005.

33. Mei, J. P. and K. Li, "Electromagnetic field from a horizontal electric dipole on the surface of a high lossy medium coated with a uniaxial layer," *Progress In Electromagnetics Research*, PIER 73, 71–91, 2007.

34. Xu, Y. H., W. Ren, L. Liu, and K. Li, "Electromagnetic field of a horizontal electric dipole in the presence of a four-layered region," *Progress In Electromagnetics Research*, PIER 81, 371–391, 2008.

35. Xu, Y. H., K. Li, and L. Liu, "Electromagnetic field of a horizontal electric dipole in the presence of a four-layered region," *Progress*

In Electromagnetics Research, PIER 81, 371–391, 2008

36. Xu, Y. H., W. Ren, L. Liu, and K. Li, "Electromagnetic field of a horizontal electric dipole in the presence of a four-layered region," *Progress In Electromagnetics Research*, PIER 81, 371–391, 2008.

37. Liu, L. and K. Li, "Radiation from a vertical electric dipole in the presence of a three-layered region," *IEEE Trans. Antennas and Propagat.*, Vol. 55, No. 12, 3469–3475, 2007.

38. Hoffman, R. K., *Handbook of Microwave Integrated Circuits*, Sec. 3.2, Artech House, Norwood, MA, 1987.

39. Gradshteyn, I. S. and I. M. Ryzhik, *Table of Integrals, Series, and Products*, Academic Press, New York, 1980.