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ELECTROMAGNETIC FIELD FROM A VERTICAL ELECTRIC DIPOLE IN A FOUR-LAYERED REGION

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Abstract—In this paper, the region of interest consists of a perfect conductor, coated with the two layer dielectrics under the air. The completed analytical formulas have been derived for the electromagnetic field due to a vertical electric dipole in the four-layered region when both the source point and observation point are located in the upper dielectric layer. Similar to the three-layered case, the trapped surface wave, which is contributed by the sums of residues of the poles, can also be excited efficiently by a vertical electric dipole in the four-layered region. The lateral wave is determined by the integrations along the branch cuts.

1. INTRODUCTION

The electromagnetic field of a dipole source in a layered region has been visited by many investigators in the past decades [1–35]. In the pioneering works by Wait [1–5], the Sommerfeld integrals for the electromagnetic field in the layered region were evaluated by using asymptotic methods, contour integration, and branch cuts. Further developments were carried out by other pioneers. In particular, the electromagnetic fields due to horizontal and vertical electric dipoles in the two- and three-layered media were treated by King et al. [8–13]. Lately, in a series of works by Li et al. [20–22], the dyadic Green's function technique is applied to examine the electromagnetic field in a four-layered forest environment.

In 1990's, Wait [14] and Mahmoud [16] wrote comments on the work by King and Sandler [15] and regarded that the trapped surface wave, varying as $\rho^{-1/2}$ in the far-field region, should not be overlooked at three-layered case. In the 2004 Collin's paper [27], the analysis supports the conclusions reached by Wait and Mahmoud. Lately, several investigators have revisited the old problem and drawn conclusions that the trapped surface wave, which is determined by the sums of residues of the poles, can be excited efficiently by a dipole source in the presence of a three-layered region [31–34]. It is concluded, naturally, that the trapped surface wave can also be excited efficiently by a dipole source in the four-layered region. In the available references [31–34], the term being contributed by the sums of residues of the poles, is named the surface wave, and the electromagnetic field of a point source in a multi-layered region is examined in detail.

In the former paper [36], the complete formulas are derived for the electromagnetic field of a vertical electric dipole in the presence of a four-layered region. However, when both dipole source and observation point are located in the second layer, because of multi-refection, the problem becomes more complex. In what follows, we will attempt to derive the completed formulas of the electromagnetic field generated by a vertical electric dipole in the four-layered region. The region of interest consists of a perfect conductor, coated with the two layer dielectrics under the air and both the source point and observation point are located in the upper dielectric layer. In Section 2, the integrated formulas of the electromagnetic field are derived by using Fourier transform technique. In Section 3, both the trapped surface wave and the lateral wave are evaluated. It is noted that the trapped surface wave and the lateral wave are determined by the residues of the poles and the integrations of the branch cuts, respectively. In Section 4, computations and discussions are carried out. It is concluded that the far field is determined primarily by the trapped surface wave in the four-layered region when both the the dipole point and the observation point are on or near the boundary between Regions 1 and 2. In Section 5, some conclusions are drawn.

2. THE INTEGRATED FORMULAS FOR THE ELECTROMAGNETIC FIELD BY USING FOURIER TRANSFORM TECHNIQUE

The relevant geometry and Cartesian coordinate system are illustrated in Fig. 1, where a vertical electric dipole in the $\hat{z}$ direction is located at $(0, 0, d)$. The space above the two-layered dielectrics is Region 0 $(z \geq h)$ occupied by the air. The upper dielectric layer is Region 1

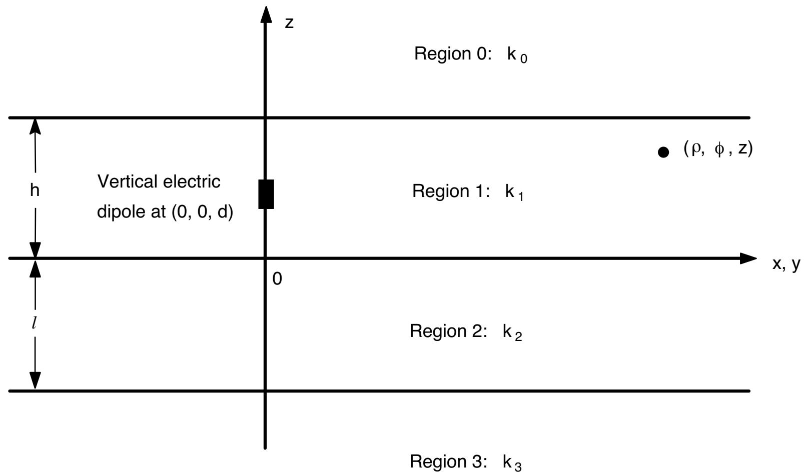

Figure 1. Geometry of a vertical electric dipole in the four-layered region.

($0 \leq z \leq h$) characterized by the permeability μ_0 and permittivity ϵ_1. The lower dielectric layer is Region 2 ($-l \leq z \leq 0$) characterized by the permeability μ_0 and ϵ_2. The rest space is Region 3 ($z \leq -l$) occupied by a perfect conductor or a dielectric characterized by the permeability μ_0 and permittivity ϵ_3. With the time dependence of $e^{-i\omega t}$, Maxwell equations can be written as follows:

$$\bigtriangledown \times \mathbf{E}_j = i\omega \mathbf{B}_j \tag{1}$$

$$\bigtriangledown \times \mathbf{B}_j = -i\frac{k_j^2 \mathbf{E}_j}{\omega} + \mu_0 \mathbf{J} \tag{2}$$

where

$$k_j = \omega\sqrt{\mu_0 \epsilon_j}; \quad j = 0, 1, 2, 3 \tag{3}$$

$$\mathbf{J} = \hat{z} I dl \delta(x)\delta(y)\delta(z-d) \tag{4}$$

is the externally maintained current in the active dipole.

The integrated formulas of the field in the four-layered region may be derived by using Fourier transform technique. Let

$$\mathbf{E}(x, y, z) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i(\epsilon x + \eta y)} \widetilde{\mathbf{E}}(\xi, \eta, z) d\xi d\eta. \tag{5}$$

Similar transforms apply to $\mathbf{B}$ and $\mathbf{J}$. Then, it follows that

$$\left(\frac{d}{dz^2} + \gamma_1^2\right) \widetilde{B}_{1x} = -i\eta\mu_0\delta(z-d) \tag{6}$$

$$\left(\frac{d}{dz^2}+\gamma_j^2\right)\widetilde{B}_{jx} = 0 \tag{7}$$

where $\gamma_j = \sqrt{k_j^2-\varepsilon^2-\eta^2}$, $j = 0,1,2,3$ and $\mathrm{Im}\gamma_j \geq 0$. The rest five components can be expressed in terms of $\widetilde{B}_{jx}$.

$$\widetilde{E}_{jx} = -i\frac{\omega}{k_j^2}\frac{\partial \widetilde{B}_{jy}}{\partial z} = i\frac{\omega}{k_j^2}\frac{\xi}{\eta}\frac{\partial \widetilde{B}_{jx}}{\partial z} \tag{8}$$

$$\widetilde{E}_{jy} = i\frac{\omega}{k_j^2}\frac{\partial \widetilde{B}_{jx}}{\partial z} \tag{9}$$

$$\widetilde{E}_{jz} = \frac{\omega}{\eta k_j^2}\left(\frac{d}{dz^2}+k_j^2\right)\widetilde{B}_{jx} \tag{10}$$

$$\widetilde{B}_{jy} = -\frac{\xi}{\eta}\widetilde{B}_{jx} \tag{11}$$

$$\widetilde{B}_{jz} = 0. \tag{12}$$

Because the dipole source is in Region 1, the solutions for the four layers can be written as

$$\widetilde{B}_{0x} = C_3 e^{i\gamma_0 z} \tag{13}$$

$$\widetilde{B}_{1x} = C_1 e^{i\gamma_1 z}+C_2 e^{-i\gamma_1 z}-\frac{\eta\mu_0}{2\gamma_1}e^{i\gamma_1|z-d|} \tag{14}$$

$$\widetilde{B}_{2x} = C_4 e^{i\gamma_2 z}+C_5 e^{-i\gamma_2 z} \tag{15}$$

$$\widetilde{B}_{3x} = C_6 e^{-i\gamma_3 z}. \tag{16}$$

The boundary conditions for the components $\widetilde{B}_{jx}$ and $\widetilde{E}_{jy}$ lead to the following equations.

$$C_1 e^{i\gamma_1 h}+C_2 e^{-i\gamma_1 h}-\frac{\eta\mu_0}{2\gamma_1}e^{i\gamma_1(h-d)} = C_3 e^{i\gamma_0 h} \tag{17}$$

$$\frac{\gamma_1}{k_1^2}\left[C_1 e^{i\gamma_1 h}-C_2 e^{-i\gamma_1 h}-\frac{\eta\mu_0}{2\gamma_1}e^{i\gamma_1(h-d)}\right] = \frac{\gamma_0}{k_0^2}C_3 e^{i\gamma_0 h} \tag{18}$$

$$C_1+C_2-\frac{\eta\mu_0}{2\gamma_1}e^{i\gamma_1 d} = C_4+C_5 \tag{19}$$

$$\frac{\gamma_1}{k_1^2}\left(C_1-C_2+\frac{\eta\mu_0}{2\gamma_1}e^{i\gamma_1 d}\right) = (C_4-C_5)\frac{\gamma_2}{k_2^2} \tag{20}$$

$$C_4 e^{-i\gamma_2 l}+C_5 e^{i\gamma_2 l} = C_6 e^{i\gamma_3 l} \tag{21}$$

$$\left(C_4 e^{-i\gamma_2 l}-C_5 e^{i\gamma_2 l}\right)\frac{\gamma_2}{k_2^2} = C_6 e^{i\gamma_3 l}\frac{-\gamma_3}{k_3^2}. \tag{22}$$

With (21) and (22), we have

$$\frac{\gamma_2}{k_2^2}\left(C_4e^{-i\gamma_2 l}-C_5e^{i\gamma_2 l}\right)=\frac{-\gamma_3}{k_3^2}\left(C_4e^{-i\gamma_2 l}+C_5e^{i\gamma_2 l}\right) \tag{23}$$

then,

$$C_4e^{-i\gamma_2 l}=\frac{\frac{\gamma_2}{k_2^2}-\frac{\gamma_3}{k_3^2}}{\frac{\gamma_2}{k_2^2}+\frac{\gamma_3}{k_3^2}}C_5e^{i\gamma_2 l}. \tag{24}$$

With (17) and (18), we have

$$\begin{aligned}&\frac{\gamma_1}{k_1^2}\left[C_1e^{i\gamma_1 h}-C_2e^{-i\gamma_1 h}-\frac{\eta\mu_0}{2\gamma_1}e^{i\gamma_1(h-d)}\right]\\ =&\ \frac{\gamma_0}{k_0^2}\left[C_1e^{i\gamma_1 h}+C_2e^{-i\gamma_1 h}-\frac{\eta\mu_0}{2\gamma_1}e^{i\gamma_1(h-d)}\right]\end{aligned} \tag{25}$$

$$\left(\frac{\gamma_0}{k_0^2}+\frac{\gamma_1}{k_1^2}\right)C_2e^{-i\gamma_1 h}=\left(\frac{\gamma_1}{k_1^2}-\frac{\gamma_0}{k_0^2}\right)C_1e^{i\gamma_1 h}-\left(\frac{\gamma_1}{k_1^2}-\frac{\gamma_0}{k_0^2}\right)\frac{\eta\mu_0}{2\gamma_1}e^{i\gamma_1(h-d)}. \tag{26}$$

