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APPLICATION OF ELECTROMAGNETISM-LIKE ALGORITHM TO PHASE-ONLY SYNTHESES OF ANTENNA ARRAYS

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Abstract—In this paper, a novel stochastic approach, i.e., the electromagnetism-like (EM-like) algorithm, is applied to phase-only syntheses of antenna arrays. The goal is to minimize the pattern sidelobe under null-steering constraints by phase-only adjusting. The mechanism of EM-like algorithm results from the Coulomb's Law of Electromagnetics. It does not require gradient calculations, and can automatically converge at a good solution through the virtual charge of each particle. Simulation results show that the EM-like algorithm can well treat the phase-only optimization of antenna arrays. Although the null-steering constraints will affect the optimization in sidelobe reduction, it can be easily included in the EM-like based optimization scheme. The EM-like algorithm can be applied not only to problems of antenna arrays, but also to many other nonlinear optimization problems in electromagnetic waves.

1. INTRODUCTION

The synthesis of an antenna array plays an important role in radar and communication systems [1, 2]. In general, such types of problems are strongly nonlinear. There are two types of approaches for treating syntheses of antenna arrays, i.e., the gradient-based and stochastic approaches. Although the gradient-based methods are efficient, they sometimes get stuck in local optimum. Alternatively, the stochastic methods become good candidates to treat complex nonlinear optimization problems with large search spaces. In recent years, stochastic approaches have attracted interests of researchers in syntheses of antenna arrays. This is because they can achieve nearly global optimum without any gradient operation.

Famous stochastic techniques include the genetic algorithms (GA) [3], simulated annealing (SA) algorithm [4], particle swarm optimization (PSO) [5], ant colony optimization (ACO) [6], etc.

In this paper, we utilize a novel algorithm, i.e., the electromagnetism-like (EM-like) algorithm [7] to treat optimum syntheses of antenna arrays. The phase-only syntheses of linear antenna arrays are given by the EM-like algorithm. In other words, the optimization problem is treated by assuming that all elements have the same exciting magnitudes and only the phase-control is allowable. The EM-like algorithm is a random-search based algorithm for finding the globally optimum solution. The mechanism of EM-like algorithm results from the Coulomb's Law of Electromagnetics. The concept of EM-like algorithm is to produce one group of random solutions from the feasible domain, and regard each solution as a virtually charged particle. The virtual charges among particles will generate resultant electromagnetic forces, and then the particles move automatically according to the resultant electromagnetic forces between particles. Besides, the charge of each particle is determined by a cost function representing the optimization goals of problems. The resultant forces between particles are divided into two kinds, i.e., attraction or repulsion. The virtual electromagnetic force is calculated from the Coulomb's law and superposition principle. In the mechanism, a larger charge will have larger attraction or repulsion, and the resultant force will be small as two particles are far from each other. In other words, the resultant forces are determined by the charges and distances between particles. In addition, the EM-like algorithm can improve the current optimum solution through local search and then modify the feasibility for global search. The EM-like algorithm has no gradient operation. It can be used in decimal system directly. Neither coding nor encoding procedure (as in GA) is required in the EM-like algorithm. Moreover, this method has the advantages of SA, i.e., movement of particle is gradually slow in latter stages of iteration. In general, this method is similar to PSO, but requires fewer particles than PSO. Based on above points, we are convinced that the EM-like algorithm is suitable for our phase-only syntheses of antenna arrays. The good convergence of the EM-like algorithm has been verified in [8]. The formulations, simulation results and conclusions are given in the following.

2. FORMULATIONS

2.1. Electromagnetism-like Algorithm

The EM-like algorithm is a stochastic evolutionary technique based on the electromagnetic theory in physics. This method includes four main stages [7], i.e., initialization, local search, calculation, and movement, respectively. These stages are interpreted in the following.

