Progress In Electromagnetics Research C, Vol. 3, 225–245, 2008

THEORETICAL ANALYSIS OF BIT ERROR RATE OF SATELLITE COMMUNICATION IN KA-BAND UNDER SPOT DANCING AND DECREASE IN SPATIAL COHERENCE CAUSED BY ATMOSPHERIC TURBULENCE

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Abstract—We study the influence of atmospheric turbulence on satellite communication by the theoretical analysis of propagation characteristics of electromagnetic waves through inhomogeneous random media. The analysis is done by using the moment of wave fields given on the basis of a multiple scattering method.

We numerically analyze the degree of the spatial coherence (DOC) of electromagnetic waves on a receiving antenna and the bit error rate (BER) of the Geostationary Earth Orbit (GEO) satellite communication in Ka-band at low elevation angles on the assumption that the spatial coherence of received waves decreases and spot dancing only occurs. In this analysis, we consider the Gaussian and the Kolmogorov models for the correlation function of inhomogeneous random media. From the numerical analysis, we find that the increase in BER for the uplink communication is caused by the decrease in the average intensity due to spot dancing of received beam waves and that the increase in BER for the downlink communication is caused by the decrease in DOC of received beam waves. Furthermore, we find that the decrease in DOC of received waves and the increase in BER becomes much more in the Kolmogorov model than in the Gaussian model.

1. INTRODUCTION

In satellite communication, random fluctuation of the dielectric constant of the atmosphere affects propagation characteristics of electromagnetic waves. In particular, satellite communication in high frequencies, such as the Geostationary Earth Orbit (GEO) satellite communication in Ka-band, is significantly affected by atmospheric turbulence at low elevation angles. It causes spreading of the beam wave, decrease in the spatial coherence of received waves, spot dancing of beam waves and scintillation of received intensities. These effects result in degrade of satellite link quality, such as the increase in bit error rate (BER).

Many studies on influences of atmospheric turbulence on satellite communication in high frequencies have been made theoretically [1–5]. In some of these studies, the influences are analyzed as the problem of wave propagation through inhomogeneous random media. The analysis is done by using the moment of wave fields given on the basis of a multiple scattering method [6, 7]. Using this method, BER of the GEO satellite communication in Ka-band under spot dancing has been analyzed numerically, where BER is derived from the integration of the average intensity on a receiving antenna [5].

However, in case that the spatial coherence of received waves is decreased and the spatial coherence radius is not much larger than a radius of the aperture of a receiving antenna, it is not enough to analyze BER by using the integration of the average intensity. In this case, BER has to be analyzed by using the mutual coherence function which includes effects of the spatial coherence of received waves.

In this paper, we numerically analyze the BER derived from the received power using the mutual coherence function on a receiving antenna as well as the BER derived from the integration of the average intensity on a receiving antenna, as shown in Reference [5]. From a result of the analysis, we consider influences of spot dancing and spatial coherence of received waves caused by atmospheric turbulence on BER of the GEO satellite communication in Ka-band at low elevation angles.

2. FORMULATION

2.1. Second Moment of Wave Fields

We assume that an inhomogeneous random medium, which represents atmospheric turbulence, is characterized by the fluctuation of the dielectric constant. The dielectric constant ε, the magnetic permeability μ and the conductivity σ are expressed as

$$\varepsilon = \varepsilon_0 \left[1 + \delta\varepsilon(\mathbf{r}, z)\right] \tag{1}$$
$$\mu = \mu_0 \tag{2}$$
$$\sigma = 0, \tag{3}$$

where $\mathbf{r} = \mathbf{i}_\mathrm{x} x + \mathbf{i}_\mathrm{y} y$ ($\mathbf{i}_\mathrm{x}$ and $\mathbf{i}_\mathrm{y}$ denote the unit vectors of x and y coordinates), ε_0 and μ_0 are the dielectric constant and the magnetic permeability for free space, respectively. $\delta\varepsilon(\mathbf{r}, z)$ is a Gaussian random function with the properties:

$$\langle \delta\varepsilon(\mathbf{r}, z)\rangle = 0 \tag{4}$$
$$\langle \delta\varepsilon(\mathbf{r}_1, z_1) \cdot \delta\varepsilon(\mathbf{r}_2, z_2)\rangle = B(\mathbf{r}_-, z_+, z_-), \tag{5}$$

where $\mathbf{r}_- = \mathbf{r}_1 - \mathbf{r}_2$, $z_+ = (z_1 + z_2)/2$, $z_- = z_1 - z_2$ and the bracket notation $\langle\cdot\rangle$ denotes an ensemble average of the quantity inside the brackets. Thus the medium fluctuates inhomogeneously in the z direction and homogeneously in the $\mathbf{r}$ direction. Moreover, we assume that for any z,

$$B(\mathbf{0}, z, 0) \ll 1, \quad kl(z) \gg 1, \tag{6}$$

where $k = 2\pi f/c$ is the wave number for free space (f is frequency and c is velocity of light), and $l(z)$ is the local correlation length of $\delta\varepsilon(\mathbf{r}, z)$. The medium changes little the state of polarization of the wave under the condition (6), and the present analysis can be made in the scalar approximation. In addition, the forward scattering and the small angle approximations can be applied.

We represent $u(\mathbf{r}, z)$ as a successively forward scattered wave with $\exp(-jwt)$ time dependence in the inhomogeneous random medium. The second moment of $u(\mathbf{r}, z)$ is given as the solution to the moment equation [7] by:

$$\begin{aligned} M_{11}(\mathbf{r}_+, \mathbf{r}_-, z) &= \langle u(\mathbf{r}_1, z) u^*(\mathbf{r}_2, z)\rangle \\ &= \frac{1}{(2\pi)^2} \int \mathrm{d}\boldsymbol{\kappa}_+ \hat{M}_{11}^{\mathrm{in}}(\boldsymbol{\kappa}_+, \mathbf{r}_-, z) \\ &\quad \cdot \exp\left[j\boldsymbol{\kappa} \cdot \mathbf{r}_- - \frac{k^2}{4} \int_0^z dz_1 \int_0^{z-z_1} dz_2 \right. \\ &\quad \left. \cdot D\left(\mathbf{r}_- - \frac{z_2}{k}\boldsymbol{\kappa},\, z - z_2 - \frac{z_1}{2},\, z_1\right)\right], \end{aligned} \tag{7}$$

where $\mathbf{r}_+ = (\mathbf{r}_1 + \mathbf{r}_2)/2$, $\mathbf{r}_- = \mathbf{r}_1 - \mathbf{r}_2$, and

$$D\,(\mathbf{r},\, z_1,\, z_2) \;=\; 2\,[B\,(\mathbf{0},\, z_1,\, z_2) - B\,(\mathbf{r},\, z_1,\, z_2)] \tag{8}$$

$$\hat{M}_{11}^{\mathrm{in}}(\boldsymbol{\kappa}_+,\, \mathbf{r}_-,\, z) \;=\; \int \mathrm{d}\mathbf{r}_+ M_{11}^{\mathrm{in}}(\mathbf{r}_+,\, \mathbf{r}_-,\, z) \exp\left(-j\boldsymbol{\kappa}_+ \cdot \mathbf{r}_+\right) \tag{9}$$

$$M_{11}^{\mathrm{in}}(\mathbf{r}_+,\, \mathbf{r}_-,\, z) \;=\; u_{\mathrm{in}}(\mathbf{r}_1,\, z) u_{\mathrm{in}}^*(\mathbf{r}_2,\, z). \tag{10}$$

$u_{\mathrm{in}}(\mathbf{r},\, z)$ represents a transmitted wave which is a wave function in free space, where $\delta\varepsilon(\mathbf{r},\, z) = 0$.