Multiplying $e^{-i\gamma_2 l}$ to both sides of (19) leads to

$$\begin{aligned}\left(C_1+C_2-\frac{\eta\mu_0}{2\gamma_1}e^{i\gamma_1 d}\right)e^{-i\gamma_2 l}&=\left(C_4e^{-i\gamma_2 l}+C_5e^{-i\gamma_2 l}\right)\\ &=\left(\frac{\frac{\gamma_2}{k_2^2}-\frac{\gamma_3}{k_3^2}}{\frac{\gamma_2}{k_2^2}+\frac{\gamma_3}{k_3^2}}e^{i\gamma_2 l}+e^{-i\gamma_2 l}\right)C_5.\end{aligned} \tag{27}$$

Similarly, multiplying $e^{-i\gamma_2 l}$ to both sides of (20) yields to

$$\begin{aligned}\frac{\gamma_1}{k_1^2}\left(C_1-C_2+\frac{\eta\mu_0}{2\gamma_1}e^{i\gamma_1 d}\right)e^{-i\gamma_2 l}&=\frac{\gamma_2}{k_2^2}\left(C_4e^{-i\gamma_2 l}-C_5e^{-i\gamma_2 l}\right)\\ &=\frac{\gamma_2}{k_2^2}\left(\frac{\frac{\gamma_2}{k_2^2}-\frac{\gamma_3}{k_3^2}}{\frac{\gamma_2}{k_2^2}+\frac{\gamma_3}{k_3^2}}e^{i\gamma_2 l}-e^{-i\gamma_2 l}\right)C_5.\end{aligned} \tag{28}$$

From (27) and (28), it follows that

$$\begin{aligned}&\frac{\gamma_1}{k_1^2}\left(C_1-C_2+\frac{\eta\mu_0}{2\gamma_1}e^{i\gamma_1 d}\right)\left[\left(\frac{\gamma_2}{k_2^2}-\frac{\gamma_3}{k_3^2}\right)e^{i\gamma_2 l}+\left(\frac{\gamma_2}{k_2^2}+\frac{\gamma_3}{k_3^2}\right)e^{-i\gamma_2 l}\right]\\ &=\frac{\gamma_2}{k_2^2}\left(C_1+C_2-\frac{\eta\mu_0}{2\gamma_1}e^{i\gamma_1 d}\right)\times\left[\left(\frac{\gamma_2}{k_2^2}-\frac{\gamma_3}{k_3^2}\right)e^{i\gamma_2 l}-\left(\frac{\gamma_2}{k_2^2}+\frac{\gamma_3}{k_3^2}\right)e^{-i\gamma_2 l}\right].\end{aligned} \tag{29}$$

Then, we write

$$\begin{aligned} & C_1\left(\frac{\gamma_1}{k_1^2} - i\frac{\gamma_3\gamma_1 k_2^2}{k_3^2 k_1^2 \gamma_2}\tan\gamma_2 l + \frac{\gamma_3}{k_3^2} - i\frac{\gamma_2}{k_2^2}\tan\gamma_2 l\right) \\ & +\frac{\eta\mu_0}{2\gamma_1}e^{i\gamma_1 d}\left(\frac{\gamma_1}{k_1^2} - i\frac{\gamma_3\gamma_1 k_2^2}{k_3^2 k_1^2 \gamma_2}\tan\gamma_2 l + i\frac{\gamma_2}{k_2^2}\tan\gamma_2 l - \frac{\gamma_3}{k_3^2}\right) \\ = \; & C_2\cdot\left(\frac{\gamma_1}{k_1^2} - i\frac{\gamma_3\gamma_1 k_2^2}{k_3^2 k_1^2 \gamma_2}\tan\gamma_2 l + i\frac{\gamma_2}{k_2^2}\tan\gamma_2 l - \frac{\gamma_3}{k_3^2}\right). \end{aligned} \tag{30}$$

In this paper, the case of interest is that Region 3 is a perfect conductor. We assume

$$m = \lim_{k_3\to\infty}\left(\frac{\gamma_1}{k_1^2} - i\frac{\gamma_3\gamma_1 k_2^2}{k_3^2 k_1^2\gamma_2}\tan\gamma_2 l\right) = \frac{\gamma_1}{k_1^2} \tag{31}$$

$$n = \lim_{k_3\to\infty}\left(\frac{\gamma_3}{k_3^2} - i\frac{\gamma_2}{k_2^2}\tan\gamma_2 l\right) = -i\frac{\gamma_2}{k_2^2}\tan\gamma_2 l \tag{32}$$

(30) can be rewritten as

$$C_1(m+n) + \frac{\eta\mu_0}{2\gamma_1}e^{i\gamma_1 d}(m-n) = C_2(m-n) \tag{33}$$

$$\left(\frac{\gamma_1}{k_1^2}-\frac{\gamma_0}{k_0^2}\right)e^{i\gamma_1 h}C_1 - \left(\frac{\gamma_1}{k_1^2}-\frac{\gamma_0}{k_0^2}\right)\frac{\eta\mu_0}{2\gamma_1}e^{i\gamma_1(h-d)} = \left(\frac{\gamma_1}{k_1^2}+\frac{\gamma_0}{k_0^2}\right)e^{-i\gamma_1 h}C_2. \tag{34}$$

With (33) and (34), it is obtained readily.

$$C_1 = -\frac{\eta\mu_0}{2\gamma_1}\cdot Q \tag{35}$$

where

$$\begin{aligned} Q = \; & \frac{(m-n)}{\left(m\frac{\gamma_0}{k_0^2} + n\frac{\gamma_1}{k_1^2}\right) - i\tan\gamma_1 h\left(m\frac{\gamma_1}{k_1^2} + n\frac{\gamma_0}{k_0^2}\right)} \\ & \cdot\left[\frac{\gamma_1}{k_1^2}\cos\gamma_1 d + i\frac{\gamma_0}{k_0^2}\sin\gamma_1 d + \left(\frac{\gamma_1}{k_1^2}\sin\gamma_1 d - i\frac{\gamma_0}{k_0^2}\cos\gamma_1 d\right)\tan\gamma_1 h\right]. \end{aligned} \tag{36}$$

Similarly,

$$C_2 = -\frac{\eta\mu_0}{2\gamma_1}e^{i\gamma_1 2h}P \tag{37}$$

where

$$P = \frac{\left(\frac{\gamma_1}{k_1^2} - \frac{\gamma_0}{k_0^2}\right)\left[(m\cos\gamma_1 d - in\sin\gamma_1 d)\cdot(1 - i\tan\gamma_1 h)\right]}{\left[m\left(\frac{\gamma_0}{k_0^2} - i\frac{\gamma_1}{k_1^2}\tan\gamma_1 h\right) + n\left(\frac{\gamma_1}{k_1^2} - i\frac{\gamma_0}{k_0^2}\tan\gamma_1 h\right)\right]}. \quad (38)$$

Then, we have

$$\begin{aligned}\widetilde{B}_{1x} &= -\frac{\eta\mu_0}{2\gamma_1}Qe^{i\gamma_1 z} - \frac{\eta\mu_0}{2\gamma_1}e^{i2\gamma_1 h}Pe^{-i\gamma_1 z} - \frac{\eta\mu_0}{2\gamma_1}e^{i\gamma_1|z-d|} \\ &= -\frac{\eta\mu_0}{2\gamma_1}\left[Qe^{i\gamma_1 z} + Pe^{i\gamma_1(2h-z)} + e^{i\gamma_1|z-d|}\right]. \quad (39)\end{aligned}$$

From the relations in (8)–(12), we have

$$\widetilde{B}_{1y} = \frac{\varepsilon\mu_0}{2\gamma_1}\left[Qe^{i\gamma_1 z} + Pe^{i\gamma_1(2h-z)} + e^{i\gamma_1|z-d|}\right] \quad (40)$$

$$\widetilde{B}_{1z} = 0 \quad (41)$$

$$\widetilde{E}_{1x} = \frac{\omega\varepsilon\mu_0}{2k_1^2}\left[Qe^{i\gamma_1 z} - Pe^{i\gamma_1(2h-z)} \pm e^{i\gamma_1|z-d|}\right] \quad (42)$$

$$\widetilde{E}_{1y} = \frac{\eta}{\varepsilon}\widetilde{E}_{1x} = \frac{\omega\eta\mu_0}{2k_1^2}\left[Qe^{i\gamma_1 z} - Pe^{i\gamma_1(2h-z)} \pm e^{i\gamma_1|z-d|}\right] \quad (43)$$

$$\begin{aligned}\widetilde{E}_{1z} &= \frac{\omega}{\eta k_1^2}\left(\frac{d}{dz^2} + k_1^2\right)\widetilde{B}_{1x} \\ &= -\frac{\omega\mu_0}{2\gamma_1 k_1^2}\lambda^2\left[Qe^{i\gamma_1 z} + Pe^{i\gamma_1(2h-z)} + e^{i\gamma_1|z-d|}\right]. \quad (44)\end{aligned}$$

It is now convenient to express the field components in the cylindrical coordinates ρ, ϕ, z with the relations

$$x = \rho\cos\phi, y = \rho\sin\phi \quad (45)$$

$$\xi = \lambda\cos\phi', \eta = \lambda\sin\phi' \quad (46)$$

and the integrated representations of the Bessel functions, viz.,

$$J_n(\lambda\rho) = \frac{i^{-n}}{2\pi}\int_0^{2\pi} e^{i(\lambda\rho\cos\theta + n\theta)}d\theta. \quad (47)$$

From (39)–(44), using the Fourier integrals like (5) and the following relations

$$E_{1\rho} = E_{1x}\cos\phi + E_{1y}\sin\phi \quad (48)$$

$$B_{1\phi} = -B_{1x}\sin\phi + B_{1y}\cos\phi \quad (49)$$

the field components in Region 1 may be written as follows:

$$E_{1\rho} = -\frac{i\omega\mu_0}{4\pi k_1^2}\left[\int_0^\infty \mp e^{i\gamma_1|z-d|}\lambda^2 J_1(\lambda\rho)d\lambda - \int_0^\infty Qe^{i\gamma_1 z}\lambda^2 J_1(\lambda\rho)d\lambda + \int_0^\infty Pe^{i\gamma_1(2h-z)}\lambda^2 J_0(\lambda\rho)d\lambda\right] \quad (50)$$

$$E_{1z} = -\frac{\omega\mu_0}{4\pi k_1^2}\left[\int_0^\infty e^{i\gamma_1|z-d|}\gamma_1^{-1}J_0(\lambda\rho)\lambda^3 d\lambda + \int_0^\infty Qe^{i\gamma_1 z}\gamma_1^{-1}J_0(\lambda\rho)\lambda^3 d\lambda + \int_0^\infty Pe^{i\gamma_1(2h-z)}\gamma_1^{-1}J_0(\lambda\rho)\lambda^3 d\lambda\right] \quad (51)$$

$$B_{1\phi} = \frac{i\mu_0}{4\pi}\left[\int_0^\infty e^{i\gamma_1|z-d|}\lambda^2\gamma_1^{-1}J_1(\lambda\rho)d\lambda + \int_0^\infty Qe^{i\gamma_1 z}\lambda^2\gamma_1^{-1}J_1(\lambda\rho)d\lambda + \int_0^\infty Pe^{i\gamma_1(2h-z)}\lambda^2\gamma_1^{-1}J_1(\lambda\rho)d\lambda\right] \quad (52)$$

where the upper sign in (50) is for the region $z \geq d$, and the lower sign for $0 \leq z \leq d$. In order to see useful physical insights, and taking into account the relationship $H_n^{(1)}(-\lambda\rho) = H_n^{(2)}(\lambda\rho)(-1)^{n+1}$, it is convenient to rewrite the integrated formulas in the following forms.