Stage 1) Initialization

Some parameters must be set before utilizing the EM-like algorithm to optimize a problem. These parameters include the population size, the criterion (e.g., iteration number), the dimension of the solution, and the feasible region of each dimension. In general, the population size should be greater than three because the movement of particle involves the resultant forces of particles. The criterion dominates the execution of EM-like iteration. In other words, the iteration will be terminated as the predetermined loop or allowable error is met. The dimension of the solution is determined according to the cost function and the region of each dimension is determined from the problem.

Initially, the EM-like algorithm creates one group of initial solution randomly. Each solution is regarded as a virtually charged particle. All particles are assumed to be uniformly distributed between the upper and lower bounds. The optimum particle of the population will then be found according to the cost function. The cost function is determined from the problem.

Stage 2) Local search

Local search can be divided into three kinds, i.e., no local search, local search only on current better particle and local search on all particles. Theoretically, the local search is expected to find a better solution, especially when it is applied to all particles. However, the local search is usually time-consuming. Therefore, this local search can be neglected in some cases. In this study, the EM-like algorithm is implemented with local search on the current better particle. The reason for choosing this way is to balance the reliability of optimization against its executing time.

Stage 3) Calculation

The virtual charge of the i-th particle is determined by the cost function value, and is calculated by

$$q^i = \exp\{-\frac{f(p^i) - f(p^{opt})}{\sum_{k=1}^{m}[f(p^k) - f(p^{opt})]}\}, \quad \forall i, \tag{1}$$

where $f(\cdot)$ denotes the cost function and m denotes the population size. The $f(p^{opt})$ denotes the best cost function value. The particle with largest charge (i.e., best cost function value) is called the "optimum particle". A particle will have stronger attraction, as it appears near the optimum particle. The particle attracts other particles with better cost function values, and repels other particles with worse cost function values.

The resultant force on the i-th particle is determined from the Coulomb's law and superposition principle, and is given as

$$F^i = \sum_{\substack{j=1 \\ j \neq i}}^{m} \begin{cases} (p^j - p^i)\dfrac{q^i q^j}{\|p^j - p^i\|^2} & \text{if} \quad f(p^j) < f(p^i) \\ (p^i - p^j)\dfrac{q^i q^j}{\|p^j - p^i\|^2} & \text{if} \quad f(p^j) \geq f(p^i) \end{cases}, \quad \forall i, \tag{2}$$

where $f(p^j) < f(p^i)$ represents attraction and $f(p^j) \geq f(p^i)$ represents repulsion. From (2), we conclude that the resultant force between particles is proportional to the product of the charges and is in inverse proportion to the distance between the particles. Of course, a particle will not produce the force to affect itself. In general, the force in (2) is normalized as

$$F^i = \frac{F^i}{\|F^i\|}, \quad \forall i. \tag{3}$$

for simplicity.

Stage 4) Movement

The update of each particle depends on the resultant force, and is given as

$$p^i = \begin{cases} p^i + \alpha \cdot F^i(b_{upper} - p^i) & \text{if} \quad F^i > 0 \\ p^i + \alpha \cdot F^i(p^i - b_{lower}) & \text{if} \quad F^i \leq 0 \end{cases}, \\ i = 1, 2, ..., m; \quad i \neq best, \tag{4}$$

where b_{upper} is the upper bound, b_{lower} is the lower bound, and α is a random value uniformly distributed between zero and one. The particle moves toward the upper bound by a random step length α as the resultant force is positive, whereas it moves toward the lower bound as the resultant force is negative. In the mechanism, the optimum particle of the population does not move, because it has the best cost function value and then attracts all other particles.

