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PATTERN SYNTHESIS OF PLANAR ANTENNA ARRAY VIA CONVEX OPTIMIZATION FOR AIRBORNE FORWARD LOOKING RADAR

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Abstract—When airborne forward looking planar antenna is used to detect ground moving target, targets may be masked by strong clutter due to high sidelobes of the antenna pattern. In this paper, transmitting pattern is synthesized via convex optimization in order to suppress clutter from ground. Transmitting pattern has a low sidelobe illuminating short ranges and a high sidelobe focused into sky and remote ranges, which results in a relative small beamwidth in the elevation plane. In the azimuthal plane, transmitting pattern can form some notches in some fixed directions where strong clutter and interference exist. With insufficient training data due to a dispersion of clutter spectrum along range, adaptive receiving pattern with low sidelobes can be obtained by convex optimization when detecting remote targets. Simulation results show that transmitting and receiving patterns can effectively be designed via convex optimization for airborne forward looking radar.

1. INTRODUCTION

Antenna array for airborne radar is used by space-time adaptive processing (STAP) technology to reject clutter and accomplish ground moving target indication (GMTI) [1]. STAP is applicable for side looking radar, where clutter Doppler frequency is independent of range. The clutter echoes from many range cell can be utilized as training samples to estimate accurately the clutter covariance matrix of the range cell under test and form an adaptive receiving beam.

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However, unlike side looking radar, forward looking radar has a range-dependence clutter spectrum, resulting from the nonzero orientation of array axis relative to the aircraft velocity vector. So for forward looking radar, clutter echoes on different range cells have distinct statistical properties and thus are not suitable for estimating the clutter covariance matrix of the range cell under test. This is called insufficient training sample problem [2]. This problem causes receiving antenna pattern to suffer high sidelobes.

Several approaches via compensation of clutter spectrum have been proposed to obtain sufficient samples [3, 4]. Unfortunately, these approaches require the accurate motion parameters of aircraft, and become invalid in the presence of ambiguous ranges. A kind of diagonal loading method is presented in [5, 6]. Diagonal loading can remedy the pattern distortion with small training sample size. However, the main shortcoming of diagonal loading is that it is not clear how to obtain the optimal value of the diagonal loading factor. In particular, when a window function, such as chebyshev, is applied to the array to obtain super low sidelobes, a big diagonal loading factor will lead to the notch position shift, whereas a small diagonal loading factor will result in undesired sidelobe levels. In this paper an adaptive receiving pattern with super sidelobes is given with small training sample size.

As is well known, when airborne radar detects remote targets, strong clutter echoes from short ranges results from high sidelobes of the antenna pattern. For airborne forward looking radar working at high-pulse repetition frequency (HPRF) mode, the range is highly ambiguous and the clutter return of each range cell is the superposition of many clutter echoes coming from different ranges. A solution to suppress these clutter echoes is that super low sidelobes of transmitting pattern is formed and focused on short ranges in the elevation plane. The cost of super low sidelobes is that the beamwidth of antenna pattern is widened. Therefore, the proposed transmitting pattern should have a super low sidelobe illuminating short ranges and a relative high sidelobe directed at sky or remote ranges. The antenna pattern with such asymmetric sidelobes has a relative narrow beam. In the azimuthal plane, transmitting pattern should have some notches resisting strong clutter echoes or interferences in some fixed directions based on some knowledge about scenario terrain. This will weaken clutter echoes and interferences and mitigates the signal processing load.

Pattern synthesis is the process of choosing the antenna parameters to obtain desired radiation characteristics, such as the specific position of the nulls, the desired sidelobe level and beamwidth of antenna pattern. A lot of methods have been developed for the synthesis of the linear and planar arrays [7, 8]. In particular, the techniques of placing nulls in the antenna patterns are given in the literature [9–11]. Most of these methods optimize only one parameter and are not suitable for the synthesis of airborne forward looking planar antenna. Recently there are several intelligent optimization methods for array antenna synthesis such as Neural Network [12], Genetic Algorithm [13], Particle Swarm [14, 15], and Colony algorithm [16]. Nevertheless, these intelligent algorithms have large computational complexity, thus can not online run.

In this paper, transmitting and receiving patterns is synthesized via convex optimization. Convex optimization has attracted considerable attention [17, 18]. Several methods have been proposed to design transmitting pattern with asymmetric sidelobes [19, 20]. These methods agree in the fact that only uniform antenna array can be optimized. Whereas, convex optimization is applicable for arbitrary arrays. And convex optimization can control the peak value of sidelobes precisely. Moreover, compared to the Genetic Algorithm [21], convex optimization can guarantee to achieve the global optimum and also can be solved with high efficient algorithms, e.g., the famous interior-point method (IPM) [22].