$$E_{1\rho} = E_{1\rho}^{(1)} + E_{1\rho}^{(2)} + E_{1\rho}^{(3)} \quad (53)$$

$$E_{1z} = E_{1z}^{(1)} + E_{1z}^{(2)} + E_{1z}^{(3)} \quad (54)$$

$$B_{1\phi} = B_{1\phi}^{(1)} + B_{1\phi}^{(2)} + B_{1\phi}^{(3)} \quad (55)$$

where

$$E_{1\rho}^{(1)} = -\frac{i}{8\pi\omega\epsilon_1}\int_{-\infty}^{\infty} \mp e^{i\gamma_1|z-d|}H_1^{(1)}(\lambda\rho)\lambda^2 d\lambda \quad (56)$$

$$E_{1\rho}^{(2)} = \frac{i}{8\pi\omega\epsilon_1}\int_{-\infty}^{\infty} Qe^{i\gamma_1 z}H_1^{(1)}(\lambda\rho)\lambda^2 d\lambda \quad (57)$$

$$E_{1\rho}^{(3)} = \frac{i}{8\pi\omega\epsilon_1}\int_{-\infty}^{\infty} -Pe^{i\gamma_1(2h-z)}H_1^{(1)}(\lambda\rho)\lambda^2 d\lambda \quad (58)$$

$$E_{1z}^{(1)} = -\frac{1}{8\pi\omega\epsilon_1}\int_{-\infty}^{\infty} e^{i\gamma_1|z-d|}\gamma_1^{-1}H_0^{(1)}(\lambda\rho)\lambda^3 d\lambda \quad (59)$$

$$E_{1z}^{(2)} = -\frac{1}{8\pi\omega\epsilon_1}\int_{-\infty}^{\infty} Qe^{i\gamma_1 z}\gamma_1^{-1}H_0^{(1)}(\lambda\rho)\lambda^3 d\lambda \quad (60)$$

$$E_{1z}^{(3)} = -\frac{1}{8\pi\omega\epsilon_1}\int_{-\infty}^{\infty} Pe^{i\gamma_1(2h-z)}\gamma_1^{-1}H_0^{(1)}(\lambda\rho)\lambda^3 d\lambda \quad (61)$$

$$B_{1\phi}^{(1)} = \frac{i\mu_0}{8\pi}\int_{-\infty}^{\infty} e^{i\gamma_1|z-d|}\gamma_1^{-1}H_1^{(1)}(\lambda\rho)\lambda^2 d\lambda \tag{62}$$

$$B_{1\varphi}^{(2)} = \frac{i\mu_0}{8\pi}\int_{-\infty}^{\infty} Qe^{i\gamma_1 z}\gamma_1^{-1}H_1^{(1)}(\lambda\rho)\lambda^2 d\lambda \tag{63}$$

$$B_{1\varphi}^{(3)} = \frac{i\mu_0}{8\pi}\int_{-\infty}^{\infty} Pe^{i\gamma_1(2h-z)}\gamma_1^{-1}H_1^{(1)}(\lambda\rho)\lambda^2 d.\lambda \tag{64}$$

It is seen that (56), (59), and (62) stand for the direct wave, which have been evaluated in the monograph by King, Owens, and Wu [8]. When $k_0 = k_1$, the above formulas can be reduced to those for three-layered case addressed in [29]. Obviously, the above integrals including the Bessel functions $J_i(\lambda\rho)$ or $H_i^{(1)}(\lambda\rho)$ $(i = 0, 1)$ with high oscillatory, these integrals converge very slowly. It is necessary to evaluate the above integrals including Q and P by using analytical techniques.

3. EVALUATIONS FOR THE INTEGRALS

In order to evaluate the six integrals $E_{1\rho}^{(2)}$, $E_{1\rho}^{(3)}$, $E_{1z}^{(2)}$, $E_{1z}^{(3)}$, $B_{1\phi}^{(2)}$, and $B_{1\phi}^{(3)}$, it is necessary to shift the contour around the branch lines at $\lambda = k_0$, $\lambda = k_1$, and $\lambda = k_2$. The configuration of the poles and the branch cuts is shown in Fig. 2. The main tasks in this section are to determine the poles and to evaluate the integrations along the branch cuts Γ_0, Γ_1, and Γ_2.

The pole equation reads in the following form.

$$\begin{aligned} f(\lambda) = &\frac{\gamma_1}{k_1^2}\frac{\gamma_0}{k_0^2} - i\frac{\gamma_2}{k_2^2}\tan\gamma_2 l \cdot \frac{\gamma_1}{k_1^2} \\ &-i\left(\frac{\gamma_1^2}{k_1^4}\tan\gamma_1 h - i\frac{\gamma_0}{k_0^2}\frac{\gamma_2}{k_2^2}\tan\gamma_2 l\tan\gamma_1 h\right) = 0. \end{aligned} \tag{65}$$

Comparing with the corresponding three-layered case as addressed in [29], the pole equation becomes more complex. It will be analyzed in the following four cases.

In the first case of positive real λ with $\lambda < k_0$, then γ_0, γ_1, and γ_2 are positive real numbers. Then, we have

$$\frac{\gamma_1\gamma_0}{k_1^2k_0^2} - \frac{\gamma_0\gamma_2}{k_0^2k_2^2}\tan\gamma_2 l\tan\gamma_1 h - i\left(\frac{\gamma_1\gamma_2}{k_1^2k_2^2}\tan\gamma_2 l + \frac{\gamma_1^2}{k_1^4}\tan\gamma_1 h\right) = 0. \tag{66}$$

Obviously, no pole exists in the interval $\lambda < k_0$.

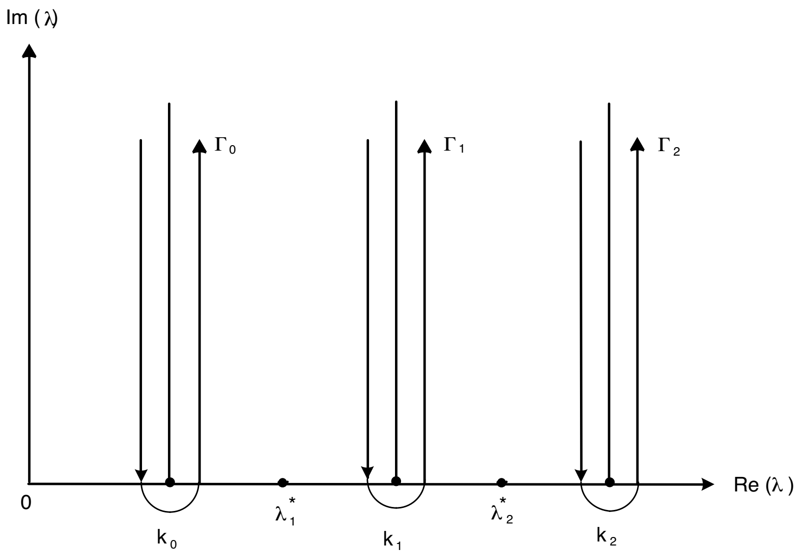

Figure 2. The configuration of the poles and the branch cuts.

In the second case with $k_0 < \lambda < k_1$, $\gamma_0 = i\sqrt{\lambda^2 - k_0^2} = i\gamma_0'$, γ_0', γ_1, and γ_2 are positive real numbers. Then, we obtain

$$\frac{\gamma_1\gamma_0'}{k_0^2k_1^2} - \frac{\gamma_1\gamma_2\tan\gamma_2 l}{k_1^2k_2^2} - \frac{\gamma_0'\gamma_2\tan\gamma_2 l\tan\gamma_1 h}{k_0^2k_2^2} - \frac{\gamma_1^2\tan\gamma_1 h}{k_1^4} = 0. \quad (67)$$

The poles can be determined by (67).

In the third case with $k_1 < \lambda < k_2$, $\gamma_i = i\sqrt{\lambda^2 - k_i^2} = i\gamma_i'$ $(i = 0, 1)$. γ_0', γ_1', and γ_2 are positive real numbers. Then, we get

$$-\frac{\gamma_1'\gamma_0'}{k_1^2k_0^2} + \frac{\gamma_1'\gamma_2\tan\gamma_2 l}{k_1^2k_2^2} + \frac{\gamma_0'\gamma_2}{k_0^2k_2^2}\tan\gamma_2 l\tanh\gamma_1' h - \frac{{\gamma_1'}^2}{k_1^4}\tanh\gamma_1' h = 0. \quad (68)$$

The poles can be determined by (68).

In the fourth case with $\lambda > k_2$, $\gamma_i = i\sqrt{\lambda^2 - k_i^2} = i\gamma_i'$ $(i = 0, 1, 2)$. γ_0', γ_1', and γ_2' are positive real numbers. Then, we write

$$-\frac{\gamma_1'\gamma_0'}{k_1^2k_0^2} - \frac{\gamma_1'\gamma_2'}{k_1^2k_2^2}\tanh\gamma_2' l - \frac{\gamma_0'\gamma_2'}{k_0^2k_2^2}\tanh\gamma_2' l\tanh\gamma_1' h - \frac{\gamma^{2\prime}{}_1}{k_1^4}\tanh\gamma_1' h = 0. \quad (69)$$

From (69), it is found that no pole existed in the interval $\lambda > k_2$.