2.2. Uniformly Spaced Linear Array

In this study, we consider the synthesis of a linear array, as shown in Fig. 1. The linear array has $2N$ elements and is equally spaced with spacing of 0.5λ, where λ is the wavelength. Assume each element is isotropic and there exist no mutual coupling effects within the array. In addition, the array elements in Fig. 1 are assumed to be symmetric as

$$\phi_i = \phi_{2N+1-i}, \quad i = 1, 2, \ldots, N, \tag{5}$$

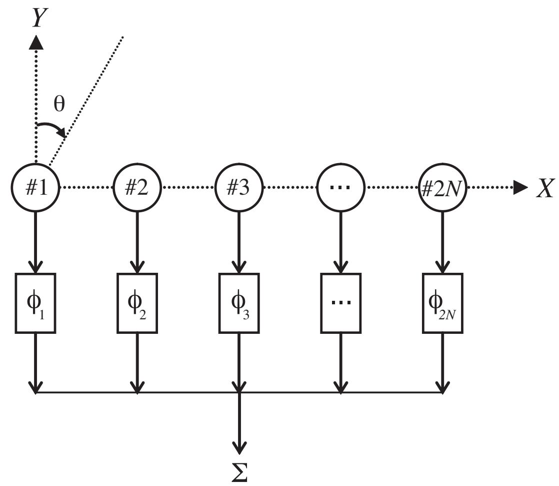

Figure 1. Schematic of a $2N$-element linear array.

where ϕ_i denotes the phase of the i-th array element. Note that the proposed scheme has no limitations on array geometry, number of array elements and mutual coupling effects. The reason why we utilized such a simple array is that it is easy to illustrate the proposed scheme and compare the results with those of other existing approaches. For such a symmetric array, the array factor is given as [9]

$$AF(\theta, \overline{\phi}) = 2\sum_{i=1}^{N} e^{j\phi_i} \cos[(i-0.5)\pi \sin\theta], \tag{6}$$

where θ denotes the azimuth angle. For convenience, (6) is usually rewritten in decibels as

$$AF_{dB}(\theta, \overline{\phi}) = 20 \cdot \log \left| \frac{AF(\theta, \overline{\phi})}{AF(\theta_o, \overline{\phi})} \right|, \tag{7}$$

where θ_o is the direction of main beam. In this study, we assume the main beam is at $\theta_o = 0°$. The phases of array elements in (6) constitute a phase vector as

$$\phi = [\phi_1, \phi_2, \ldots, \phi_N]^T. \tag{8}$$

The goal becomes to adjust the phases of array elements in (8) to achieve the desired array performance, i.e., to optimize (7). So we use the phases of array elements, i.e., (8), to compute the cost function of each particle. In this study, the optimization goals of array synthesis include the sidelobe reduction and null steering. Therefore, we define the cost function of each particle as

$$f(p) = \kappa_1 \cdot f_{SL}(\overline{\phi}) + \kappa_2 \cdot f_{NS}(\overline{\phi}), \tag{9}$$

where κ_1 and κ_2 are two parameters representing the weights of the two goals. The first term of the right-hand side in (9) is for suppressing the maximal sidelobe level of array pattern, and is given as

$$f_{SL}(\overline{\phi}) = \max\{AF_{dB}(\theta, \overline{\phi})\}_{\theta \in S}, \tag{10}$$

where S denotes the feasible region of sidelobe excluding the main beam. The second term of the right-hand side in (9) is for null steering, and is given as

$$f_{NS}(\overline{\phi}) = k \cdot \sum_k AF_{dB}(\theta^k_{null}, \overline{\phi}), \tag{11}$$

where θ^k_{null} denotes the direction of k-th null. The nulls are symmetrical in this problem. To compare the performance of different arrays with a given power limitation, the relative array efficiency is defined as [9]

$$\eta = \frac{P}{P_o}, \tag{12}$$

where P and P_o are the power density of main beam for simulated array and for uniformly excited array, respectively.

3. SIMULATION RESULTS

In this study, we consider the linear and equally spaced array, as shown in Fig. 1. The element spacing is fixed as 0.5λ. The array is synthesized using the EM-like algorithm described above. In EM-like procedures, the population size is chosen as 20 and the maximum iteration loops are chosen as 20000. The parameters κ_1 and κ_2 in (9) are chosen as 1 and 0.1. In this paper, the feasible region of phase control is chosen to be in the range of $[-180°, 180°]$. In addition, the minimum manageable amount of phase difference and the resolution of azimuth angle are both assumed to be $0.1°$. In order to illustrate the proposed scheme, different examples are simulated below. The following simulation was implemented by using the hardware of personal computer with Intel Core-2 6420 CPU (2.13 GHz), and the software of Fortran 90 in Absoft ProFortran 6.2.