2. TRANSMITTING PATTERN SYNTHESIS

Consider the airborne phased-array antenna with a rectangular planar array composed of $M\,(\text{rows}) \times N\,(\text{columns})$ omnidirectional elements with interelement spacing $d = \lambda/2$, where λ is the working wavelength. In Fig. 1, θ is azimuth angle, φ elevation angle and ψ cone angle. The time delay of signal propagation for the kth array element relative to

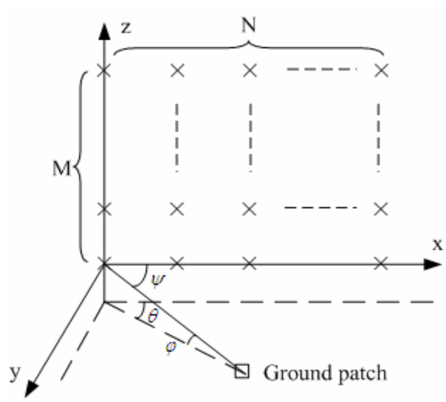

Figure 1. Planar array geometry for airborne radar.

the reference element can be computed by

$$\tau_k = \frac{1}{c}(x_k \cos\psi + z_k \sin\varphi) \tag{1}$$

The transmitting pattern of the planar array is given by

$$\begin{aligned} F(\psi,\varphi) &= \sum_{k=1}^{MN} w_k e^{-j2\pi F_c \tau_k} = \sum_{k=1}^{MN} w_k e^{-j\frac{2\pi}{\lambda}(x_k \cos\psi + z_k \sin\varphi)} \\ &= \sum_{m=1}^{M}\sum_{n=1}^{N} w_{mn} e^{j\frac{2\pi d}{\lambda}[(n-1)\cos\psi + (m-1)\sin\varphi]} \end{aligned} \tag{2}$$

where F_c is the center frequency of transmitting signal. $k = mn$ and w_{mn} (i.e., w_k) is the excitation current of the mnth element, whose magnitude and phase determine the radiation pattern.

Let $w = [w_{11}, w_{12}, \dots, w_{MN}]^T$ and

$$\begin{aligned} a(\psi,\varphi) &= \left[\begin{array}{cccc} 1 & e^{j\frac{2\pi d}{\lambda}\sin\varphi} & \dots & e^{j\frac{2\pi d}{\lambda}(M-1)\sin\varphi} \end{array}\right]^T \\ &\otimes \left[\begin{array}{cccc} 1 & e^{j\frac{2\pi d}{\lambda}\cos\psi} & \dots & e^{j\frac{2\pi d}{\lambda}(N-1)\cos\psi} \end{array}\right]^T \end{aligned} \tag{3}$$

then (2) is rewritten as:

$$F(\psi,\varphi) = w^H a(\psi,\varphi) \tag{4}$$

Assume the center of the radiation pattern is directed at the ground patch (φ_0, θ_0) and has a half-beamwidth of Ω_{ML} in the elevation plane. According to the geometry relationship between antenna and the ground patch, we have $\cos\psi_0 = \cos\theta_0 \cos\varphi_0$. The bounds for φ are $-90^\circ \leq \varphi \leq 90^\circ$. The angle region of clutter coming from ground is $\Omega_{ML} \leq \varphi \leq 90^\circ$. So the super low sidelobe of antenna pattern should correspond to this region. Whereas the sidelobes located at the region $(-90^\circ, -\Omega_{ML})$ corresponds to remote ground patches or sky, where clutter echoes have weak power. Therefore, we can raise appropriately these sidelobe levels in $(-90^\circ, -\Omega_{ML})$ to obtain a relative narrow beamwidth, which will not bring more clutter echoes and interferences. In this case, a radiation pattern with asymmetric sidelobes occurs in the elevation plane. The region $(\Omega_{ML}, 90^\circ)$ are chosen and a uniform grid is used to obtain φ_i, $i = 1, 2, \dots, L$, where L is the number of samples. In order to control the sidelobe levels of transmitting pattern t, we add constraints to the radiation pattern in the regions $(\Omega_{ML}, 90^\circ)$

and $(-90^\circ, -\Omega_{ML})$, respectively in the following.

$$\begin{aligned} &\left|W^H a(\varphi_i,\psi_i)\right|^2 \le U_i \quad \varphi_i \in (\Omega_{ML}, 90^\circ) \\ \text{and } &\left|W^H a(\varphi_i,\psi_i)\right|^2 \le V_i \quad \varphi_i \in (-90^\circ, -\Omega_{ML}) \end{aligned} \tag{5}$$

where U_i and V_i are the specific sidelobe levels in the corresponding regions $V_i \le U_i$.