From the above analysis, it is concluded that the poles may exist in the intervals $k_0 < \lambda_j^* < k_2$, which can be determined by using Newton method as addressed in [29]. Then, the integrals $E_{1z}^{(2)}$ and $E_{1z}^{(3)}$ can be expressed as follows:

$$\begin{aligned} E_{1\rho}^{(2)} &= -\frac{\omega\mu_0}{4k_1^2}\sum_j Q'(\lambda_j^*)e^{i\gamma_{1j}^* z}H_1^{(1)}(\lambda_j^*\rho)\lambda^{*2} \\ &\quad +\frac{i}{8\pi\omega\epsilon_1}\int_{\Gamma_0+\Gamma_1+\Gamma_2} Qe^{i\gamma_1 z}H_0^{(1)}(\lambda\rho)\lambda^2 d\lambda \end{aligned} \tag{70}$$

$$\begin{aligned} E_{1\rho}^{(3)} &= \frac{\omega\mu_0}{4k_1^2}\sum_j P(\lambda_j^*)e^{i\gamma_{1j}^* z}H_1^{(1)}(\lambda_j^*\rho)\lambda^{*2} \\ &\quad -\frac{1}{8\pi\omega\epsilon_1}\int_{\Gamma_0+\Gamma_1+\Gamma_2} Pe^{i\gamma_1(2h-z)}H_0^{(1)}(\lambda\rho)\lambda^2 d\lambda. \end{aligned} \tag{71}$$

where

$$\begin{aligned} Q(\lambda_j^*) &= \left(\frac{\gamma_{1j}^*}{k_1^2}-n^*\right)\cdot\left(\frac{\gamma_{1j}^*}{k_1^2}\cos\gamma_{1j}^* d+\frac{\gamma_{1j}^*}{k_1^2}\tan\gamma_{1j}^* h\sin\gamma_{1j}^* d\right. \\ &\quad \left.-i\frac{\gamma_{0j}^*}{k_0^2}\tan\gamma_{1j}^* h\cos\gamma_{1j}^* d+i\frac{\gamma_{0j}^*}{k_0^2}\sin\gamma_{1j}^* d\right)/q'(\lambda^*) \end{aligned} \tag{72}$$

$$P(\lambda_j^*) = \frac{\left(\frac{\gamma_{1j}^*}{k_1^2}-\frac{\gamma_{0j}^*}{k_0^2}\right)\cdot\left(\frac{\gamma_{1j}^*}{k_1^2}\cos\gamma_{1j}^* d - in^*\sin\gamma_{1j}^* d\right)\cdot\left(1-i\tan\gamma_{1j}^* h\right)}{q'(\lambda_j^*)} \tag{73}$$

$$q(\lambda) = -i\frac{{\gamma_1}^2}{k_1^4}\tan\gamma_1 h - i\frac{\gamma_0}{k_0^2}n^*\tan\gamma_1 h+\frac{\gamma_1 n^*}{k_1^2}+\frac{\gamma_1\gamma_0}{k_1^2 k_0^2} \tag{74}$$

$$\begin{aligned} q'(\lambda) &= \lambda\left[\frac{i}{k_1^4}\left(2\tan\gamma_1 h+h\gamma_1\sec^2\gamma_1 h\right)\right. \\ &\quad +\frac{1}{k_0^2k_2^2}\left(\frac{\gamma_2}{\gamma_0}\tan\gamma_2 l\tan\gamma_1 h+\frac{\gamma_0}{\gamma_2}\tan\gamma_2 l\tan\gamma_1 h\right. \\ &\quad \left.+\gamma_0 l\sec^2\gamma_2 l\tan\gamma_1 h+\frac{\gamma_0\gamma_2 h}{\gamma_1}\tan\gamma_2 l\sec^2\gamma_1 h\right) \\ &\quad +i\frac{1}{k_1^2k_2^2}\left(\frac{\gamma_2\tan\gamma_2 l}{\gamma_1}+\frac{\gamma_1}{\gamma_2}\tan\gamma_2 l+\gamma_1 l\sec^2\gamma_2 l\right) \\ &\quad \left.-\frac{1}{k_1^2k_0^2}\left(\frac{\gamma_0}{\gamma_1}+\frac{\gamma_1}{\gamma_0}\right)\right] \end{aligned} \tag{75}$$

$$\gamma_{ij}^* = \sqrt{k_i^2-\lambda_j^{*2}}, \qquad i=0,1 \tag{76}$$

$$n^* = -i\frac{\gamma_2^*}{k_2^2}\tan\gamma_2^* l. \tag{77}$$

Because both the integrands $P(\lambda)$ and $Q(\lambda)$ are even functions of γ_2, the integrals in (70) and (71) along the branch cut Γ_2 are zero. Next, we will evaluate the integrals in (70) and (71) along the branch cuts Γ_1 and Γ_0.

Taking into account the conditions of $k_1\rho \gg 1$ and $(z+d) \ll \rho$, the dominant contribution of the integral along the branch line Γ_1 comes from the vicinity of k_1. Let $\lambda = k_1(1+i\tau^2)$, γ_0, γ_1, and γ_2 at the vicinity of k_0 can be approximated as follows:

$$\gamma_{01} = \sqrt{k_0^2-\lambda^2} \approx i\sqrt{k_1^2-k_0^2} \tag{78}$$

$$\gamma_{11} = \sqrt{k_1^2-\lambda^2} \approx \sqrt{2}k_1 e^{i\frac{3\pi}{4}}\tau \tag{79}$$

$$\gamma_{21} = \sqrt{k_2^2-\lambda^2} \approx \sqrt{k_2^2-k_0^2}. \tag{80}$$

Considering the case of interest that both h and d are not very large, we arrive at the following expressions.

$$\cos\gamma_{11}d \approx 1; \quad \tan\gamma_{11}h \approx \gamma_{11}h; \quad \sin\gamma_{11}d \approx \gamma_{11}d. \tag{81}$$

Substituting (81) into (36), and neglecting the high-order terms of γ_{11}, we have

$$Q = \left(\tau + \frac{ik_1\sqrt{k_2^2-k_1^2}\tan\sqrt{k_2^2-k_1^2}l}{\sqrt{2}k_2^2 e^{i\frac{3\pi}{4}}}\right)\frac{\left(\frac{1}{k_1^2}+\frac{i\gamma_{01}}{k_0^2}d - i\frac{\gamma_{01}}{k_0^2}h\right)\sqrt{2}e^{i\frac{3\pi}{4}}}{k_1\left(-i\frac{\gamma_{01}}{k_0^2}n_1 h + \frac{n_1}{k_1^2}+\frac{\gamma_{01}}{k_1^2 k_0^2}\right)} \tag{82}$$

where

$$n_1 = -i\frac{\sqrt{k_2^2-k_1^2}}{k_2^2}\tan\sqrt{k_2^2-k_1^2}l. \tag{83}$$

Let

$$A_{\rho p k_1} = \frac{ik_1\sqrt{k_2^2-k_1^2}\tan\sqrt{k_2^2-k_1^2}l}{\sqrt{2}k_2^2 e^{i\frac{3\pi}{4}}} \tag{84}$$

$$B_{\rho p k_1} = \frac{\left(\frac{1}{k_1^2}+\frac{i\gamma_{01}}{k_0^2}d - i\frac{\gamma_{01}}{k_0^2}h\right)\sqrt{2}e^{i\frac{3\pi}{4}}}{k_1\left(-i\frac{\gamma_{01}}{k_0^2}n_1 h + \frac{n_1}{k_1^2}+\frac{\gamma_{01}}{k_0^2 k_1^2}\right)} \tag{85}$$

then, we write

$$\begin{aligned}
&\frac{i}{8\pi\omega\epsilon_1}\int_{\Gamma_1} Qe^{i\gamma_1 z}H_1^{(1)}(\lambda\rho)\lambda^2 d\lambda \\
&= \frac{i\omega\mu_0}{8\pi k_1^2}\int_{-\infty}^{\infty} Qe^{i\sqrt{2}k_1 e^{i\frac{3\pi}{4}}\tau z}\sqrt{\frac{2}{\pi k_1\rho}}e^{i\left(k_1\rho-\frac{3\pi}{4}\right)}e^{-k_1\rho\tau^2}\cdot k_1^2\cdot i2k_1\tau d\tau \\
&= \frac{-\omega\mu_0 k_1}{4\pi}\sqrt{\frac{2}{\pi k_1\rho}}e^{i\left(k_1\rho-\frac{3\pi}{4}+\frac{k_1 z^2}{2\rho}\right)}\int_{-\infty}^{\infty}\tau(\tau+A_{\rho p k_1}) \\
&\quad \times B_{\rho p k_1}e^{-k_1\rho\left(\tau-\frac{i}{\sqrt{2}\rho}e^{i\frac{3\pi}{4}}z\right)^2}d\tau. \qquad (86)
\end{aligned}$$

Considering the condition $\rho \gg z$, we find

$$e^{ik_1\rho+i\frac{k_1 z^2}{2\rho}} \approx e^{ik_1\sqrt{\rho^2+z^2}}. \qquad (87)$$

With the changes of the variable $\tau = t + i\frac{z}{\sqrt{2}\rho}e^{i\frac{3\pi}{4}}$, we have

$$\begin{aligned}
&\frac{i}{8\pi\omega\epsilon_1}\int_{\Gamma_1} Qe^{i\gamma_1 z}H_1^{(1)}(\lambda\rho)\lambda^2 d\lambda \\
&= \frac{\omega\mu_0 k_1}{4\pi}\sqrt{\frac{2}{\pi k_1\rho}}e^{i\left(k_1\sqrt{\rho^2+z^2}+\frac{\pi}{4}\right)}B_{\rho p k_1}\int_{-\infty}^{\infty}\left(t+i\frac{z}{\sqrt{2}\rho}e^{i\frac{3\pi}{4}}\right) \\
&\quad \cdot\left(t+i\frac{z}{\sqrt{2}\rho}e^{i\frac{3\pi}{4}}+A_{\rho p k_1}\right)e^{-k_1\rho t^2}dt \\
&= \frac{\omega\mu_0}{2\sqrt{2}\pi\rho}e^{i\left(k_1\sqrt{\rho^2+z^2}+\frac{\pi}{4}\right)}\cdot B_{\rho p k_1}\left[\frac{1}{k_1\rho}+i\left(\frac{z^2}{2\rho^2}+A_{\rho p k_1}\frac{z}{\sqrt{2}\rho}e^{i\frac{3\pi}{4}}\right)\right]. \qquad (88)
\end{aligned}$$

Similarly, we have

$$\begin{aligned}
P &= \frac{\left(\frac{\gamma_{11}}{k_1^2}-\frac{\gamma_{01}}{k_0^2}\right)\cdot\left(\frac{\gamma_{11}}{k_1^2}-in_1\gamma_{11}d\right)}{-i\frac{\gamma_{01}}{k_0^2}n_1\gamma_{11}h+\frac{\gamma_{11}n_1}{k_1^2}+\frac{\gamma_{11}\gamma_{01}}{k_1^2k_0^2}} = \frac{\left(\frac{\gamma_{11}}{k_1^2}-\frac{\gamma_{01}}{k_0^2}\right)\cdot\left(\frac{1}{k_1^2}-in_1 d\right)}{-i\frac{\gamma_{01}}{k_0^2}n_1 h+\frac{n_1}{k_1^2}+\frac{\gamma_{01}}{k_1^2k_0^2}} \\
&= \left(\tau-\frac{\gamma_{01}k_1 e^{-i\frac{3\pi}{4}}}{\sqrt{2}}\right)\cdot\frac{\sqrt{2}e^{i\frac{3\pi}{4}}\left(\frac{1}{k_1^2}-in_1 d\right)}{k_1\left(-i\frac{\gamma_{01}}{k_0^2}n_1 h+\frac{n_1}{k_1^2}+\frac{\gamma_{01}}{k_0^2k_1^2}\right)}. \qquad (89)
\end{aligned}$$

Let

$$A_{\rho q k_1} = -\frac{\gamma_{01} k_1 e^{-i\frac{3\pi}{4}}}{\sqrt{2} k_0^2} \tag{90}$$

$$B_{\rho q k_1} = \frac{\sqrt{2} e^{i\frac{3\pi}{4}} \left(\frac{1}{k_1^2} - i n_1 d\right)}{k_1 \left(-i\frac{\gamma_{01}}{k_0^2} n_1 h + \frac{n_1}{k_1^2} + \frac{\gamma_{01}}{k_0^2 k_1^2}\right)}. \tag{91}$$

With the similar procedures, it is obtained readily.

$$\begin{aligned} &\frac{-i}{8\pi\omega\epsilon_1} \int_{\Gamma_1} P e^{i\gamma_1(2h-z)} H_1^{(1)}(\lambda\rho) \lambda^2 d\lambda \\ = &-\frac{\omega\mu_0}{2\sqrt{2}\pi\rho} e^{i\left(k_1\sqrt{\rho^2+(2h-z)^2}+\frac{\pi}{4}\right)} \cdot B_{\rho q k_1} \\ &\times \left[\frac{1}{k_1\rho} + i\left(\frac{z^2}{2\rho^2} + A_{\rho q k_1} \frac{z}{\sqrt{2}\rho} e^{i\frac{3\pi}{4}}\right)\right]. \end{aligned} \tag{92}$$