3.1. Maximal Sidelobe Minimization

In the first example, the array synthesis for maximal sidelobe reduction is given. Different numbers of array elements are considered. The final optimum results for the case of 20-element array are shown in Fig. 2. The dashed line is the pattern of the uniformly excited array, i.e., no array synthesis. The solid line is the final optimum pattern by phase-only synthesis. From Fig. 2, it shows that the maximal sidelobe reduction is improved by 2.95 dB (from −13.19 dB to −16.14 dB). This optimum result is slightly better than that of [9] by about 0.3 dB.

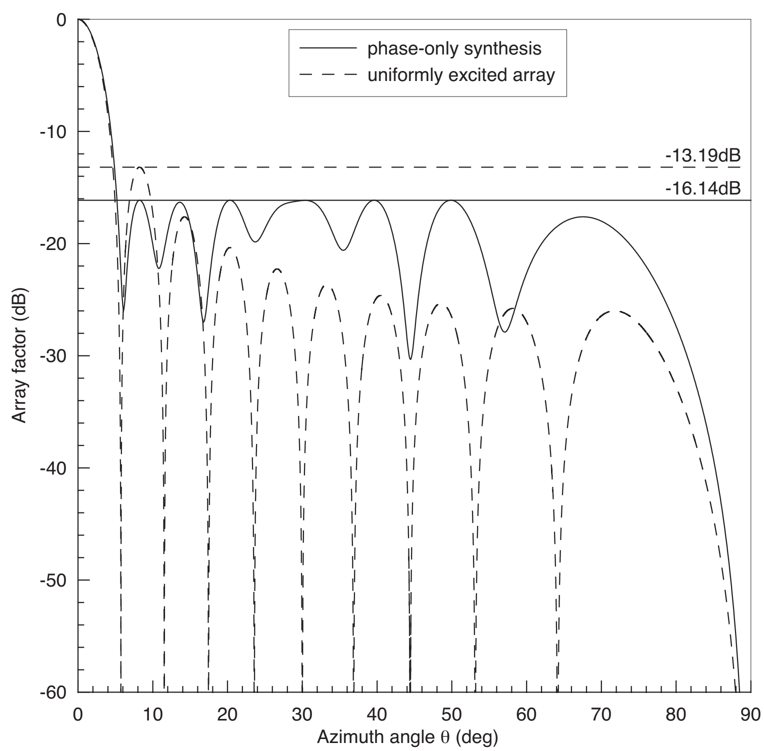

Figure 2. Final optimum array patterns for the uniformly excited array and the phase-only synthesis.

Figure 3 shows the optimum reduction of maximal sidelobe and the array efficiency versus the number of array elements. Note that the horizontal axis of Fig. 3 is in logarithmic scale. For comparison, the results calculated by the steepest descent method (SDM) [9] are also given in Fig. 3. From Fig. 3, we find that the results by these two approaches are consistent in most cases. In addition, the maximal sidelobe reduction by EM-like algorithm is slightly better than that by SDM in the case of 20-element array. From Fig. 3, we find that the optimum reduction of maximal sidelobe and the array efficiency are both highly correlated with the logarithm of array element number. Therefore, one can easily predict the results for other cases from the simulated data in Fig. 3.