If we have some knowledge about scenario terrain, such as buildings, mountain, then some notches of transmitting pattern should be formed in the directions of strong reflection patches. We assume that these directions are $\psi_1, \psi_{2,}, \ldots, \psi_Q$.

Based on the constrained minimum white noise gain criterion, transmitting pattern synthesis problem with the above constraints can be represented as the below optimization problem:

$$\begin{aligned} &\min_w \|w\| \\ &\text{subject to } W^H a(\varphi_0,\psi_0) = 1 \\ &\qquad \left|W^H a(\varphi_i,\psi_i)\right|^2 \le U_i, \quad \varphi_i \in (\Omega_{ML}, 90^\circ) \\ &\qquad \left|W^H a(\varphi_i,\psi_i)\right|^2 \le V_i \quad \varphi_i \in (-90^\circ, -\Omega_{ML}) \\ &\qquad \left|W^H a(\varphi,\psi_k)\right|^2 \le D_k, \quad k = 1,\ldots,Q \end{aligned} \tag{6}$$

where $\|\cdot\|$ denotes L_2 norm. D_{ij} is the designed sidelobe level corresponding to strong reflection patches. A single linear equality constraint in (6) aims at keeping the unit gain in the direction of the desired signal. There are some quadratic inequality constraints in (6). Therefore, the optimization problem is convex. The SeDuMi MATLAB toolbox [23] is used to solve this convex optimization problem.

3. RECEIVING PATTERN SYNTHESIS

To mitigate signal processing complexity, generally we synthesize each column of the array by setting excitation current on them, and then apply a set of adaptive weights to the output of the equivalent horizontal linear array with N "elements" (column subarray). The pattern of the nth column subarray is given by

$$g_n(\varphi) = \sum_{m=1}^{M} I_m e^{j\frac{2\pi d}{\lambda}(m-1)(\sin\varphi - \sin\varphi_0)} \quad n = 1, 2, \ldots, N \tag{7}$$

where $I_m e^{-j\frac{2\pi d}{\lambda}(m-1)\sin\varphi_0}$ is the fixed excitation current in the mth row subarray.

Consequently the receiving pattern of the planar antenna is written as

$$G(\psi,\varphi)=\sum_{n=1}^{N}g_n(\varphi)w_r e^{j\frac{2\pi d}{\lambda}(n-1)\cos\psi}=w_r^H a_r(\psi) \tag{8}$$

where $w_r=\begin{bmatrix} w_{r1} & \dots & w_{rN}\end{bmatrix}^T$ is adaptive weight vector applied to each column subarray. And the receiving steering vector is

$$a_r(\psi)=\begin{bmatrix} 1 & e^{j\frac{2\pi d}{\lambda}\cos\psi} & \dots & e^{j\frac{2\pi d}{\lambda}(N-1)\cos\psi}\end{bmatrix}^T \tag{9}$$

For the minimum variance distortionless response (MVDR) method [24], the average output power is minimized when maintaining the response of the array in the signal direction. In order to suppress more clutter echoes and interferences, we should add a constraint to MVDR method to obtain super low sidelobes. Therefore, the receiving pattern synthesis can be represented as

$$\begin{aligned}&\min_{w} w_r^H \mathbf{R}_x w_r\\ &\quad \text{subject to } W_r^H a_r(\psi_0)=1\\ &\qquad\qquad \left|W_r^H a_r(\psi)\right|^2\le\gamma_{ij},\psi\notin(-\Theta_{ML},\Theta_{ML})\end{aligned} \tag{10}$$

where $\mathbf{R}_x$ is the covariance matrix of clutter-plus-noise, Θ_{ML} is half-beamwidth in azimuthal plane. γ_{ij} is the required sidelobe level of the receiving pattern.