In the next step, we consider the branch cut Γ_0. Let

$$A_{pk_0} = \left| \frac{k_0 \left(\frac{\gamma_{10}\gamma_{20}}{k_1^2 k_2^2} \tan\gamma_{20} l + \frac{\gamma_{10}^2}{k_1^4} \tan\gamma_{10} h\right)}{\sqrt{2}\left(\frac{\gamma_{10}}{k_1^2} - \frac{\gamma_{20} \tan\gamma_{10} h \tan\gamma_{20} l}{k_2^2}\right)} \right| \tag{93}$$

$$\begin{aligned} B_{pk_0} = &\frac{k_0 e^{-i\frac{3\pi}{4}} \left(\frac{\gamma_{10}}{k_1^2} - n_0\right)}{\sqrt{2}\left(\frac{\gamma_{10}}{k_1^2} - i n_0 \tan\gamma_{10} h\right)^2} \\ &\left[\left(\frac{\gamma_{10}}{k_1^2} \cos\gamma_{10} d + \frac{\gamma_{10}}{k_1^2} \tan\gamma_{10} h \sin\gamma_{10} d\right)\right. \\ &\left(\frac{\gamma_{10}}{k_1^2} - i n_0 \tan\gamma_{10} h\right) - i(\sin\gamma_{10} d - \tan\gamma_{10} h \cos\gamma_{10} d) \\ &\left.\cdot\left(\frac{\gamma_{10}}{k_1^2} n_0 - i\frac{\gamma_{10}^2}{k_1^4} \tan\gamma_{10} h\right)\right] \end{aligned} \tag{94}$$

$$\gamma_{10} = \sqrt{k_1^2 - k_0^2}; \tag{95}$$

$$\gamma_{20} = \sqrt{k_2^2 - k_0^2}; \tag{96}$$

$$n_0 = -i\frac{\gamma_{20}}{k_2^2}\tan\gamma_{20}l. \tag{97}$$

Then, we write

$$Q=\frac{i(\sin\gamma_{10}d-\tan\gamma_{10}h\cos\gamma_{10}d)\cdot\left(\dfrac{\gamma_{10}}{k_1^2}-n_0\right)}{\dfrac{\gamma_{10}}{k_1^2}-in_0\tan\gamma_{10}h}+\frac{B_{pk_0}}{\tau\pm A_{pk_0}} \tag{98}$$

$$\begin{aligned}&\frac{i}{8\pi\omega\epsilon_1}\int_{\Gamma_0}Qe^{i\gamma_1 z}H_1^{(1)}(\lambda\rho)\lambda^2 d\lambda\\ =\ &\frac{i\omega\mu_0}{8\pi k_1^2}e^{i\gamma_{10}z}e^{i\left(k_0\rho-\frac{3\pi}{4}\right)}\sqrt{\frac{2}{\pi k_0\rho}}\int_{-\infty}^{\infty}2ik_0^3\tau e^{-k_0\rho\tau^2}\frac{B_{pk_0}}{\tau\pm A_{pk_0}e^{i\frac{3\pi}{4}}}d\tau\\ =\ &-\frac{\omega\mu_0k_0^3}{4\pi k_1^2}e^{i\left(\gamma_{10}z+k_0\rho-\frac{3\pi}{4}\right)}\sqrt{\frac{2}{\pi k_0\rho}}B_{pk_0}\\ &\times\left[\sqrt{\frac{\pi}{k_0\rho}}+\int_{-\infty}^{\infty}\frac{\left(A_{pk_0}e^{i\frac{3\pi}{4}}\right)^2}{\tau^2-\left(A_{pk_0}e^{i\frac{3\pi}{4}}\right)^2}e^{-k_0\rho\tau^2}d\tau\right].\end{aligned} \tag{99}$$

In terms of the variable $t=\sqrt{k_0\rho}\tau$, and use is made of the formula (pp.609) in [37], the result becomes

$$\begin{aligned}&\frac{i}{8\pi\omega\epsilon_1}\int_{\Gamma_0}Qe^{i\gamma_1 z}H_1^{(1)}(\lambda\rho)\lambda^2 d\lambda\\ =\ &-\frac{\omega\mu_0k_0^3}{4\pi k_1^2}e^{i\left(\gamma_{10}z+k_0\rho-\frac{3\pi}{4}\right)}\sqrt{\frac{2}{\pi k_0\rho}}B_{pk_0}\\ &\cdot\left[\sqrt{\frac{\pi}{k_0\rho}}+\int_{-\infty}^{\infty}\frac{A_{pk_0}e^{i\frac{3\pi}{4}}\left(\sqrt{k_0\rho}A_{pk_0}e^{i\frac{3\pi}{4}}\right)}{t^2-\left(\sqrt{k_0\rho}A_{pk_0}e^{i\frac{3\pi}{4}}\right)^2}dt\right]\\ =\ &-\frac{\omega\mu_0k_0^3}{4\pi k_1^2}e^{i\left(\gamma_{10}z+k_0\rho-\frac{3\pi}{4}\right)}\sqrt{\frac{2}{\pi k_0\rho}}B_{pk_0}\\ &\cdot\left[\sqrt{\frac{\pi}{k_0\rho}}+\pi A_{pk_0}\mathrm{erfc}\left(e^{i\frac{\pi}{4}}\sqrt{k_0\rho}A_{pk_0}\right)e^{i\left(\frac{5\pi}{4}+k_0\rho A_{pk_0}^2\right)}\right].\end{aligned} \tag{100}$$

Similarly, it is also obtained readily.

$$\frac{-i}{8\pi\omega\epsilon_1}\int_{\Gamma_0}Pe^{i\gamma_1(2h-z)}H_1^{(1)}(\lambda\rho)\lambda^2 d\lambda=\frac{\omega\mu_0k_0^3}{4\pi k_1^2}e^{i\left[\gamma_{10}(2h-z)+k_0\rho-\frac{3\pi}{4}\right]}$$

$$\times B_{qk_0}\sqrt{\frac{2}{\pi k_0\rho}}\left[\sqrt{\frac{\pi}{k_0\rho}}+\pi A_{qk_0}\mathrm{erfc}\left(e^{i\frac{\pi}{4}}\sqrt{k_0\rho}A_{qk_0}\right)e^{i\left(\frac{5\pi}{4}+k_0\rho A_{qk_0}^2\right)}\right] \quad (101)$$

where

$$A_{qk_0} = \left|\frac{k_0\left(\dfrac{\gamma_{10}\gamma_{20}\tan\gamma_{20}l}{k_2^2k_1^2}+\dfrac{\gamma_{10}^2}{k_1^4}\tan\gamma_{10}h\right)}{\sqrt{2}\left(\dfrac{\gamma_{10}}{k_1^2}-\dfrac{\gamma_{20}}{k_2^2}\tan\gamma_{10}h\tan\gamma_{20}l\right)}\right| \quad (102)$$

$$B_{qk_0} = k_0e^{-i\frac{3\pi}{4}}\left(\frac{\gamma_{10}}{k_1^2}\cos\gamma_{10}d-in_0\sin\gamma_{10}d\right)\cdot(1-i\tan\gamma_{10}h)$$
$$\cdot\frac{\dfrac{\gamma_{10}^2}{k_1^4}-i\dfrac{\gamma_{10}}{k_1^2}n_0\tan\gamma_{10}h+\dfrac{\gamma_{10}}{k_1^2}n_0-i\dfrac{\gamma_{10}^2}{k_1^4}\tan\gamma_{10}h}{\sqrt{2}\left(\dfrac{\gamma_{10}}{k_1^2}-in_0\tan\gamma_{10}h\right)^2}. \quad (103)$$

Substituting (88) and (100) into (70), we write

$$E_{1\rho}^{(2)} = -\frac{\omega\mu_0}{4k_1^2}\sum_j Q(\lambda_j^*)e^{i\gamma_{1j}^*z}H_0^{(1)}(\lambda_j^*\rho)\lambda_j^{*2}+\frac{\omega\mu_0e^{i\left(k_1\sqrt{\rho^2+z^2}+\frac{\pi}{4}\right)}}{2\sqrt{2}\pi\rho}$$
$$\cdot B_{\rho pk_1}\left[\frac{1}{k_1\rho}+i\left(\frac{z^2}{2\rho^2}+A_{\rho pk_1}\frac{z}{\sqrt{2}\rho}e^{i\frac{3\pi}{4}}\right)\right]$$
$$-\frac{\omega\mu_0k_0^3}{4\pi k_1^2}e^{i\left(\gamma_{10}z+k_0\rho-\frac{3\pi}{4}\right)}\sqrt{\frac{2}{\pi k_0\rho}}B_{pk_0}\left[\sqrt{\frac{\pi}{k_0\rho}}\right.$$
$$\left.+i\pi A_{pk_0}e^{i\frac{3\pi}{4}}e^{ik_0\rho A_{pk_0}^2}\mathrm{erfc}\left(e^{i\frac{\pi}{4}}\sqrt{k_0\rho}A_{pk_0}\right)\right]. \quad (104)$$

Similarly, Substituting (92) and (101) into (71), we write

$$E_{1\rho}^{(3)} = \frac{\omega\mu_0}{4k_1^2}\sum_j P(\lambda_j^*)e^{i\gamma_{1j}^*z}H_0^{(1)}(\lambda_j^*\rho)\lambda_j^{*2}-\frac{\omega\mu_0e^{i\left(k_1\sqrt{\rho^2+(2h-z)^2}+\frac{\pi}{4}\right)}}{2\sqrt{2}\pi\rho}$$
$$\cdot B_{\rho qk_1}\left[\frac{1}{k_1\rho}+i\left(\frac{z^2}{2\rho^2}+A_{\rho qk_1}\frac{z}{\sqrt{2}\rho}e^{i\frac{3\pi}{4}}\right)\right]$$
$$+\frac{\omega\mu_0k_0^3}{4\pi k_1^2}e^{i[\gamma_{10}(2h-z)+k_0\rho-\frac{3\pi}{4}]}\cdot\sqrt{\frac{2}{\pi k_0\rho}}B_{qk_0}$$

$$\times\left[\sqrt{\frac{\pi}{k_0\rho}}+\pi A_{qk_0}\text{erfc}\left(e^{i\frac{\pi}{4}}\sqrt{k_0\rho}A_{qk_0}\right)e^{i\left(\frac{5\pi}{4}+k_0\rho A_{qk_0}^2\right)}\right]. \quad (105)$$