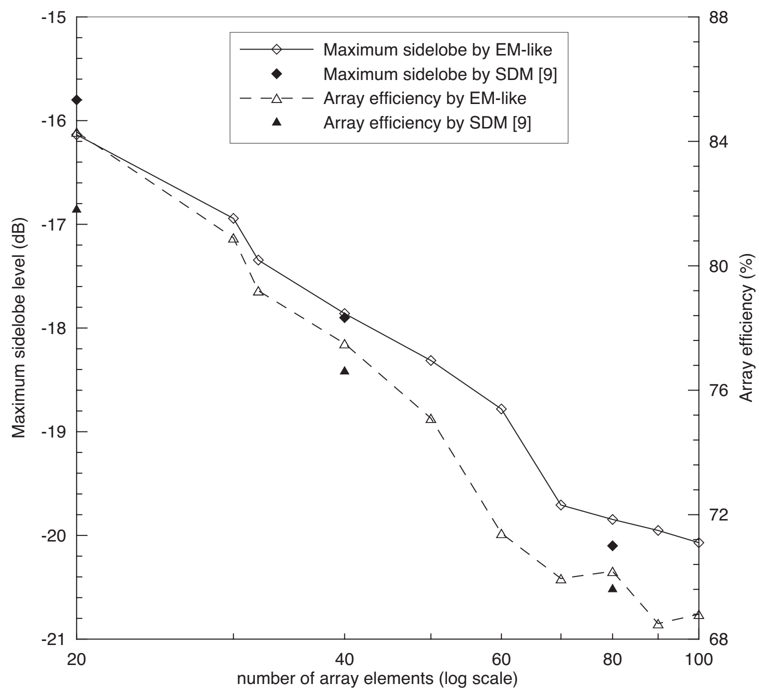

Figure 3. Final optimum results for maximum sidelobe level and array efficiency with respect to the number of array elements.

3.2. Unidirectional Null Steering

The second example of phase-only synthesis discusses the optimum sidelobe reduction under the constraint of null steering at a specific direction. In the simulation, the pattern value fewer than $-60\,\text{dB}$ is viewed as the null. Fig. 4 and Fig. 5 show the final optimum array patterns of 32-element and 40-element arrays, respectively. The dashed line in Fig. 4 represents the pattern of the 32-element array by phase-only synthesis without the constraint of null steering, and the final maximal sidelobe is about $-17.34\,\text{dB}$ at $58.73°$. The half-power beamwidth, first null beamwidth, and array efficiency are about 3.34°, 7.94°, and 79.2%, respectively. As the constraint of unidirectional null steering at ±9% is imposed on the problem, the final optimum maximal sidelobe of the 32-element array becomes −13.03 dB (sacrificed by 4.31 dB), as illustrated by the solid line of Fig. 4. With such a null-steering constraint, the half-power beamwidth, first null beamwidth, and array efficiency are changed to 3.23°, 7.42°, and 71.7%, respectively. Fig. 5 shows the results for the case of 40-element array. As the null-steering constraint is imposed on the problem, the final optimum sidelobe reduction is sacrificed by 1.69 dB (from −17.86 dB to −16.17 dB). In addition, the half-power beamwidth, first null beamwidth, and array efficiency are changed from 2.75° to 2.65°, from 6.48° to 6.3°, and from 77% to 77.5%, respectively.

