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# INVESTIGATION OF LIGHT AMPLIFICATION IN SI-NANOCRYSTAL-ER DOPED OPTICAL FIBER

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**Abstract**—In this article the details of Erbium doped Fiber Amplifier (EDFA) in presence of Silicon nanocrystal (Si-Nc) is investigated. In this analysis Si-Nc and Er ions in matrix of fiber are assumed two and five levels respectively. For this structure rate equation for transient and steady state analysis is considered. The range of $\mathrm{Er}^{(+3)}$ concentration and pump intensity at 488 nm are considered $[10^{17} - 10^{21}]1/\mathrm{cm}^3$ and $[10^{17} - 10^{23}]\,\mathrm{photon/cm}^2 \cdot \mathrm{sec}$ respectively in this paper. Based on numerical simulation of this problem we observed that with increasing of concentration of the Si-Nc the fiber length for given optical gain is decreased. For example in the case of 40 dB optical gain, we calculated fiber length to be $1.1 \times 10^5$ cm without Si-Nc and $5 \times 10^2$ cm for $\sigma_{CCa} = 1 \times 10^{-17}\,\mathrm{cm}^2$ (confined carrier absorption cross-section) and 5 cm for $\sigma_{CCa} = 1 \times 10^{-19}\,\mathrm{cm}^2$ respectively. Our simulations show that for second level with increasing pump intensity the population rise and fall times are decreased. But, for third level the population rise time is decreased and fall time is depends to level of Er ion interactions. We observed that with increasing Er ions optical net gain is increased and finally has a maximum. After this special value of Er ions because of increasing in interaction between ions the net gain finally begin to decrease.

In this analysis we let to Er ions have inhomogeneous distribution. Also, we observed that in this case with increasing of the distribution peak, the net gain is increased, interaction between ions is increased, the coupling coefficient to the Si-Ncs is increased and losses due to Si-Ncs are decreased. Finally effect of $K_{exiton}$ (maximum number of exciton in single Si-Nc) on amplification process is considered and we observed that in the case of $K_{exiton} = 2$ the optical gain considerably and introduced losses due to Si-Nc are increased.

## 1. INTRODUCTION

High-speed data communication and processing are basic industrial and scientific demand recently. Optical method is one of best alternatives for doing these tasks. Main physical medium for optical communication and processing is optical fiber. Transmission bit rate in optical communication is higher than other data transmission methods. But, this is still far from the fundamental limit to the information transfer rate, and future systems are expected to reach data transfer rates of several turbits per second in a single fiber, for transmission over large distance the optical signal needs to be amplified at regular intervals in order to maintain sufficient light intensity. For loss compensation in optical communications we need optical amplifiers which there are some alternatives such as semiconductor optical amplifiers (SOA) and Erbium doped fiber amplifier (EDFA). Traditionally in fiber optic communications this is done using erbium doped fiber amplifier that operates at a wavelength of 1.55 μm. This wavelength is currently one of the standard wavelengths used in optical telecommunication [1, 2]. $\mathrm{Er}^{(+3)}$ doped optical amplifiers are an important component in optical telecommunication networks. The operation of these amplifiers relies on an optical transition of $\mathrm{Er}^{(+3)}$ at 1.55 μm, which is in the region of optimum transmission of silica based glass fiber. The transition responsible for the amplified 1.55 μm light signal occurs within the partially filled 4f shell of $\mathrm{Er}^{(+3)}$, which is electrically shielded from its surrounding by filled 5s and 5p shells. EDFA is better than SOA because of easy connection between amplifier and fiber. Also, optical fiber amplifier usually has higher nonlinear refractive index which has critical role in optical functional blocks design. Applications including optical fiber based sensors are other important category that optical fibers play important role. Traditional EDFA have some disadvantages such as large fiber length for obtaining high gain, requiring high quality lasers for pumping, small band width, and small cross-section area. For example the optical cross sections for these transitions in common EDFA are small, typically on the order of $10^{-21}\,\mathrm{cm}^2$, because optical transitions between the 4f levels are parity forbidden. Consequently, rather high pump intensities are needed to reach population inversion, typically on order of $10^1\mathrm{kw/cm}^2$. In high quality optical engineering these disadvantages are considerable. For removing main disadvantages there are some proposals. One of important idea is using nanotechnology for improving amplification process. It was shown that the adding silicon nanocrystals to Er doped $\mathrm{SiO}_2$ strongly enhances the effective Er absorption cross section [3–5]. Also, it was shown that nanocrystals doped into fiber causes the excitations of Er ions compared traditional EDFA through a strong coupling mechanism. This energy transfer process could enable the fabrication of an optical amplifier operating at 1.55 μm that is optically excited at pump intensities as low as a few mw/cm$^2$. We think that beside of the reported results, adding Si-Ncs into Er-doped optical fiber can decrease fiber length for given gain. Also, Si-Ncs have large absorption cross-section area which is so interesting property for increasing the absorption coefficient of the pump wave, concluding to decrease of pump loss or increase of quantum efficiency. Also, because of large quantum confinement of Si-Nc the appeared band gap is increased compared bulk $SiO_2$ matrix. This property concludes to absorption of visible light. Due to this effect amplifier bandwidth also can be increased [6–9].

In [10–12] the authors concentrated on Si-Nc-Er doped fiber amplifier generally and in these references main focusing done on quantum dot and optical properties of Si-Nc was investigated. Using this idea some features of optical amplifier was extracted and distinguished properties were illustrated. Also, a traditional EDFA and Si-Nc Er-doped fiber amplifier was compared. In these references effect of maximum number of excitons in Si-Nc on amplification process didn't discussed.

In [13] modeling of experimentally realized Si-Nc Er doped fiber amplifier was done. In this paper a phenomenological model was presented based on an energy level scheme taking into account the strong coupling between each Si nanocrystal and the neighboring Er ions and considering the interactions between pairs of Er ions too, such as the concentration quenching effect and the cooperative up-conversion mechanism. This is an interesting paper, but some critical points such as inhomogeneous distribution of Er-ions didn't addressed. In practice inhomogeneous distribution usually occurred and complete description of experimental results should consider this subject.

Optical losses in presence of Si-Nc Er doped fiber amplifier were discussed in [14, 15]. In these papers scattering loss and optical loss due to inhomogeneity in manufacturing step of waveguide were discussed. Results of this paper can be used for modeling of optical amplifier precisely.

Energy transfer between Si-Nc and Er ions and time constant of energy coupling was discussed in [16]. This paper presented experimental result of silica thin films containing Si nanocrystals and Er ions prepared by ion implementation. Results of this paper can be used for transient analysis of optical amplifier in presence of nanocrystals.

Finally, gain limiting factors in Si-Nc Er doped fiber amplifier was discussed and addressed in [14]. Presented materials in this paper is interesting for finding the root of gain limitation in this structure.

As we reviewed some of interesting papers including EDFA and Si-Nc Er doped fiber amplifiers, there are some of important key factors in light amplification process that didn't addressed in these papers. One of these subjects is inhomogeneous Er distribution. Maximum number of exciton in Si-Nc also didn't addressed. There are some other factors which are important for modeling of optical amplifiers and there isn't suitable paper for covering these subjects.

For this reason in this paper we like study in detail effect of Si-Nc-Er doped fiber amplifier for obtaining qualified EDFA for precision engineering in optical domain. In this work we concentrate on rate equation for extracting all related parameters of optical amplifier in presence of Si-Ncs. The organization of this paper is as follows. Mathematical background and operation principle of the proposed structure for optical amplifier is presented in Section 2. Simulation results is discussed and presented in Section 3. Finally the paper ends with a short conclusion.