Considering the contributions of the residues of the poles and those of the integrations of the branch cuts, the integrals $E_{1z}^{(2)}$, $E_{1z}^{(3)}$, $B_{1\phi}^{(2)}$, and $B_{1\phi}^{(3)}$ can be expressed as follows:

$$\begin{aligned}
E_{1z}^{(2)} &= -i\frac{\omega\mu_0}{4k_1^2}\sum_j Q(\lambda_j^*)e^{i\gamma_{1j}^* z}H_0^{(1)}(\lambda_j^*\rho)\lambda_j^{*3}\gamma_{1j}^{*\,-1} \\
&\quad -\frac{1}{8\pi\omega\epsilon_1}\int_{\Gamma_0+\Gamma_1+\Gamma_2} Qe^{i\gamma_1 z}\gamma_1^{-1}H_0^{(1)}(\lambda\rho)\lambda^3 d\lambda \qquad (106)\\
E_{1z}^{(3)} &= -i\frac{\omega\mu_0}{4k_1^2}\sum_j P(\lambda_j^*)e^{i\gamma_{1j}^* z}H_0^{(1)}(\lambda_j^*\rho)\lambda_j^{*3}\gamma_{1j}^{*\,-1} \\
&\quad -\frac{1}{8\pi\omega\epsilon_1}\int_{\Gamma_0+\Gamma_1+\Gamma_2} Pe^{i\gamma_1(2h-z)}\gamma_1^{-1}H_0^{(1)}(\lambda\rho)\lambda^3 d\lambda \qquad (107)\\
B_{1\phi}^{(2)} &= -\frac{\mu_0}{4}\sum_j Q(\lambda_j^*)e^{i\gamma_{1j}^* z}H_1^{(1)}(\lambda_j^*\rho)\lambda_j^{*2}\gamma_{1j}^{*\,-1} \\
&\quad +\frac{i\mu_0}{8\pi}\int_{\Gamma_0+\Gamma_1+\Gamma_2} Qe^{i\gamma_1 z}\gamma_1^{-1}H_1^{(1)}(\lambda\rho)\lambda^2 d\lambda \qquad (108)\\
B_{1\phi}^{(3)} &= -\frac{\mu_0}{4}\sum_j P(\lambda_j^*)e^{i\gamma_{1j}^*(2h-z)}H_1^{(1)}(\lambda_j^*\rho)\lambda_j^{*2}\gamma_{1j}^{*\,-1} \\
&\quad +\frac{i\mu_0}{8\pi}\int_{\Gamma_0+\Gamma_1+\Gamma_2} Pe^{i\gamma_1(2h-z)}\gamma_1^{-1}H_1^{(1)}(\lambda\rho)\lambda^2 d\lambda. \qquad (109)
\end{aligned}$$

Because the evaluations of the integrals in (106)–(109) along the branch cut Γ_2 are zero, it is necessary to evaluate the integrations along the branch cuts Γ_1 and Γ_0.

Following the similar procedures, we arrive at the following expression.

$$\begin{aligned}
&\frac{i\mu_0}{8\pi}\int_{\Gamma_1} Qe^{i\gamma_1 z}H_1^{(1)(\lambda\rho)}\lambda^2\gamma_1^{-1}d\lambda = \frac{i\mu_0 k_1^2}{8\pi} \\
&\times\int_{-\infty}^{\infty} Qe^{ik_1\sqrt{2}e^{i\frac{3\pi}{4}}\tau z}\sqrt{\frac{2}{\pi k_1\rho}}e^{i\left(k_1\rho-\frac{3\pi}{4}\right)}e^{-k_1\rho\tau^2}\frac{2ik_1\tau}{\sqrt{2}k_1 e^{i\frac{3\pi}{4}}\tau}d\tau \\
= &\ \frac{i\mu_0 k_1^2}{4\pi}\sqrt{\frac{1}{\pi k_1\rho}}e^{ik_1\sqrt{\rho^2+z^2}}\int_{-\infty}^{\infty} Qe^{-k_1\rho(\tau-\frac{iz}{\sqrt{2}}e^{i\frac{3\pi}{4}})^2}d\tau. \qquad (110)
\end{aligned}$$

where

$$A_{pk_1} = \frac{n_1 k_1 e^{-i\frac{3\pi}{4}}}{\sqrt{2}} \tag{111}$$

$$B_{pk_1} = \frac{\sqrt{2}e^{i\frac{3\pi}{4}}\left(\frac{1}{k_1^2} + i\frac{\gamma_{01}}{k_0^2}d - i\frac{\gamma_{01}}{k_0^2}h\right)}{k_1\left(\frac{n_1}{k_1^2} + \frac{\gamma_{01}}{k_1^2 k_0^2} - i\frac{\gamma_{01}}{k_0^2}n_1 h\right)} \tag{112}$$

With the change of the variable $t = \tau - \frac{z}{\sqrt{2}\rho}e^{i\frac{5\pi}{4}}$, it becomes

$$\begin{aligned}
&\frac{i\mu_0}{8\pi}\int_{\Gamma_1} Qe^{i\gamma_1 z}H_1^{(1)(\lambda\rho)}\lambda^2\gamma_1^{-1}d\lambda = \frac{i\mu_0 k_1^2}{4\pi}\sqrt{\frac{1}{\pi k_1\rho}}e^{ik_1\sqrt{\rho^2+z^2}} \\
&\times\int_{-\infty}^{\infty}\left(t + \frac{z}{\sqrt{2}\rho}e^{i\frac{5\pi}{4}} - A_{pk_1}\right)B_{pk_1}e^{-k_1\rho t^2}dt \\
= &\frac{i\mu_0 k_1}{4\pi\rho}B_{pk_1}\left(\frac{z}{\sqrt{2}\rho}e^{i\frac{5\pi}{4}} - A_{pk_1}\right)e^{ik_1\sqrt{\rho^2+z^2}}.
\end{aligned} \tag{113}$$

Obviously, we can also get

$$\begin{aligned}
&-\frac{1}{8\pi\omega\epsilon_1}\int_{\Gamma_1} d\lambda Qe^{i\gamma_1 z}H_0^{(1)}(\lambda\rho)\lambda^3\gamma_{01}^{-1} \\
= &-\frac{i\omega\mu_0}{4\pi\rho}B_{pk_1}\left(\frac{z}{\sqrt{2}\rho}e^{i\frac{5\pi}{4}} - A_{pk_1}\right)e^{ik_1\sqrt{\rho^2+z^2}}.
\end{aligned} \tag{114}$$

The integrals including the factor P in (107) and (109) along the branch cut Γ_1 can be evaluated readily. They are

$$\begin{aligned}
&\frac{i\mu_0}{8\pi}\int_{\Gamma_1} Pe^{i\gamma_1(2h-z)}H_1^{(1)}(\lambda\rho)\lambda^2\gamma_1^{-1}d\lambda \\
= &\frac{i\mu_0 k_1}{4\pi\rho}B_{qk_1}\left(\frac{z}{\sqrt{2}\rho}e^{i\frac{5\pi}{4}} - A_{qk_1}\right)e^{ik_1\sqrt{\rho^2+(2h-z)^2}}
\end{aligned} \tag{115}$$

$$\begin{aligned}
&-\frac{1}{8\pi\omega\epsilon_1}\int_{\Gamma_1} d\lambda Pe^{i\gamma_1(2h-z)}H_0^{(1)}(\lambda\rho)\lambda^3\gamma_1^{-1} \\
= &\frac{-i\omega\mu_0}{4\pi\rho}B_{qk_1}\left(\frac{z}{\sqrt{2}\rho}e^{i\frac{5\pi}{4}} - A_{qk_1}\right)e^{ik_1\sqrt{\rho^2+(2h-z)^2}}
\end{aligned} \tag{116}$$

where

$$A_{qk_1} = \frac{\gamma_{01}k_1 e^{-i\frac{3\pi}{4}}}{k_0^2\sqrt{2}} \tag{117}$$

$$B_{qk_1} = \frac{\sqrt{2}e^{i\frac{3\pi}{4}}\left(\frac{1}{k_1^2} - in_1 d\right)}{k_1\left(-i\frac{\gamma_{01}}{k_0^2}n_1 h + \frac{n_1}{k_1^2} + \frac{\gamma_{01}}{k_1^2 k_0^2}\right)}. \tag{118}$$

With a similar manner in the evaluations of the integrals (70) and (71) along the branch cut Γ_0, we can obtain

$$\begin{aligned}
&\frac{i\mu_0}{8\pi}\int_{\Gamma_0} Qe^{i\gamma_1 z}H_1^{(1)}(\lambda\rho)\lambda^2\gamma_1^{-1}d\lambda \\
&\quad= -\frac{\mu_0 k_0^3}{4\pi}e^{i\left(\gamma_{10}z+k_0\rho-\frac{3\pi}{4}\right)}\gamma_{10}^{-1}\cdot\sqrt{\frac{2}{\pi k_0\rho}}B_{pk_0} \\
&\qquad\times\left[\sqrt{\frac{\pi}{k_0\rho}} + \pi A_{pk_0}\mathrm{erfc}\left(e^{i\frac{\pi}{4}}\sqrt{k_0\rho}A_{pk_0}\right)e^{i\left(\frac{5\pi}{4}+k_0\rho A_{pk_0}^2\right)}\right] &(119)\\
&\frac{i\mu_0}{8\pi}\int_{\Gamma_0} Pe^{i\gamma_1(2h-z)}H_1^{(1)}(\lambda\rho)\lambda^2\gamma_1^{-1}d\lambda \\
&\quad= -\frac{\mu_0 k_0^3}{4\pi}\gamma_{10}^{-1}e^{i[\gamma_{10}(2h-z)+k_0\rho-\frac{3\pi}{4}]}\cdot\sqrt{\frac{2}{\pi k_0\rho}}B_{qk_0} \\
&\qquad\times\left[\sqrt{\frac{\pi}{k_0\rho}} + \pi A_{qk_0}\mathrm{erfc}\left(e^{i\frac{\pi}{4}}\sqrt{k_0\rho}A_{qk_0}\right)e^{i\left(\frac{5\pi}{4}+k_0\rho A_{qk_0}^2\right)}\right] &(120)\\
&-\frac{1}{8\pi\omega\epsilon_1}\int_{\Gamma_0} Qe^{i\gamma_1 z}H_0^{(1)}(\lambda\rho)\lambda^3\gamma_1^{-1}d\lambda \\
&\quad= \frac{\omega\mu_0 k_0^4\gamma_{10}^{-1}}{4\pi k_1^2}e^{i\left(\gamma_{10}z+k_0\rho-\frac{3\pi}{4}\right)}\cdot\sqrt{\frac{2}{\pi k_0\rho}}B_{pk_0} \\
&\qquad\times\left[\sqrt{\frac{\pi}{k_0\rho}} + \pi A_{pk_0}\mathrm{erfc}\left(e^{i\frac{\pi}{4}}\sqrt{k_0\rho}A_{pk_0}\right)e^{i\left(\frac{5\pi}{4}+k_0\rho A_{pk_0}^2\right)}\right] &(121)\\
&-\frac{1}{8\pi\omega\epsilon_1}\int_{\Gamma_0} Pe^{i\gamma_1(2h-z)}H_0^{(1)}(\lambda\rho)\lambda^3\gamma_1^{-1}d\lambda \\
&\quad= \frac{\omega\mu_0 k_0^4}{4\pi k_1^2}\gamma_{10}^{-1}e^{i[\gamma_{10}(2h-z)+k_0\rho-\frac{3\pi}{4}]}\cdot\sqrt{\frac{2}{\pi k_0\rho}}B_{qk_0} \\
&\qquad\times\left[\sqrt{\frac{\pi}{k_0\rho}} + \pi A_{qk_0}\mathrm{erfc}\left(e^{i\frac{\pi}{4}}\sqrt{k_0\rho}A_{qk_0}\right)e^{i\left(\frac{5\pi}{4}+k_0\rho A_{qk_0}^2\right)}\right]. &(122)
\end{aligned}$$