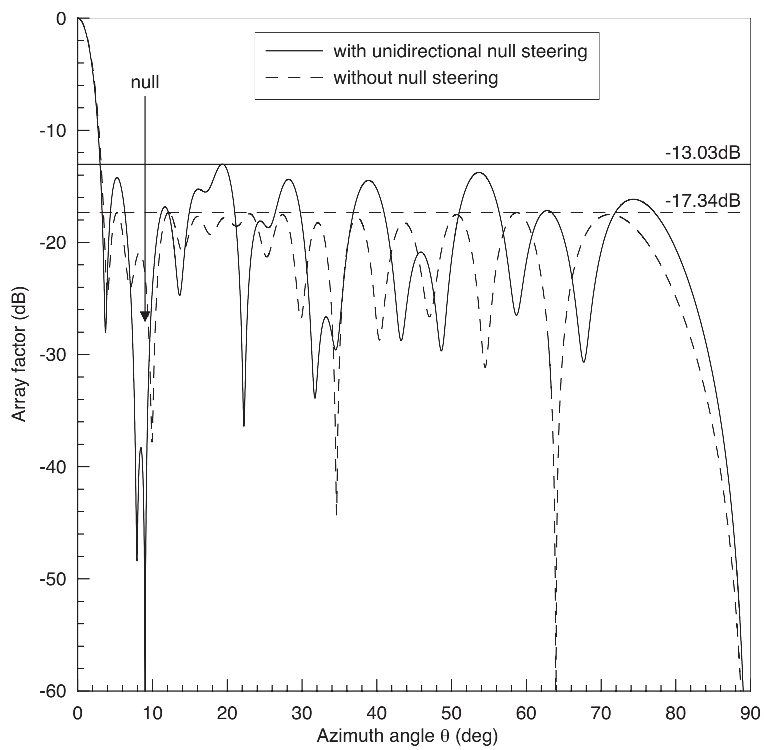

Figure 4. Final optimum array patterns of the 32-element array by phase-only synthesis with unidirectional null steering and without null steering.

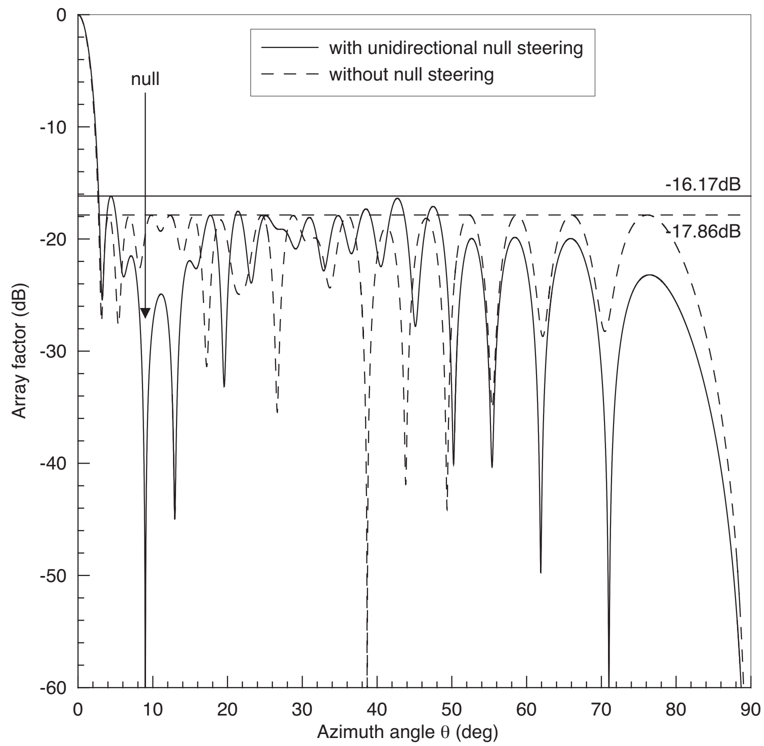

Figure 5. Final optimum array patterns of the 40-element array by phase-only synthesis with unidirectional null steering and without null steering.

3.3. Bidirectional Null Steering

The third example of phase-only synthesis discusses the sidelobe reduction under the constraint of null steering at two specific directions, i.e., bidirectional null steering. Fig. 6 shows the final optimum pattern of 20-element array with the constraint of bidirectional null steering. The solid line in Fig. 6 represents the results that the two directions of null steering are chosen as $\pm 14°$ and $\pm 20.5°$. The optimum maximal sidelobe is -11.37 dB at 5.42°. The half-power beamwidth, first null beamwidth, and array efficiency are 5.51°, 13.4°, and 69.1%, respectively. The dashed line in Fig. 6 represents the results that the two directions of null steering are chosen as $\pm 33.5°$ and $\pm 40°$. The optimum maximal sidelobe is -12.41 dB at 8.4°. The half-power beamwidth, first null beamwidth, and array efficiency are 5.42°, 12.2°, and 75%, respectively.