## 2. BASIC PRINCIPLE AND MATHEMATICAL BACKGROUND

In this section we consider basic operation of added Si-Nc Er doped fiber amplification and mathematical description of optical fiber amplifier. Also, based on the developed mathematical formalism effects of different parameters on optical amplification characteristics are studied. Fig. 1 describes the principle of operation of possibly new type of optical amplifier based on silicon nanocrystal and Er doped fiber.

In such system, input signal at 1.55 μm are amplified thanks to the stimulated emission of Er ions which are indirectly pumped via silicon nanocrystal. Indeed, each excitable Er ions within the silica matrix is excited into its first excited state by means of a fast and effective energy transfer from optically pump silicon nanocrystal. Since absorption cross-section area for Si-Nc is larger $[10^{-17}-10^{-18}]\,\mathrm{cm}^2$ than Er ions $10^{-21}\,\mathrm{cm}^2$ alone (1000 times) then the photoluminescence from Er ion which can then strongly amplify the incoming signal at 1.55 μm by stimulated emission is strongly increased [10–12].

### 2.1. Rate Equation

For description and detailed analysis of Si-Nc Er doped fiber [17, 18] amplifier time development equations such as rate equations are

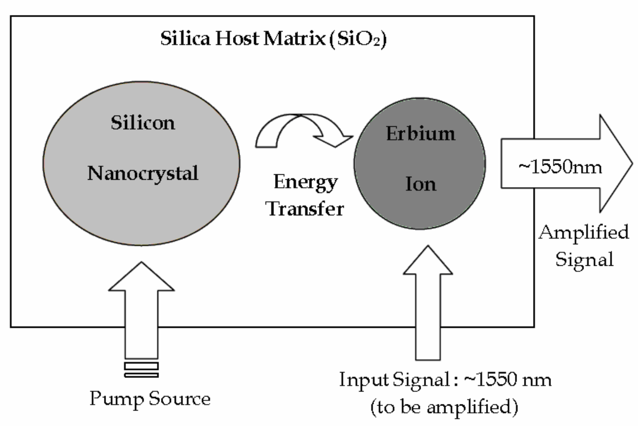

**Figure 1.** Schematic diagram of silicon nanocrystal and Er doped amplifier [11].

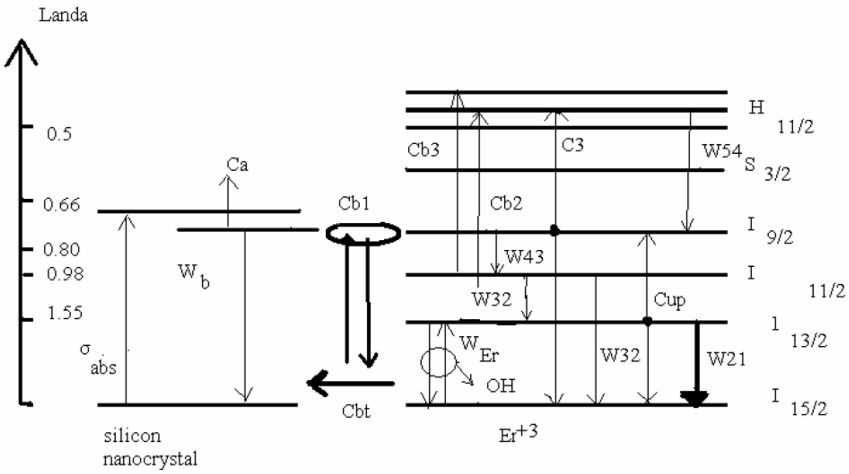

**Figure 2.** Energy level scheme for the system of interaction Silicon Nanocrystal and Er ions.

considered. For these subject energy levels of the amplifier is considered in Fig. 2 [13].

In this analysis silicon nanocrystal behavior can be described by a three level system. Since the reported experimental results show that the trapping is very fast process compared to the typical decay time in optical amplifiers, with accepting a small error two-level case is good approximation for silicon nanocrystals. The appeared Si-Nc parameters such as $\sigma_{abs}$, $C_a$ and $W_b$ are the effective excitation cross section, auger recombination coefficient and the exciton total recombination rate respectively. According to spectroscopic study $\mathrm{Er}^{(+3)}$ is schematized as a five level system. In this formalism $W_{ij}$, $C_{b1}$, $C_{bi}$, $C_{bt}$, $W_{er}$, $C_{up}$ and $C_3$ are the total transition rate from level $i$ to level $j(i > j)$, the coupling between the silicon nanocrystal and the ground state of $\mathrm{Er}^{(+3)}$ (it is therefore responsible for the energy transfer between the silicon nanocrystals and ion), the excited state absorption (ESA) from level $i(i > 2)$, energy back transfer to the first level of the nanocrystal, the concentration quenching effect (due to the energy migration all over the sample introduced by the energy transfer between two nearby $\mathrm{Er}^{(+3)}$ ions occurred between ground and first excited states), the co-operative up conversion coefficient describing the interaction of two nearby $\mathrm{Er}^{(+3)}$ ions which are both in the first excited state and co-operative up-conversion coefficient of interacting electrons in nearby $\mathrm{Er}^{(+3)}$ ions in the second excited state respectively. For full description of $C_{up}$ it should be mentioned that one of electrons in the two ions will return to the ground state and giving its energy to the other which will be excited to the level $I_{9/2}$. In the case of $C_3$ the interaction generates one ion state in the level 5 and other in the ground state.

Also, in the following rate equations the parameters $K_{exiton}$, $\sigma_{dir}$, $N_0$, $N_{Totaler}$ and $C_{Si}(1 \text{ or } 0)$ are the maximum number of the exciton within a single nanocrystal, the effective absorption cross section for $\mathrm{Er}^{(+3)}$ions, the total silicon nanocrystal, the total $\mathrm{Er}^{(+3)}$concentration and a constant ($C_{Si} = 1$) illustrating Si-Nc in Er doped fiber respectively [13]. In the following based on continuity equation the rate equations for the proposed system are given.

$$\frac{dN_b}{dt} = \sigma_{abs}\phi(N_0K_{exciton} - N_b) - (W_bN_b) - C_{si}\sum_{i=1}^{3} C_{bi}N_bN_i \quad (1)$$

$$\frac{dN_a}{dt} = \sigma_{abs}\phi(-N_0K_{exciton} + N_b) + (W_bN_b) + C_{si}\sum_{i=1}^{3} C_{bi}N_bN_i \quad (2)$$

$$\frac{dN_5}{dt} = (C_{dir}\sigma_{dir}\phi N_3) + (C_3N_3^2) - (W_{54}N_5) + C_{si}\sum_{i=2}^{3} C_{bi}N_bN_i \quad (3)$$

$$\frac{dN_4}{dt} = (W_{54}N_5) + (C_{up}N_2^2) - (W_{43}N_4) + (C_{si}C_{b1}N_1N_b) \quad (4)$$

$$\frac{dN_3}{dt} = (W_{43}N_4) - (W_{32} + W_{31})N_3 - (2C_3N_3^2)$$

$$
\begin{aligned}
&-(C_{si}C_{b3}N_bN_3) - (C_{si}C_aN_bN_3) && (5)\\
\frac{dN_2}{dt} &= (W_{32}N_3) - (W_{21}+W_{er})N_2 - (2C_{up}N_2^2)\\
&-(C_{si}C_{b2}N_bN_2) - (C_{si}C_aN_bN_2) && (6)\\
\frac{dN_1}{dt} &= (W_{21}+W_{er})N_2 + (C_{up}N_2^2) + (N_2+N_3)C_aN_bC_{si}\\
&+(W_{31}N_3) + (C_3N_3^2) - (C_{si}C_{bt}N_bN_1) - (\sigma_{dir}\phi N_1C_{si}) && (7)
\end{aligned}
$$

In above equations, $W_{31} \ll W_{21}$, where $N_{a,b}$, $N_i$ and $\phi$ are population of level $a$ and $b$ in nanocrystal, population in different levels of $\mathrm{Er}^{(+3)}$ ions and pump intensity respectively.