Here, A_{pk_0}, A_{qk_0}, B_{pk_0}, and B_{qk_0} are defined by (93), (102), (94), and (103), respectively. Substituting (113)–(116) and (119)–(122) into

(106)–(109), we have

$$\begin{aligned}
E_{1z}^{(2)} &= -i\frac{\omega\mu_0}{4k_1^2}\sum_j Q(\lambda_j^*)e^{i\gamma_{1j}^* z}H_0^{(1)}(\lambda_j^*\rho)\lambda_j^{*3}\gamma_{1j}^{*\,-1} \\
&\quad -\frac{i\omega\mu_0}{4\pi\rho}B_{pk_1}\cdot\left(\frac{z}{\sqrt{2}\rho}e^{i\frac{5\pi}{4}}-A_{pk_1}\right)e^{ik_1\sqrt{\rho^2+z^2}} \\
&\quad -\frac{\omega\mu_0 k_0^4\gamma_{10}^{-1}}{4\pi k_1^2}e^{i\left(\gamma_{10}z+k_0\rho-\frac{3\pi}{4}\right)}\sqrt{\frac{2}{\pi k_0\rho}}\cdot B_{pk_0} \\
&\quad \times\left[\sqrt{\frac{\pi}{k_0\rho}}+\pi A_{pk_0}\mathrm{erfc}\left(e^{i\frac{\pi}{4}}\sqrt{k_0\rho}A_{pk_0}\right)e^{i\left(\frac{5\pi}{4}+k_0\rho A_{pk_0}^2\right)}\right] \quad (123) \\
E_{1z}^{(3)} &= -i\frac{\omega\mu_0}{4k_1^2}\sum_j P(\lambda_j^*)e^{i\gamma_{1j}^* z}H_0^{(1)}(\lambda_j^*\rho)\lambda_j^{*3}\gamma_{1j}^{*\,-1} \\
&\quad -\frac{i\omega\mu_0}{4\pi\rho}B_{qk_1}\cdot\left(\frac{z}{\sqrt{2}\rho}e^{i\frac{5\pi}{4}}-A_{qk_1}\right)e^{ik_1\sqrt{\rho^2+(2h-z)^2}} \\
&\quad +\frac{\omega\mu_0 k_0^4}{4\pi k_1^2}\gamma_{10}^{-1}e^{i[\gamma_{10}(2h-z)+k_0\rho-\frac{3\pi}{4}]}\cdot\sqrt{\frac{2}{\pi k_0\rho}}B_{qk_0} \\
&\quad \times\left[\sqrt{\frac{\pi}{k_0\rho}}+\pi A_{qk_0}\mathrm{erfc}\left(e^{i\frac{\pi}{4}}\sqrt{k_0\rho}A_{qk_0}\right)e^{i\left(\frac{5\pi}{4}+k_0\rho A_{qk_0}^2\right)}\right] \quad (124) \\
B_{1\phi}^{(2)} &= -\frac{\mu_0}{4}\sum_j Q(\lambda_j^*)e^{i\gamma_{1j}^*(2h-z)}H_1^{(1)}(\lambda_j^*\rho)\lambda_j^{*2}\gamma_{1j}^{*\,-1} \\
&\quad +\frac{i\mu_0 k_1}{4\pi\rho}B_{pk_1}\cdot\left(\frac{z}{\sqrt{2}\rho}e^{i\frac{5\pi}{4}}-A_{pk_1}\right)e^{ik_1\sqrt{\rho^2+z^2}} \\
&\quad -\frac{\mu_0 k_0^3}{4\pi}e^{i\left(\gamma_{10}z+k_0\rho-\frac{3\pi}{4}\right)}\gamma_{10}^{-1}\sqrt{\frac{2}{\pi k_0\rho}}\cdot B_{pk_0} \\
&\quad \times\left[\sqrt{\frac{\pi}{k_0\rho}}+\pi A_{pk_0}\mathrm{erfc}\left(e^{i\frac{\pi}{4}}\sqrt{k_0\rho}A_{pk_0}\right)e^{i\left(\frac{5\pi}{4}+k_0\rho A_{pk_0}^2\right)}\right] \quad (125) \\
B_{1\phi}^{(3)} &= -\frac{\mu_0}{4}\sum_j P(\lambda_j^*)e^{i\gamma_{1j}^*(2h-z)}H_1^{(1)}(\lambda_j^*\rho)\lambda_j^{*2}\gamma_{1j}^{*\,-1} \\
&\quad +\frac{i\mu_0 k_1}{4\pi\rho}B_{qk_1}\cdot\left(\frac{z}{\sqrt{2}\rho}e^{i\frac{5\pi}{4}}-A_{qk_1}\right)e^{ik_1\sqrt{\rho^2+(2h-z)^2}} \\
&\quad -\frac{\mu_0 k_0^3}{4\pi}\gamma_{10}^{-1}e^{i[\gamma_{10}(2h-z)+k_0\rho-\frac{3\pi}{4}]}\cdot\sqrt{\frac{2}{\pi k_0\rho}}B_{qk_0}
\end{aligned}$$

$$\times\left[\sqrt{\frac{\pi}{k_0\rho}}+\pi A_{qk_0}\text{erfc}\left(e^{i\frac{\pi}{4}}\sqrt{k_0\rho}A_{qk_0}\right)e^{i\left(\frac{5\pi}{4}+k_0\rho A_{qk_0}^2\right)}\right]. \quad (126)$$

Using the above derivations and the results for the direct field addressed in [8], the final completed formulas for the three components are obtained readily. They are

$$E_{1\rho}(\rho,\phi,z) = -\frac{\omega\mu_0}{4\pi k_1}e^{ik_1\gamma_0}\left(\frac{\rho}{r_1}\right)\left(\frac{z-d}{r_1}\right)\left(\frac{ik_1}{r_1}-\frac{3}{r_1^3}-\frac{3i}{k_1r_1^3}\right) +E_{1\rho}^{(2)}+E_{1\rho}^{(3)} \quad (127)$$

$$E_{1z}(\rho,\phi,z) = \frac{\omega\mu_0}{4\pi k_1}e^{ik_1r_1}\left[\frac{ik_1}{r_1}-\frac{1}{r_1^2}-\frac{i}{k_1r_1^3}-\left(\frac{z-d}{r_1}\right)^2 \cdot\left(\frac{ik_1}{r_1}-\frac{3}{r_1^2}-\frac{3i}{k_1r_1^3}\right)\right]+E_{1z}^{(2)}+E_{1z}^{(3)} \quad (128)$$

$$B_{1\phi}(\rho,\phi,z) = -\frac{\mu_0}{4\pi}e^{ik_1r_1}\left(\frac{\rho}{r_1}\right)\left(\frac{ik_1}{r_1}-\frac{1}{r_1^2}\right)+B_{1\phi}^{(2)}+B_{1\phi}^{(3)}. \quad (129)$$

4. COMPUTATIONS AND DISCUSSIONS

From the expressions of the six integrals $E_{1\rho}^{(2)}$ in (104), $E_{1\rho}^{(3)}$ in (105), $E_{1z}^{(2)}$ in (123), $E_{1z}^{(3)}$ in (124), $B_{1\phi}^{(2)}$ in (125), and $B_{1\phi}^{(3)}$ in (126), it is seen that the first terms of them are the sums of residues of the poles λ_j^*. The terms, which are contributed by the sums of residues of the poles, are named the trapped surface wave. When $k_1 \le \lambda_j^* \le k_2$, $\gamma_{1j}^* = i\sqrt{\lambda_j^{*2}-k_1^2}$ is a positive imaginary number, that is to say, the terms of the trapped surface wave including the factor $e^{i\gamma_{1j}^*z}$ will attenuates exponentially as $e^{-\sqrt{\lambda_j^{*2}-k_1^2}z}$ in the $\hat{z}$ direction when the wave numbers λ_j^* are between k_1 and k_2. Evidently, it is also seen that the terms of the trapped surface wave have not an attenuated factor in the $\hat{z}$ direction when the wave numbers λ_j^* are between k_0 and k_1.

The wave numbers of the trapped surface wave are the poles λ_j^*, which are determined by the operating frequency f, the thicknesses h and l of the two dielectric layers, the relative permittivity ϵ_{1r} of the upper dielectric layer, and the relative permittivity ϵ_{2r} of the lower dielectric layer. The number of the poles λ_j^* can not be seen directly from the pole equation. In this paper the poles λ_j^*, which are between k_0 and k_2, can be determined by using Newton method.

If assuming that both Regions 0 and 1 are occupied the air, it is found that the factor P, which is expressed in (36), reduces to

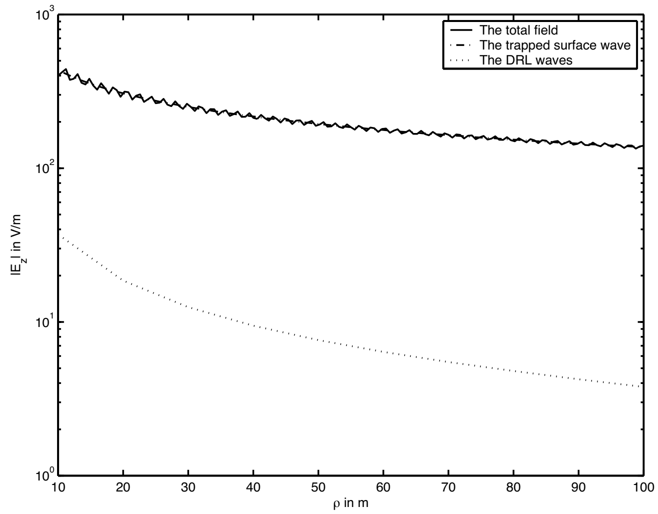

Figure 3. Electric field E_z in V/m with $f = 1\,\mathrm{GHz}$, $\epsilon_{1r} = 2.65$, $\epsilon_{2r} = 4$, $k_1h = 5$, $k_2l = 2$, and $z = d = 0$.

zero and the problem will reduces to that of the three-layered case. For conveniences in evaluating the integrals including the reflection coefficients Q and P, in this paper the terms of the ideal reflected wave are not separated with those of the lateral wave. Obviously, the integrations along the branch cuts Γ_1 and Γ_0 includes the terms of the ideal reflected wave and the lateral wave. When the conditions $k_1\rho \gg 1$ and $z + d \ll \rho$ are satisfied, the lateral waves with the wave numbers being k_0 and k_1 can be excited efficiently. Evidently, it is seen that the lateral waves propagate in Region 0 along the boundary $z = 0$ and propagate in Region 1 along the boundary $z = h$.

In Figs. 3–5, for the components E_{1z}, the total field, the trapped surface wave, and the DRL waves, which include the direct wave, the reflected wave, and the lateral wave, are computed and shown in three cases of $z = d = 0$, $k_1z = k_1d = 0.5$, and $k_1z = k_1d = 0.75$, respectively. In Figs. 7–9, the similar results for the components $E_{1\rho}$ are computed and shown, respectively. In Fig. 6, the total field for the component E_{1z} is computed and shown in three cases of $z = d = 0$, $k_1z = k_1d = 0.5$, and $k_1z = k_1d = 0.75$, respectively. Similar graphs for the components $E_{1\rho}$ are shown in Fig. 10. Computations show that there is a significant contribution from the trapped surface wave for the total field in the four-layered region when both the dipole point and the observation point are located in the upper dielectric layer under the air.