Let $\mathbf{R}_x=\mathbf{T}^H\mathbf{T}$, where $\mathbf{T}$ be the Cholesky factorization. Then the objective function of (10) becomes:

$$w_r^H\mathbf{R}_x w_r=w_r^H\mathbf{T}^H\mathbf{T}w_r=\|\mathbf{T}w\|^2 \tag{11}$$

Introducing $\|\mathbf{T}w\|\le\tau$, we can convert the above optimization problem into the following SOC program [25].

$$\begin{aligned}&\min_{w}\tau\\ &\text{subject to } \|\mathbf{T}w\|\le\tau\\ &\qquad W_r^H a_r(\psi_0)=1\\ &\qquad \left|W_r^H a_r(\psi)\right|^2\le\gamma_{ij},\psi\in(-\Theta_{ML},\Theta_{ML})\end{aligned} \tag{12}$$

The SeDuMi MATLAB toolbox is also applied to solve this problem.

4. SIMULATION RESULTS

We consider a uniform planar array of $M = 20$ and $N = 20$. And antenna elements are spaced half wavelength apart. Transmitting pattern is designed to have a beamwidth of 11°, the sidelobes of less than −30 dB illuminating short ranges and other sidelobes of less than −10 dB directed at sky and remote ranges in the elevation plane. In the azimuthal plane, the sidelobe levels are less than −30 dB and beamwidth is 14°. 100 independent simulation runs are performed to obtain final results.

In Fig. 2, we can see asymmetry sidelobes in the elevation plane. Assume that strong clutter echoes impinge on the array from $\psi = 60°$. Fig. 3 shows two dimensional radiation pattern of planar array. Obviously, a fixed deep notch with −60 dB level is obtained in the direction $\psi = 60°$.

Figure 4 plots adaptive receiving patterns by MVDR, convex optimization and diagonal loading. We use 30 snapshots to estimate the sample covariance matrix. The notch in the direction of strong clutter is obtained, as shown in Fig. 4. However, due to a limit in the number of samples, the sidelobe level is high for the MVDR method. This will increase the amount of clutter echoes and interferences. The diagonal loading method with a window function of Chebyshev can achieve low sidelobes. Whereas, not all appearing sidelobes have the same level, of which some levels exceed −30 dB, as shown in Fig. 4. It can be seen that all appearing sidelobes via the convex optimization reach exactly 30 dB below the mainlobe. Note that the cost of low sidelobes is a little dispersion of the beamwidth.

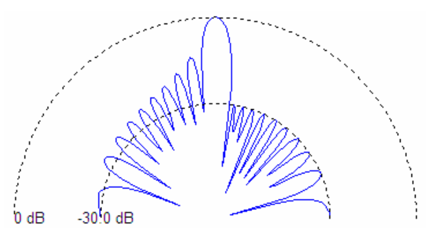

Figure 2. Elevation radiation pattern of the antenna.

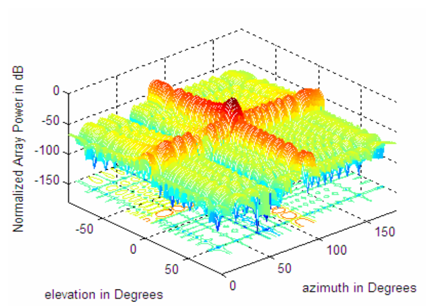

Figure 3. Transmitting pattern via convex optimization.

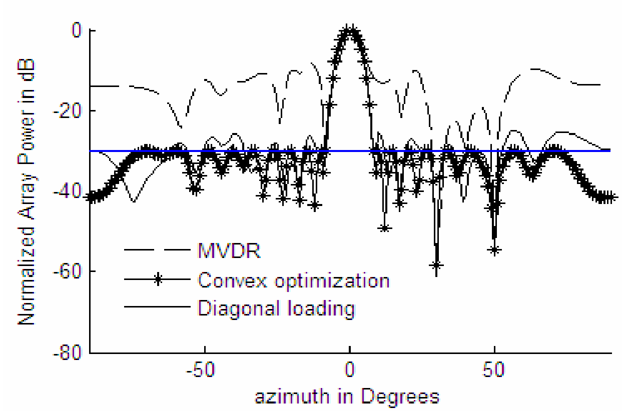

Figure 4. Adaptive receiving pattern.

5. CONCLUSIONS

In this paper, we have designed transmitting and receiving patterns to suppress clutter and interference for airborne forward looking radar. The transmitting pattern is synthesized to have asymmetric sidelobes in the elevation plane and fixed notches in the directions of strong clutter echoes or interferences in the azimuthal plane. To overcome the shortcoming of a limit in the number of samples, we use convex optimization to synthesize adaptive receiving pattern with super low sidelobes. The simulation results verify the feasibility and validity of the proposed method.

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