**Table 1.** Physical parameter taken from [13] that used in rate equations.

| Symbol | Value |
|---|---|
| $\lambda_{exc}$ | 488 nm |
| $\sigma_{abs}$ | $2 \times 10^{-16}\,\mathrm{cm}^2$ |
| $\sigma_{dir}$ | $1 \times 10^{-20}\,\mathrm{cm}^2$ |
| $W_b$ | $2 \times 10^{4}\,\mathrm{s}^{-1}$ |
| $W_{21}$ | $4.2 \times 10^{2}\,\mathrm{s}^{-1}$ |
| $W_{32}$ | $4.2 \times 10^{5}\,\mathrm{s}^{-1}$ |
| $W_{43}$ | $1 \times 10^{7}\,\mathrm{s}^{-1}$ |
| $W_{54}$ | $< 1 \times 10^{7}\,\mathrm{s}^{-1}$ |
| $C_{b1}$ | $3 \times 10^{-15}\,\mathrm{cm}^3\mathrm{s}^{-1}$ |
| $C_{up0}$ | $7 \times 10^{-17}\,\mathrm{cm}^3\mathrm{s}^{-1}$ |
| $C_{b2}$ | $< 3 \times 10^{-19}\,\mathrm{cm}^3\mathrm{s}^{-1}$ |
| $C_{b3}$ | $< 3 \times 10^{-19}\,\mathrm{cm}^3\mathrm{s}^{-1}$ |
| $C_{bt}$ | $< 3 \times 10^{-19}\,\mathrm{cm}^3\mathrm{s}^{-1}$ |
| $C_a$ | $< 3 \times 10^{-19}\,\mathrm{cm}^3\mathrm{s}^{-1}$ |
| $C_3$ | $7 \times 10^{-17}\,\mathrm{cm}^3\mathrm{s}^{-1}$ |
| $W_{er}$ | $8.1 \times 10^{-19} N_{Totaler}\,\mathrm{s}^{-1}$ |

### 2.2. Gain Limiting Factors

There are some key factors that have main effects on optical gain in the proposed structure. So, in the following we introduce and investigate them in the simulation section [14–16].

### A- Erbium Concentration and Solubility

The most important parameter for amplification is the concentration of $\mathrm{Er}^{(+3)}$ ions that are optically active. The maximum gain per unit length is determined by the product of the cross section for stimulated emission and the excited $\mathrm{Er}^{(+3)}$ concentration. Since typical value for $\sigma_{dir}$ is in the range of $[10^{-21} - 10^{-20}]\,\mathrm{cm}^2$ thus concentration of excited $\mathrm{Er}^{(+3)}$ will be $[10^{18} - 10^{20}]\,\mathrm{cm}^{-3}$. So, these values are required to obtain considerable gain. According to these reported values for $\mathrm{Er}^{(+3)}$ ions, therefore the host material must have high Er solubility.

### B- Waveguide Loss

In the proposed optical amplifier (Si-Nc $Er^{(+3)}$ doped fiber) there are some basic sources of optical loss including absorption (intrinsic and extrinsic) and radiation (Rayleigh scattering). These optical losses $[0.1 - 1]\,\mathrm{dB/cm}$ must be compensated by stimulated emission. Also, pump light due to inhomogeneous refractive index distribution is scattered and reduces the pump efficiency with propagating through fiber.

### C- Mode Overlap

For obtaining high optical gain it is necessary that optical propagating mode has maximum overlap with $\mathrm{Er}^{(+3)}$ ions distribution. Thus, high gain optical amplifier needs high refractive index difference between core and clad. For example in the case of low refractive index contrast material, a significant fraction of the optical mode travels through the undoped cladding, resulting in a lower gain per unit length.

### D- Pump Wavelength

Traditional EDFA usually uses $980\,\mathrm{nm}\,(I_{11/2})$ and $1480\,\mathrm{nm}\,(I_{13/2})$ as optical pumping wavelengths. Also, for optical pumping $488\,\mathrm{nm}$ can be used. Because of similarity in mode size between signal and pump at $1480\,\mathrm{nm}\,(I_{13/2})$, overlap between pump and signal is increased. Also, there is a main problem in this case because of small wavelength distance between pump and signal. This is stimulated emission introduced by pump light. This effect puts an upper limit to the degree of population inversion and limits optical gain. So, pumping at lower wavelengths is better from this point of view.

Pumping at lower wavelengths also have some basic problem related to increase in scattering concludes to higher optical loss and the small absorption cross section.

### E- Pump Absorption

Due to the relatively small absorption cross section of the $\mathrm{Er}^{(+3)}$

ions, high pump intensity is required to achieve population inversion for desired optical gain. This problem can be overcome by introducing a species with large optical absorption cross section. In this case energy transfer to $\text{Er}^{(+3)}$ ions can be done through this introduced element. The doped rare earth ions or silicon nanocrystals into optical fiber can illustrate these elements and operate such as sensitizer. The broad absorption band of the silicon nanocrystal makes it possible to excite Er ions in a waveguide amplifier using a broad band light source.

### F- Co-operative Up Conversion

Interaction between $\text{Er}^{(+3)}$ ions in high level concentration is important factor in gain limiting. Co-operative up-conversion is one of processes occurred in interaction between ions. In this process an excited $\text{Er}^{(+3)}$ ion de-excited by transferring its energy to a neighboring excited ion, promoting it into level $I\ I_{9/2}$ (Fig. 3(a)).

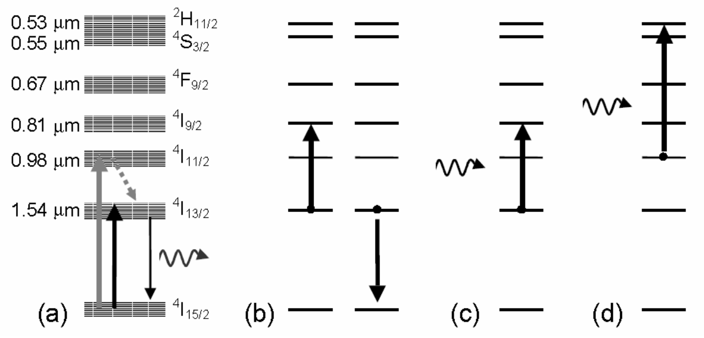

**Figure 3.** Schismatic representation of $\text{Er}^{(+3)}$ intra 4f energy level [13].

This lowers the amount of excited $\text{Er}^{(+3)}$ ions, or conversely increase the pump power needed to obtain a certain degree of inversion. Co-operative up-conversion is possible due to the presence of a resonant level at twice the energy of the first excited state. Co-operative up-conversion coefficient depends on the host material parameters and characteristics such as exact energy level positions, cross section, the dielectric constant and the typical phonon energy.