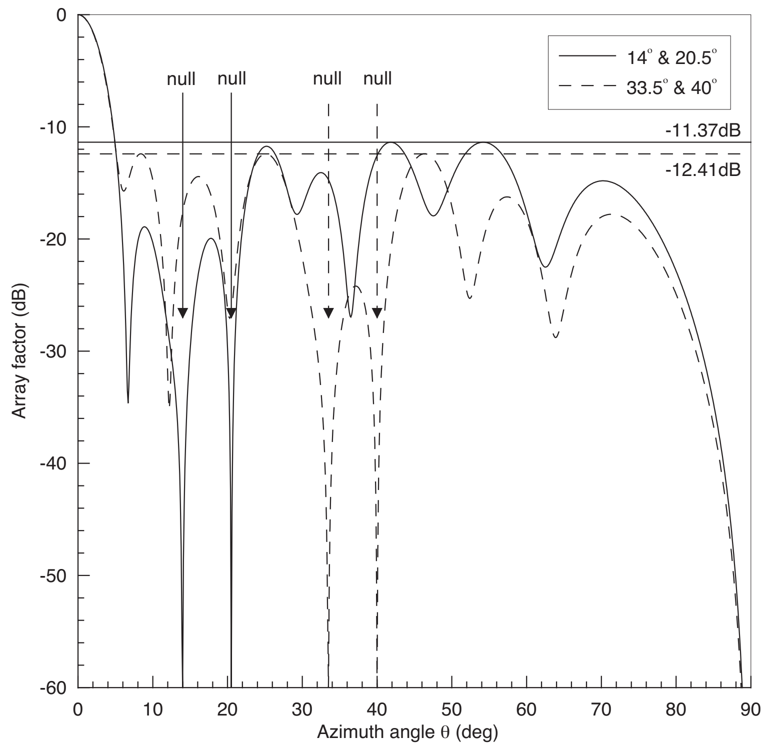

Figure 6. The final optimum pattern of 20-element array with the constraint of bidirectional null steering.

4. CONCLUSIONS

This paper utilized a novel stochastic algorithm, i.e., the electromagnetism-like (EM-like) algorithm, to implement phase-only syntheses of antenna arrays. The EM-like algorithm has been successfully combined with the array pattern formulas to achieve an efficient array synthesis scheme. Many array characteristics, such as the sidelobe reduction and the steering-null control can be easily optimized by the EM-like algorithm. Although only the change of excitation phase is considered in our simulation, other parameters such as amplitude factors can be easily added to our array synthesis scheme. Simulation results show that the constraints of null steering will degrade the optimization performance in reducing the maximal sidelobe level. Such constraints can be easily included in the iteration procedures of the EM-like algorithm. The mechanism of EM-like algorithm results from the Coulomb's Law of Electromagnetics. Due to the inherent stochastic property of EM-like algorithm, the optimization scheme in this study does not require the gradient operation and can automatically achieve nearly global optimum solutions. Although the stochastic approaches such as GA, SA, PSO, ACO algorithms can also achieve nearly global optimum without any gradient operation, the EM-like algorithm gives more related physical meanings in such a problem. The EM-like algorithm based optimization scheme is easy to implement. Therefore, it is suitable not only for problems of antenna arrays, but also for many other nonlinear optimization problems in electromagnetic waves [10–20].