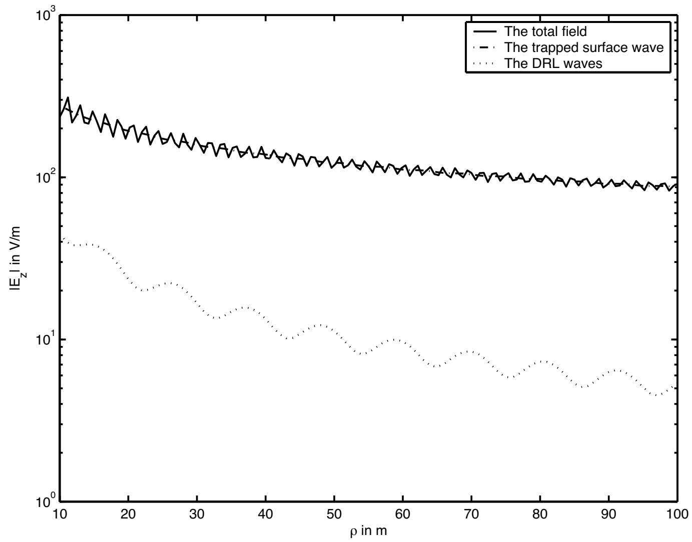

Figure 4. Electric field E_z in V/m with $f = 1\,\mathrm{GHz}$, $\epsilon_{1r} = 2.65$, $\epsilon_{2r} = 4$, $k_1h = 5$, $k_2l = 2$, and $k_1z = k_1d = 0.5$.

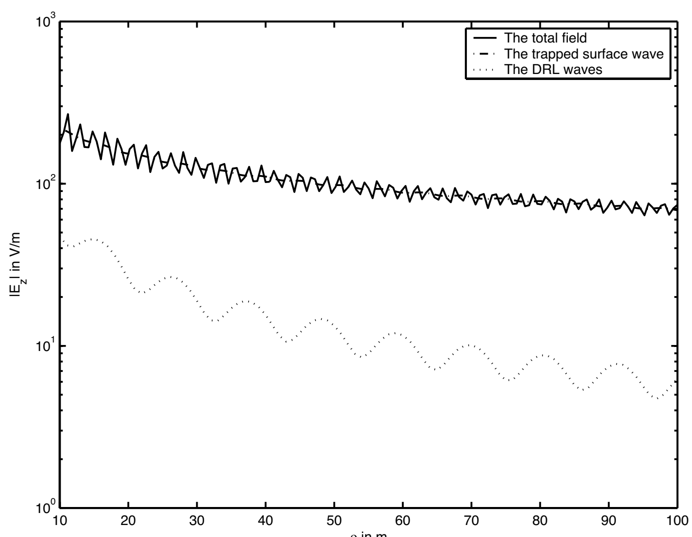

Figure 5. Electric field E_z in V/m with $f = 1\,\mathrm{GHz}$, $\epsilon_{1r} = 2.65$, $\epsilon_{2r} = 4$, $k_1h = 5$, $k_2l = 2$, and $k_1z = k_1d = 0.75$.

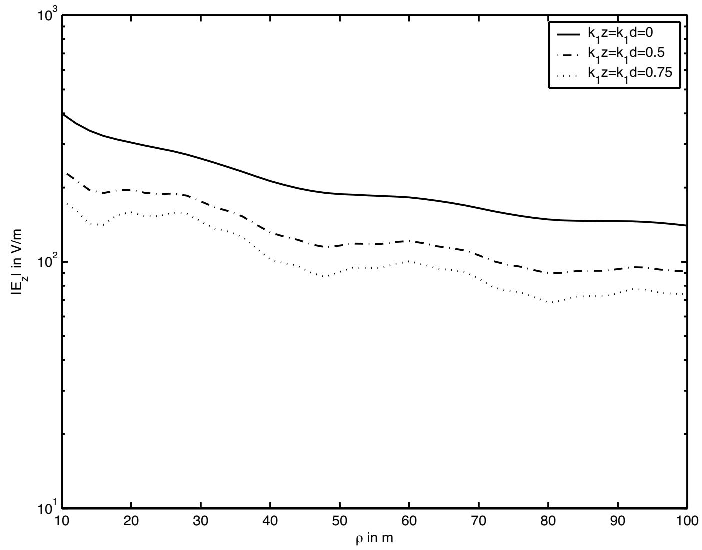

Figure 6. The total fields E_z in V/m with $f = 1\,\mathrm{GHz}$, $\epsilon_{1r} = 2.65$, $\epsilon_{2r} = 4$, $k_1 h = 5$, $k_2 l = 2$ at three cases of $z = d = 0$, $k_1 z = k_1 d = 0.5$, and $k_1 z = k_1 d = 0.75$.

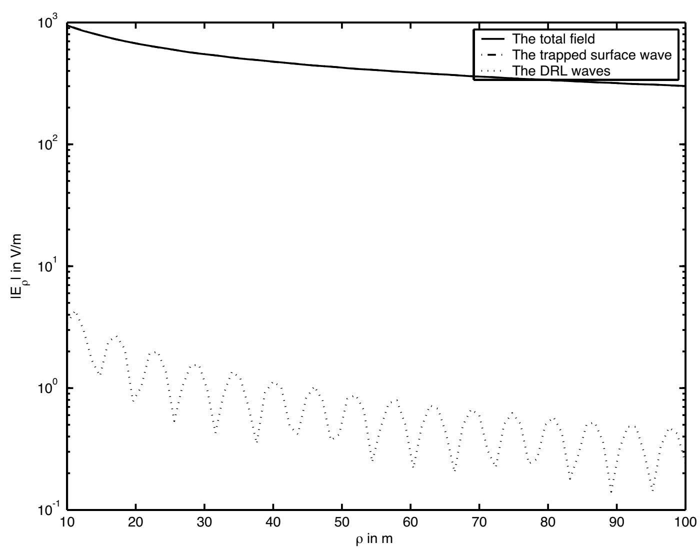

Figure 7. Electric field E_ρ in V/m with $f = 1\,\mathrm{GHz}$, $\epsilon_{1r} = 2.65$, $\epsilon_{2r} = 4$, $k_1 h = 5$, $k_2 l = 2$, and $z = d = 0$.

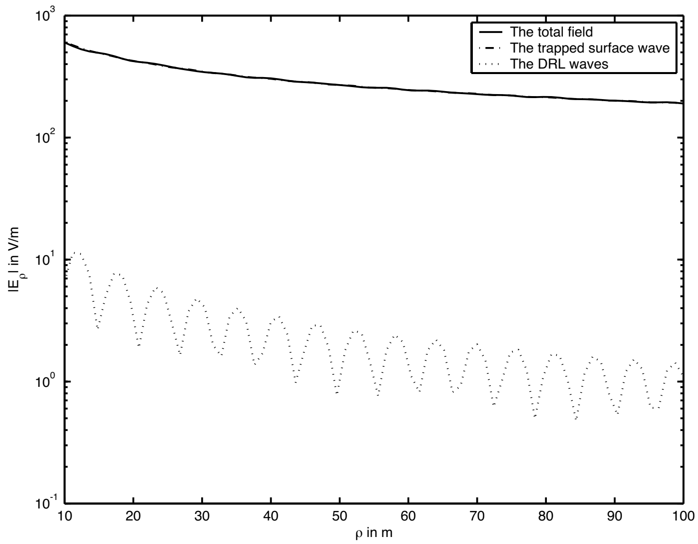

Figure 8. Electric field E_ρ in V/m with $f = 1\,\mathrm{GHz}$, $\epsilon_{1r} = 2.65$, $\epsilon_{2r} = 4$, $k_1h = 5$, $k_2l = 2$, and $k_1z = k_1d = 0.5$.

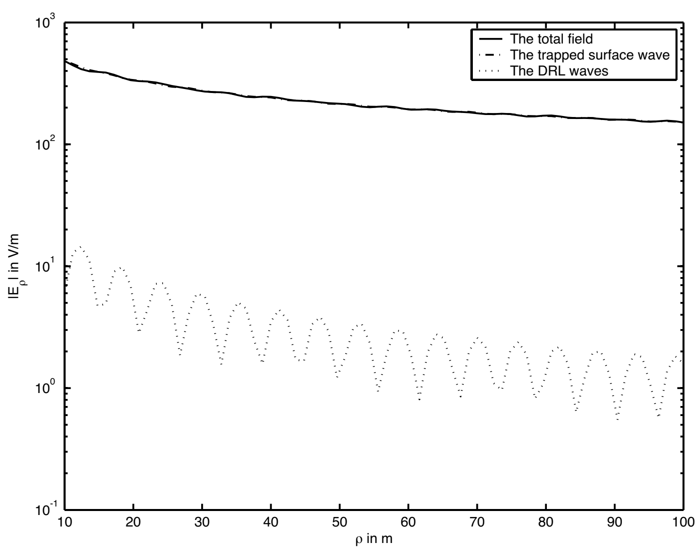

Figure 9. Electric field E_ρ in V/m with $f = 1\,\mathrm{GHz}$, $\epsilon_{1r} = 2.65$, $\epsilon_{2r} = 4$, $k_1h = 5$, $k_2l = 2$, and $k_1z = k_1d = 0.75$.

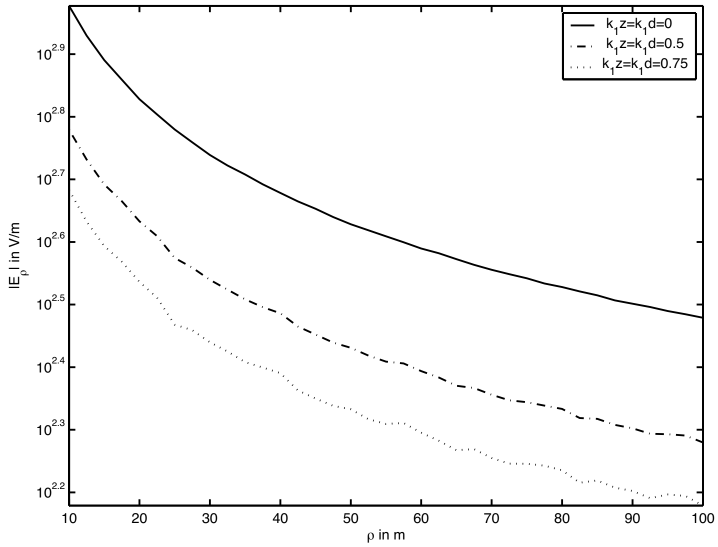

Figure 10. The total fields E_ρ in V/m with $f = 1\,\text{GHz}$, $\epsilon_{1r} = 2.65$, $\epsilon_{2r} = 4$, $k_1 h = 5$, $k_2 l = 2$ at three cases of $z = d = 0$, $k_1 z = k_1 d = 0.5$, and $k_1 z = k_1 d = 0.75$.

5. CONCLUSIONS

In the above derivations and analysis, the completed formulas have been derived for the electromagnetic field generated by a vertical electric dipole in the four-layered region when both the dipole point and observation point are located in the upper dielectric layer under the air. It is noted that the wave numbers of the trapped surface wave are between k_0 and k_2 and those of the lateral wave are k_0 and k_1. The computations and discussions show that the field components in far regions are determined primarily by the terms of the trapped surface waves in the four-layered region. Evidently, the results obtained can be reduced to those for three-layered case as addressed in [29] if we assume $k_0 = k_1$.

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