### G- Excited State Absorption

Another gain limiting effect is excited state absorption (ESA). In this effect an excited $\text{Er}^{(+3)}$ ion absorbs a signal photon (Fig. 3(b)), or a pump photon (Fig. 3(c), Fig. 3(d)), bringing it in a higher excited state. This effect concludes to lower gain and decrease pump efficiency. The effect becomes dominant especially on gain when the higher energy related states have considerable lifetimes. This effect leads to significant steady state population in the higher energy levels, which do not take part in the main optical amplification. Since ESA depends on the pump intensity, it becomes important when high pomp powers are required.

## 3. SIMULATION RESULTS

Based on the proposed rate equations in Section 2, in this section for evaluation of the proposed optical fiber amplifier in presence of Si-Nc Er doped some simulations are presented and discussed. For this task, we consider simulations in four different categories. These are

a) Transient response,
b) Traditional EDFA characteristics,
c) Effect of inhomogeneous Er ions and Si-Nc doping on optical amplifier characteristics
d) Effect of nanocrystals on optical gain and required fiber length for achieving of given gain.

In first part of simulations we considered time evaluation of normalized photoluminescence (PL) intensity, which is illustrated in Fig. 4. In

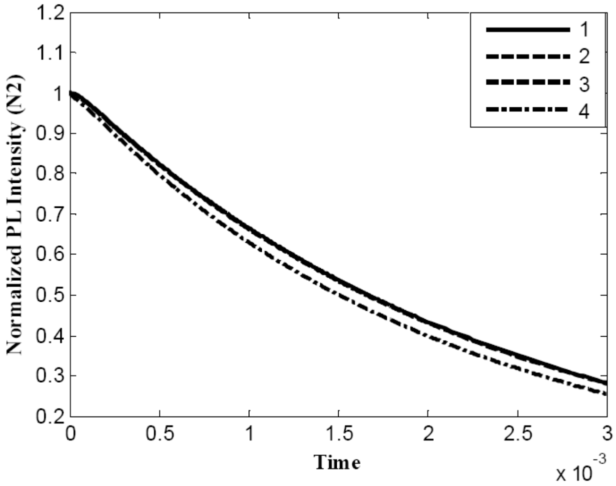

**Figure 4.** Normalized PL Intensity ($N_2$) for EDFA in presence of Si-Nc Vs. Time (sec). 1) Pump $= 1 \times 10^{17}$ Photon/cm$^2 \cdot$ sec, 2) Pump $= 1 \times 10^{18}$ Photon/cm$^2 \cdot$ sec, 3) Pump $= 5 \times 10^{18}$ Photon/cm$^2 \cdot$ sec and 4) Pump $= 1 \times 10^{20}$ Photon/cm$^2 \cdot$ sec.

this figure effect of different pump intensity on PL is considered. It is shown that with increasing the pump level the PL is decreased. For description of this effect it can point out that in high level pump intensity rate of co-operative up-conversion is increased and so PL is decreased. In this curve we assumed steady state condition and the pump turn off in $t = 0$. On the other hand this curve show decay time of the system. So, we expect that the PL decreases with increasing time.

Another important quantity illustrating the quality of the proposed optical amplifier is investigating of population in second excited state. Since the time constant of this level (3 μ sec) is so smaller than first excited state, thus population of this level shows irregular behavior. Fig. 5 shown simulation of this subject and effect of pump level is considered. Direct conclusion of this simulation comes back to unwanted population oscillation of first excited state. Also, this oscillation causes gain variation versus time which shows poor quality of designed optical amplifier.

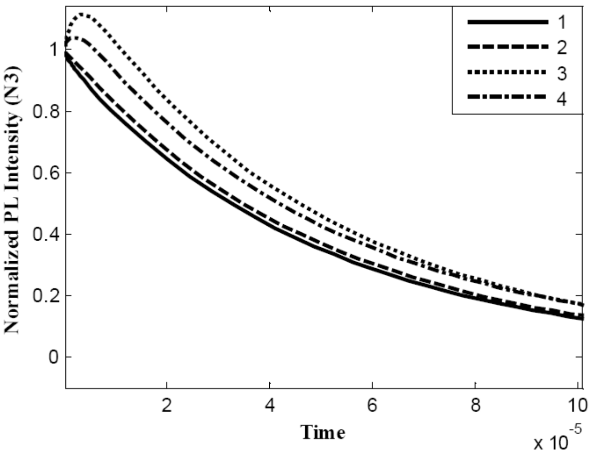

**Figure 5.** Normalized PL Intensity ($N_3$) for EDFA in presence of Si-Nc Vs. Time (sec). 1) Pump $= 1 \times 10^{17}$ Photon/cm$^2$ · sec, 2) Pump $= 1 \times 10^{18}$ Photon/cm$^2$ · sec, 3) Pump $= 5 \times 10^{18}$ Photon/cm$^2$ · sec and 4) Pump $= 1 \times 10^{20}$ Photon/cm$^2$ · sec.

Figure 6 shows the normalized photoluminescence intensity calculated at 1550 nm as a function of time after switching on the laser [19, 20] beam at $t = 0$ and for different pump power. It is observed that the rise time of photoluminescence signal is decreased with increasing of excitation power. We defined the typical experimental rise time $\tau_{on}$

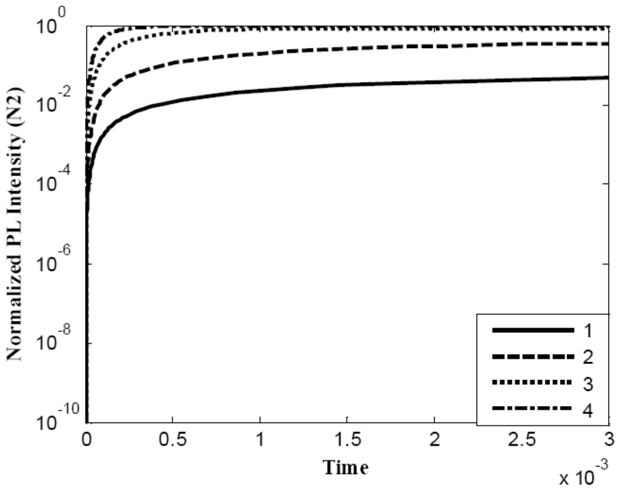

**Figure 6.** Normalized PL Intensity ($N_2$) for EDFA in presence of Si-Nc Vs. Time (sec). 1) Pump $= 1 \times 10^{17}$ Photon/cm$^2 \cdot$ sec, 2) Pump $= 1 \times 10^{18}$ Photon/cm$^2 \cdot$ sec, 3) Pump $= 5 \times 10^{18}$ Photon/cm$^2 \cdot$ sec and 4) Pump $= 1 \times 10^{20}$ Photon/cm$^2 \cdot$ sec.

as the time it takes the luminescence signal to reach the 63% of the saturation value.

In Fig. 7, the photoluminescence intensity at 980 nm is reported as a function of time, normalized for the steady state value, for different excitation pump power. It is observed that the rise time is almost independent of the pump power and approximately close to 1.5 μsec.

In Fig. 8, the number of first excited Er$^{(+3)}$ ions in steady state conditions for EDFA as a function of pump power is illustrated. Based on this simulation it is observed that by increasing the pump power the Er$^{(+3)}$ ions are excited and thus the number of excited ions is increased. Also, in high level pump power there is saturation situation related to increase of interaction between carriers. Interaction between carriers tries to deplete population density at first excited state and this effect operates opposite to increase of pump power.