REFERENCES

1. Bucci, O. M., et al., "Antenna pattern synthesis: A new general approach," *Proc. of IEEE*, Vol. 82, No. 3, 358–371, 1994.
2. Balanis, C. A., *Antenna Theory: Analysis and Design*, 3rd edition, Wiley, New York, 2005.
3. Haupt, R. L. and D. H. Werner, *Genetic Algorithms in Electromagnetics*, Wiley, New York, 2007.
4. Bevelacqua, P. J. and C. A. Balanis, "Optimizing antenna array geometry for interference suppression," *IEEE Trans. Antennas Propag.*, Vol. 55, No. 3, 637–641, 2007.
5. Bevelacqua, P. J. and C. A. Balanis, "Minimum sidelobe levels for linear arrays," *IEEE Trans. Antennas Propag.*, Vol. 55, No. 12, 3442–3449, 2007.
6. Rajo-lglesias, E. and O. Quevedo-Teruel, "Linear array synthesis using an ant-colony-optimization-based algorithm," *IEEE Antennas Propag. Mag.*, Vol. 49, No. 4, 70–79, 2007.
7. Birbil, S. I. and S. C. Fang, "An electromagnetism-like mechanism for global optimization," *J. Global Optimization*, Vol. 25, No. 3, 263–282, 2003.
8. Birbil, S. I., S. C. Fang, and R. L. Sheu, "On the convergence of a population-based global optimization algorithm," *J. Global Optimization*, Vol. 30, No. 2, 301–318, 2004.
9. Deford, J. F. and O. P. Gandhi, "Phase-only synthesis of minimum peak sidelobe patterns for linear and planar arrays," *IEEE Trans. Antennas Propagat.*, Vol. 36, No. 2, 191–201, 1998.
10. Guo, Y., Y. Xu, L. Xia, and R. Xu, "Efficient optimization of a ka-band branch waveguide power divider," *Journal of Electromagnetic Waves and Applications*, Vol. 22, No. 1, 17–26, 2008.

11. Liu, X. F., Y. C. Jiao, and F. S. Zhang, "Conformal array antenna design using modified particle swarm optimization," *Journal of Electromagnetic Waves and Applications*, Vol. 22, No. 2/3, 207–218, 2008.
12. Rattan, M., M. S. Patterh, and B. S. Sohi, "Optimization of Yagi-Uda antenna using simulated annealing," *Journal of Electromagnetic Waves and Applications*, Vol. 22, No. 2/3, 291–299, 2008.
13. Mahmoud, K. R., et al., "Performance of circular Yagi-Uda arrays for beamforming applications using particle swarm optimization algorithm," *Journal of Electromagnetic Waves and Applications*, Vol. 22, No. 2/3, 353–364, 2008.
14. Kouveliotis, N. K., et al., "Optimizing a PIFA using a genetic algorithms approach," *Journal of Electromagnetic Waves and Applications*, Vol. 22, No. 2/3, 453–461, 2008.
15. Zhou, S. G., "A new fitness function for optimizing the matching network of broadband antennas by genetic algorithm," *Journal of Electromagnetic Waves and Applications*, Vol. 22, No. 5/6, 759–765, 2008.
16. Zainud-Deen, S. H., W. M. Hassen, E. M. Ali, and K. H. Awadalla, "Breast cancer detection using a hybrid finite difference frequency domain and particle swarm optimization techniques," *Progress In Electromagnetics Research B*, Vol. 3, 35–46, 2008.
17. Lim, T. S., V. C. Koo, H. T. Ewe, and H. T. Chuah, "A SAR autofocus algorithm based on particle swarm optimization," *Progress In Electromagnetics Research B*, Vol. 1, 159–176, 2008.
18. Ngo Nyobe, E. and E. Pemha, "Shape optimization using genetic algorithms and laser beam propagation for the determination of the diffusion coefficient in a hot turbulent jet of air," *Progress In Electromagnetics Research B*, Vol. 4, 211–221, 2008.
19. Mangaraj, B. B., I. S. Misra, and A. K. Barisal, "Optimizing included angle of symmetrical V-dipoles for higher directivity using bacteria foraging optimization algorithm," *Progress In Electromagnetics Research B*, Vol. 3, 295–314, 2008.
20. Rocca, P., L. Manica, and A. Massa, "Directivity optimization in planar sub-arrayed monopulse antenna," *Progress In Electromagnetics Research Letters*, Vol. 4, 1–7, 2008.