Since optical gain is proportional to population inversion of Er$^{(+3)}$ ions between the first excited and ground states. For simple conclusion in this paper we simulated normalized population inversion $((N_2 - N_1)/N_{Totaler}$, where $N_2$ and $N_1$ are the steady state concentrations of Er$^{(+3)}$ ions in the first excited and ground states respectively and $N_{Totaler}$ is the total Er$^{(+3)}$ concentration in the

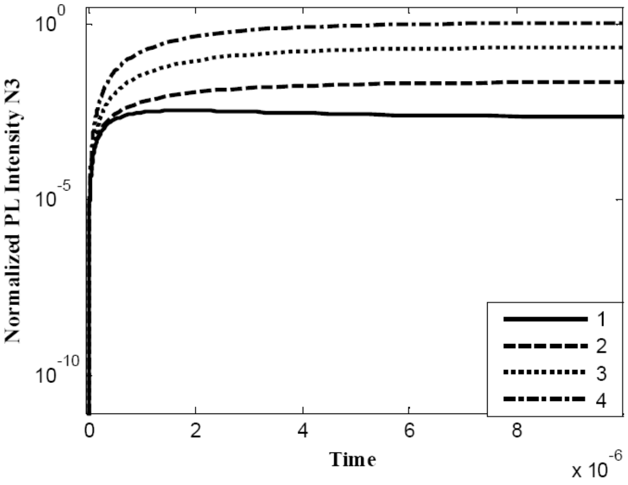

**Figure 7.** Normalized PL Intensity ($N_3$) in presence of Si-Nc Vs. Time (sec). 1) Pump $= 10^{17}$ Photon/cm$^2 \cdot$ sec, 2) Pump $= 10^{18}$ Photon/cm$^2 \cdot$ sec, 3) Pump $= 10^{20}$ Photon/cm$^2 \cdot$ sec.

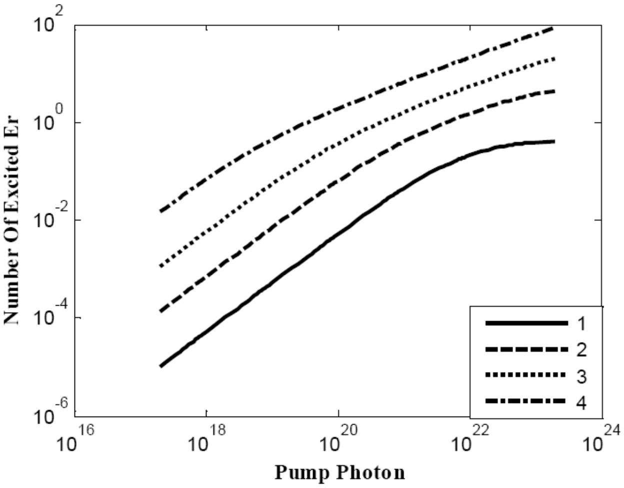

**Figure 8.** Number of Excited Er Vs. Pump Power (Photon/cm$^2 \cdot$ sec). 1). Concentration of Er $= 1 \times 10^{18}$ 1/cm$^3$, 2) Concentration of Er $= 1 \times 10^{19}$ 1/cm$^3$, 3) Concentration of Er $= 5 \times 10^{19}$ 1/cm$^3$ and 4) Concentration of Er $= 5 \times 10^{20}$ 1/cm$^3$.

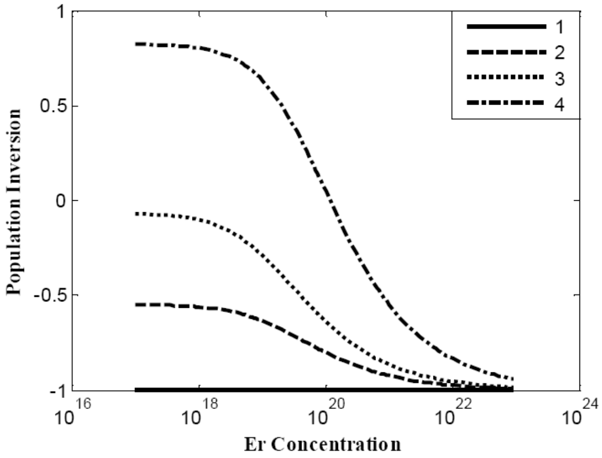

**Figure 9.** Population Inversion (first excited state) Vs. Concentration of Er Ions ($1/\mathrm{cm}^3$). 1) Pump $= 10^{17}\,\mathrm{Photon/cm^2 \cdot sec}$, 2) Pump $= 10^{20}\,\mathrm{Photon/cm^2 \cdot sec}$, 3) Pump $= 10^{21}\,\mathrm{Photon/cm^2 \cdot sec}$ and 4) Pump $= 5 \times 10^{22}\,\mathrm{Photon/cm^2 \cdot sec}$.

film). The result for EDFA is shown in Fig. 9 at a fixed high power density. It is observed that by increasing the $\mathrm{Er}^{(+3)}$ concentrations the population inversion is decreased. This effect comes back to non-radiative quenching process such as concentration quenching and up conversion effect. This subject will be so important when concentration is greater than $1 \times 10^{20} 1/\mathrm{cm}^3$.

Effect of concentration of Er ions on optical gain with pomp power as a parameter is illustrated in Fig. 10. In this analysis we assumed that optical gain can be calculated by $g = \sigma_e \cdot (N_2 - N_1)$, where $\sigma_e$ and $(N_2 - N_1)$ are the emission cross section of level $I_{13/2}$ and the total concentration of $\mathrm{Er}^{(+3)}$ ions inverted in the level respectively. In this simulation the emission cross section of $\mathrm{Er}^{(+3)}$ ions is assumed to $1 \times 10^{-19}\,\mathrm{cm}^2$ which recently determined by [14]. For small pump power positive population inversion doesn't occur and so there isn't optical net gain. With increasing the pump power level the positive population inversion can be achieved. Finally for the case of high power level because of depletion of first excited state due to increase of interaction between carriers the optical net gain begin to decrease. Also, it should mention that with increase of concentration of Er ions first optical gain is increased and finally because of strong interaction between Er ions population inversion is decreased and optical gain comes down.

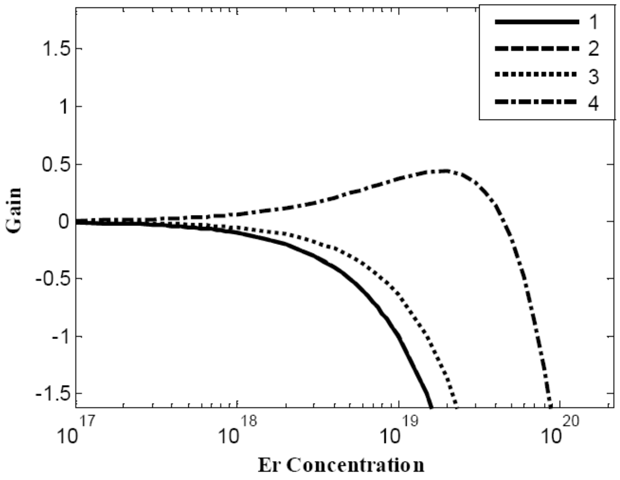

**Figure 10.** Gain Vs. Concentration of Er Ions (1/cm$^3$). 1) Pump $= 10^{17}$ Photon/cm$^2 \cdot$ sec, 2) Pump $= 10^{18}$ Photon/cm$^2 \cdot$ sec, 3) Pump $= 10^{21}$ Photon/cm$^2 \cdot$ sec and 4) Pump $= 3 \times 10^{22}$ Photon/cm$^2 \cdot$ sec.

For investigation of effect of inhomogeneous distribution of Er ions, in this paper we assumed the Gaussian distribution profile and investigated characteristics of the optical amplifier. Fig. 11(a) shows the Er ions profile. Excited Er ions in this case is illustrated in Fig. 11(b). In this situation one must consider effects of high pump power and concentration of Er ions. Interaction between Er ions is increased in middle of distribution (center of core) due to high concentration and also high pump power induces strong interaction between carriers in this position. Both of these effects try to reduce optical gain and finally saturation situation is obtained.

The normalized population inversion of Er$^{(+3)}$ ions in the sample $((N_2 - N_1)/N_{Totaler})$ at fixed power density is illustrated in Fig. 12. In the case of Gaussian distribution for Er$^{(+3)}$ ions by increasing the concentration it is observed that the inverted Er$^{(+3)}$ is decreased. This can be described based on concentration on nonradiative quenching process such as concentration quenching and up conversion. When the maximum number of exciton is increased the population inversion is increased. With increasing the $K_{exiton}$, actually the energy coupling coefficient is increased and this concludes to increase in the population inversion.

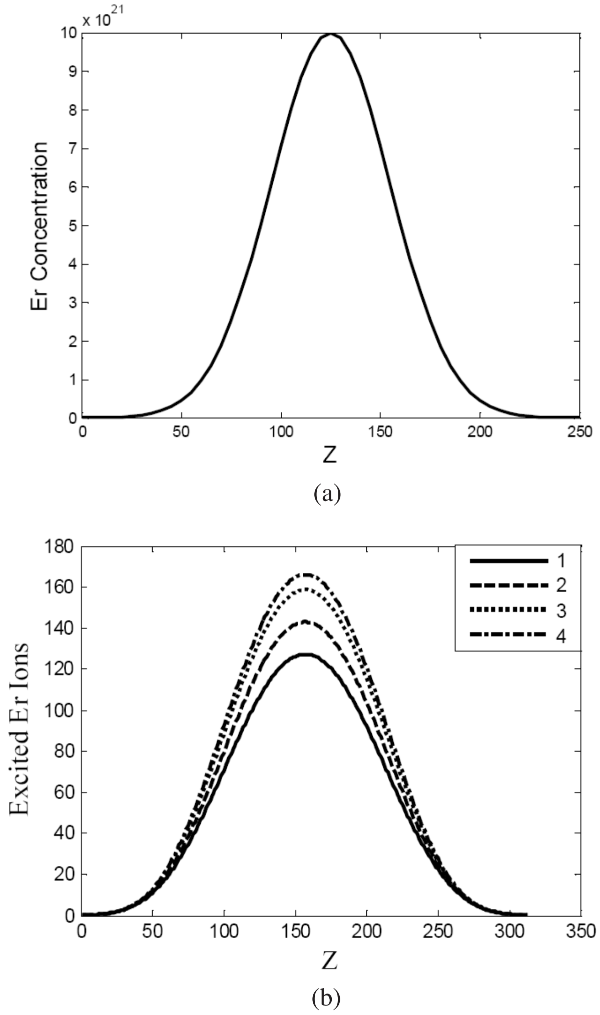

**Figure 11.** (a) Er ions concentration Vs Z (Gaussian distribution, $\sigma = 30$), (b) Excited Er Ion Vs. Z. 1) Pump $= 10^{19}$ Photon/cm$^2 \cdot$ sec, 2) Pump $= 7\times10^{20}$ Photon/cm$^2 \cdot$ sec, 3) Pump $= 10^{22}$ Photon/cm$^2 \cdot$ sec and 4) Pump $= 3 \times 10^{23}$ Photon/cm$^2 \cdot$ sec.

Effects of inhomogeneous distribution of Er$^{(+3)}$ ions, the pump power and the maximum number of exciton on excited Si-Nc investigated and result is shown in Fig. 13. It is shown that for a given density of Si-Nc ($N_0 = 1 \times 10^{19} \frac{1}{\text{cm}^3}$), for density of Er$^{(+3)}$ ions smaller than $N_0$ really the excited Si-Nc is constant. This is simple for

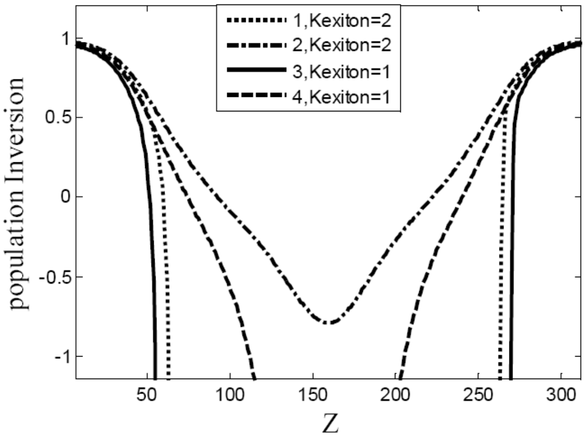

**Figure 12.** Population Inversion Vs. Z. 1) Pump = 5 × $10^{20}$Photon/cm$^2$ · sec, 2) Pump = 5 × $10^{22}$Photon/cm$^2$ · sec, 3) Pump = 5 × $10^{20}$ Photon/cm$^2$ · sec and 4) Pump = 5 × $10^{22}$ Photon/cm$^2$ · sec.

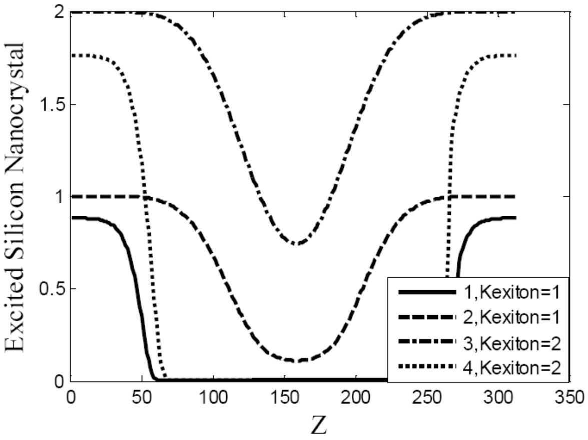

**Figure 13.** Population Inversion Vs. Z. 1) Pump = 5 × $10^{20}$ Photon/cm$^2$ · sec, 2) Pump = $10^{23}$ Photon/cm$^2$ · sec, 3) Pump = $10^{23}$ Photon/cm$^2$ · sec and 4) Pump = 5 × $10^{20}$ Photon/cm$^2$ · sec.

understanding because each Er$^{(+3)}$ coupled at least to one Si-Nc. But in the case of the density of Er$^{(+3)}$ ions larger than $N_0$, the excited Si-Nc is decreased and this is related to weak coupling coefficient due to connecting more than one Er$^{(+3)}$ ions to single Si-Nc. In the case of

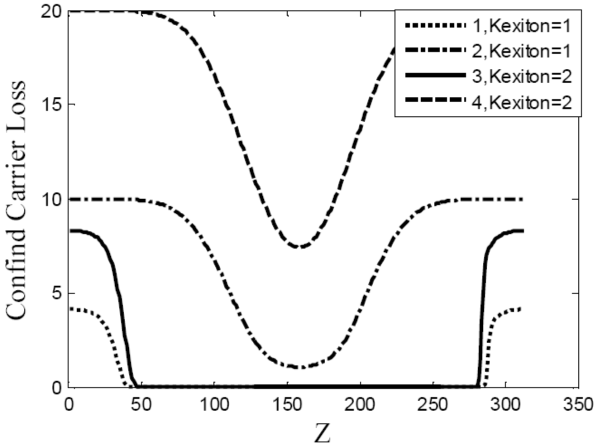

**Figure 14.** Confined Carrier Loss Vs. Z. 1) Pump = 5 × $10^{19}$ Photon/cm$^2 \cdot$ sec, 2) Pump = $10^{22}$ Photon/cm$^2 \cdot$ sec, 3) Pump = 5 × $10^{19}$ Photon/cm$^2 \cdot$ sec and 4) Pump = $10^{22}$ Photon/cm$^2 \cdot$ sec, Confined Carrier Absorption cross section = 1 × $10^{-18}$ cm$^2$.

high pump power the excited Si-Nc because of large absorption cross section area of Si-Nc is increased. So, the energy coupling between Si-Nc and Er$^{(+3)}$ ions can be easily done. When $K_{exiton} = 2$, the fraction of an excited silicon nanocrystal considerably increased.

Confined carrier absorption in presence of Si-Nc Er doped amplifiers sounds to be important. This effect conducts to absorption of optical signal (1.55 µm) and introduces considerable loss. We define confined carrier absorption loss by $\alpha_{CCa} = \sigma_{CCa} N_b$, where $\sigma_{CCa}$ is the confined carrier absorption cross section. Fig. 14 shows confined carrier absorption loss as a function of Er$^{(+3)}$ ions concentration. In this simulation we assumed that the Gaussian distribution for Er$^{(+3)}$ ions concentration. Introduced optical loss comes back to absorption of optical signal by exciton in Si-Nc and electron-hole pair dissociates and finally electron moves to the excited state of Si-Nc. So the number of photons in optical signal frequency (1.55 µm) is decreased. Considering Fig. 14, it is observed that with increasing concentration of Er$^{(+3)}$ ions the introduced confined carrier optical loss is decreased because of strong coupling of Si-Nc and Er$^{(+3)}$ ions. Thus possibility of absorption of optical signal by excitons with increasing of the concentration of Er$^{(+3)}$ ions is decreased. Also, as a final result of this simulation it is considerable to discuss about effect of the pump wave on the confined carrier absorption. It is shown that with increase of the pump power the number of generated exciton is increased and so the confined carrier absorption will be increased.

Effects of the $\mathrm{Er}^{(+3)}$ ions concentration and pump power on optical gain are illustrated in Figs. 15 and 16. With increasing of the $\mathrm{Er}^{(+3)}$ ions concentration the optical gain begins to increase and finally in the case of high concentrations it goes to decrease because of increasing in interaction between $\mathrm{Er}^{(+3)}$ ions. Also, in this situation for the $K_{exiton} = 2$ the optical gain is more than $K_{exiton} = 1$ case. It should mention that in this simulation the emission cross section of $\mathrm{Er}^{(+3)}$ ions assumed to be $1 \times 10^{-19}\,\mathrm{cm}^2$. Fig. 16 illustrates effect of pump power on optical gain. The simulated result shows that with increase of the pump power the optical gain is increased and finally a saturation situation is obtained. In the case of fixed pump power with increasing the $\mathrm{Er}^{(+3)}$ ions concentration the maximum achievable gain and optical threshold power are increased also.

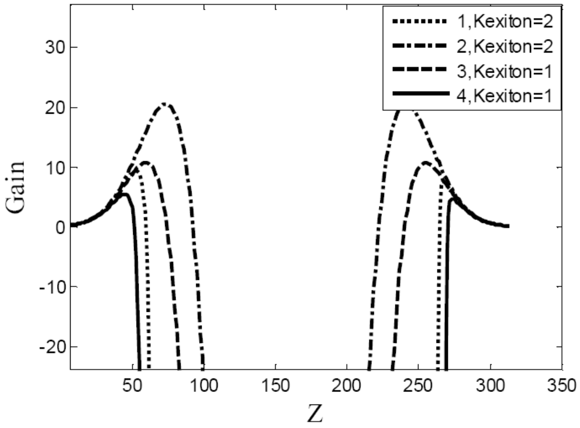

**Figure 15.** Gain Vs. Z. 1) Pump $= 5 \times 10^{20}$ Photon/cm$^2 \cdot$ sec, 2) Pump $= 10^{22}$ Photon/cm$^2 \cdot$ sec, 3) Pump $= 10^{22}$ Photon/cm$^2 \cdot$ sec and 4) Pump $= 5 \times 10^{20}$ Photon/cm$^2 \cdot$ sec.

In this part effects of the maximum number of excitons and concentration of $\mathrm{Er}^{(+3)}$ ions in the case of different confined carrier absorption loss are discussed and illustrated in Figs. 17 and 18. In Fig. 17, for large confined carrier cross section, effect of pump power is considerable on the net gain. With increasing of the pump power because of high level confined carrier cross section induced optical loss due to Si-Nc is increased more than increase in optical gain. So, the

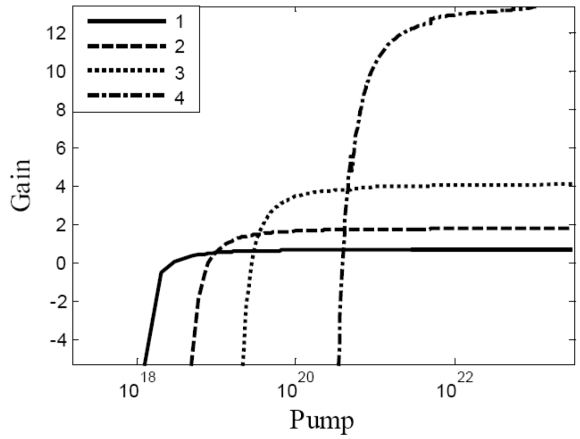

**Figure 16.** Gain vs. Pump (Photon/cm$^2 \cdot$ sec). 1) Concentration of Er $= 1 \times 10^{18}$ 1/cm$^3$, 2) Concentration of Er $= 1 \times 10^{19}$ 1/cm$^3$, 3) Concentration of Er $= 5 \times 10^{19}$ 1/cm$^3$ and 4) Concentration of Er $= 5 \times 10^{20}$ 1/cm$^3$.

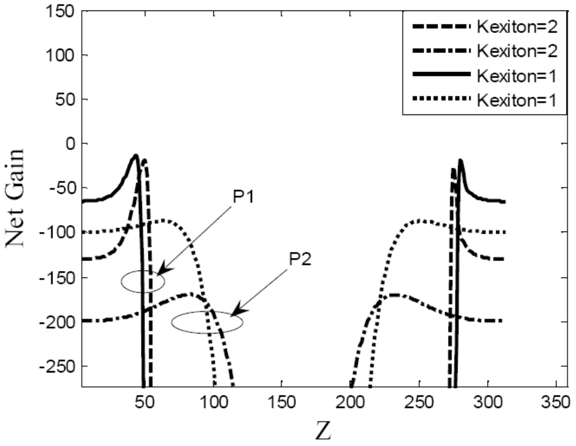

**Figure 17.** Net Gain Vs. Z. 1) $P_1 = 5 \times 10^{20}$ Photon/cm$^2 \cdot$ sec, 2) $P_2 = 1 \times 10^{22}$ Photon/cm$^2 \cdot$ sec, Confined Carrier Absorption cross section $= 1 \times 10^{-17}$ cm$^2$.

optical net gain will be decreased with increase of pump power. Also, in this case with increase of exciton number in Si-Nc the optical gain going to increase, but in the case of high level confined carrier absorption cross section induced optical loss due to Si-Nc is increased more than increase in optical gain. So, the optical net gain will be decreased with

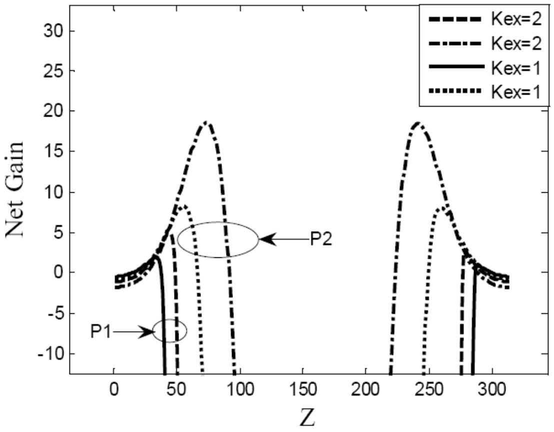

**Figure 18.** Net Gain Vs. Z. 1) $P_1 = 5 \times 10^{20}$ Photon/cm$^2 \cdot$ sec, 2)$P_2 = 1 \times 10^{22}$ Photon/cm$^2 \cdot$ sec, Confined Carrier Absorption cross section $= 1 \times 10^{-19}$ cm$^2$.

increase of number exciton.

In Fig. 18, the situation is similar to previous case generally, but because of small confined carrier cross section area effect of pump power is considerable than previous simulation (Fig. 17). Based on simulated figures it is observed that the proposed optical amplifier may have quite high gain (10 cm$^{-1}$) for $\sigma_{CCa} = 1 \times 10^{-19}$ cm$^2$. Also, in this situation in the case of $K_{exiton} = 2$ high gain is available compared previous condition ($K_{exiton} = 1$).

In the following effect of pump power on net gain for different concentration of Er ions as parameter and fixed confined carrier absorption cross section is illustrated. It is shown that with increase of the pump power the net gain begins to increase and finally comes to saturation state. Also, it is observed that with increase of the concentration of Er ions the threshold intensity for achievement of positive net gain is increased.

Finally effect of the concentration of Si-Nc on fiber length for a given optical gain and optical gain are considered. In Fig. 20, the first case is studied. It is shown that with increase of the concentration of Si-Nc the fiber length for obtaining 40 dB gain is decreased. It should mention that for higher density the required fiber length going to be constant. This is related to increase of interaction between Si-Nc. Thus the energy coupling between Si-Nc and Er ions can't be done correctly and increase of Si-Nc can't increase the population inversion. Finally in this curve is shown that with increase of the exciton number the required fiber length is decreased for a given gain.

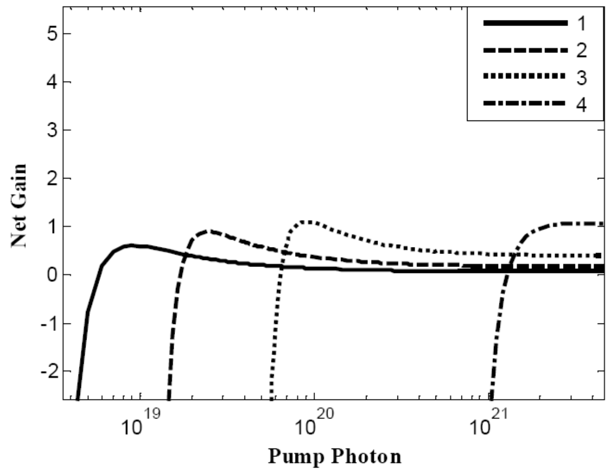

**Figure 19.** Net Gain Vs. Pump (Photon/cm$^2 \cdot$ sec). 1) Concentration of Er $= 1 \times 10^{18}$ 1/cm$^3$, 2) Concentration of Er $= 1 \times 10^{19}$ 1/cm$^3$, 3) Concentration of Er $= 5 \times 10^{19}$ 1/cm$^3$ and 4) Concentration of Er $= 5 \times 10^{20}$ 1/cm$^3$, Confined Carrier Absorption cross section $= 1 \times 10^{-18}$ cm$^2$.

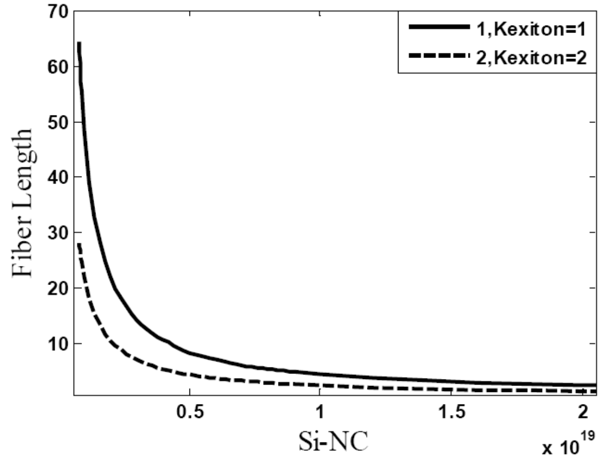

**Figure 20.** Maximum Fiber Length (cm) Vs. Concentration of Silicon Nanocrystal (1/cm$^3$). Gain $= 40$ dB, Pump $= 1 \times 10^{21}$ Photon/cm$^2 \cdot$ sec.

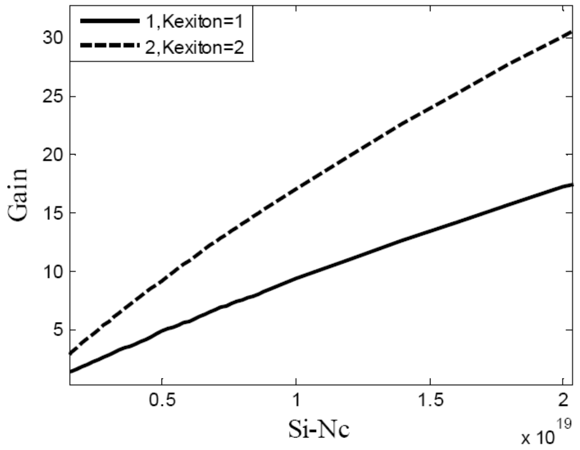

**Figure 21.** Gain Vs. Concentration of Silicon Nanocrystal ($1/\text{cm}^3$). Pump $= 1 \times 10^{22}$ Photon/$\text{cm}^2 \cdot \text{sec}$.

As a final result, we illustrated effect of the concentration of Si-Nc on optical gain for a given pump intensity and concentration of Er ions. We observed that with increase of the density of Si-Nc the optical gain is going to increase. In the high level density because of increasing of the interaction between Si-Nc the slope of increasing in the optical gain is decreased and finally a situation similar to saturation case is obtained. Also, with increase of the exciton number the slope of the optical gain is increased.

## 4. CONCLUSION

Effect of Si-Nc added to traditional EDFA has been investigated both in steady state and transient regimes. It was observed that this introduction had so interesting properties. Some of these interesting characteristics are decreasing of the fiber length for a given optical gain and increase of the optical gain in the case of fixed fiber length. Our calculations have shown that for 40 dB optical gain the requested fiber length for traditional EDFA and Si-Nc Er doped fiber amplifier are $L = 1.1 \times 10^5$ cm and $L = 50$ cm respectively. Also, in this case optical loss due to Si-Nc should be considered for optimization purposes. Generally this introduced optical loss try to decrease the optical net gain and in practice one should optimize this subject with control of different parameters such as density of Er ions, pump power and density of Si-Nc. Effect of the number of exciton in Si-Nc has considered in this paper. It was shown that with increase of the exciton number the optical gain is increased. Also, it was shown that with increase of the pump level in presence of Si-Nc both the rise and fall times are decreased.

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