2.2. Transmitted Wave Model

A transmitted wave in free space is assumed to be a Gaussian beam wave, where the transmitting antenna is located in the plane $z = 0$ and the amplitude distribution is Gaussian with the minimum spot size w_0 at $z = -z_0$ and w_0 denotes the radius at which the field amplitude falls to $1/e$ of that on the beam axis. Then, the wave field is given [8] by

$$u_{\mathrm{in}}(\mathbf{r},\, z) = (2A/\pi)^{1/2} w^{-1} \exp[-(1 - jp)r^2/w^2 + j(kz - \beta)], \tag{11}$$

where A is constant, $r = |\mathbf{r}|$ and

$$w \;=\; w_0(1 + p^2)^{1/2} \tag{12}$$

$$p \;=\; 2(z + z_0)/(kw_0^2) \tag{13}$$

$$\beta \;=\; \tan^{-1} p. \tag{14}$$

Therefore, $M_{11}^{\mathrm{in}}(\mathbf{r}_+,\, \mathbf{r}_-,\, z)$ is given by

$$M_{11}^{\mathrm{in}}(\mathbf{r}_+,\, \mathbf{r}_-,\, z) = \frac{2A}{\pi w^2} \exp\left[-\frac{2}{w^2} r_+^2 + j\frac{2p}{w^2}(\mathbf{r}_+ \cdot \mathbf{r}_-) - \frac{r_-^2}{2w^2}\right]. \tag{15}$$

2.3. Correlation Function of Random Dielectric Constant

We assume that the correlation function of random dielectric constant $B(\mathbf{r}_+,\, z_+,\, z_-)$ can be expressed in terms of infinite power series in allover $\mathbf{r}$ space as follows:

$$B(\mathbf{r}_-,\, z_+,\, z_-) \;=\; B(\mathbf{0},\, z_+,\, z_-) + \sum_{i=1}^{\infty} a_{2i}(z_+,\, z_-) r_-^{2i} \tag{16}$$

$$a_{2i}(z_+,\, z_-) \;=\; \frac{\left[\left(\nabla^2\right)^i B(\mathbf{r}_-,\, z_+,\, z_-)\right]_{r_-=0}}{\left[2^i(i!)\right]^2},\ i = 0,\, 1,\, 2,\, \ldots. \tag{17}$$

where $\nabla = \mathbf{i}_x\partial/\partial x + \mathbf{i}_y\partial/\partial y$. From (16), the structure function $D(\mathbf{r}_-, z_+, z_-)$ defined by (8) can be also expressed in terms of the infinite power series:

$$D(\mathbf{r}_-, z_+, z_-) = \sum_{i=0}^{\infty} b_{2i}(z_+, z_-) r_-^{2(i+1)}, \tag{18}$$

where

$$b_{2i}(z_+, z_-) = -2a_{2(i+1)}(z_+, z_-), \quad i = 0, 1, 2, \ldots . \tag{19}$$

It has been already shown that the r^2 term in $D(\mathbf{r}_-, z_+, z_-)$ gives rise to the ideal spot dancing of received beam waves in which the arrival position is normally distributed but each amplitude keeps the same form [6, 7].

We consider here an effect of the r^2 term only in $D(\mathbf{r}_-, z_+, z_-)$ as follows:

$$D(\mathbf{r}_-, z_+, z_-) = b_0(z_+, z_-) r_-^2. \tag{20}$$

Substituting (20) into (7), we get the second moment under the ideal spot dancing:

$$\begin{aligned} M_{11}(\mathbf{r}_+, \mathbf{r}_-, z) = & \int \mathrm{d}\mathbf{r}' M_{11}^{\mathrm{in}}(\mathbf{r}_+ - \mathbf{r}', \mathbf{r}_-, z) \\ & \cdot \frac{1}{2\pi\sigma_0^2} \exp\left[-\frac{\sigma_2^2}{2}(kr_-)^2 \right. \\ & \left. -\frac{1}{2\sigma_0^2}\left(\mathbf{r}' - j\sigma_1^2 k\mathbf{r}_-\right)^2\right], \end{aligned} \tag{21}$$

where

$$\sigma_n^2 = \frac{1}{2}\int_0^z \mathrm{d}z_1 \int_0^{z-z_1} \mathrm{d}z_2 z_2^{2-n} b_0\left(z - z_2 - \frac{z_1}{2}, z_1\right), \quad n = 0, 1, 2, \tag{22}$$

which represents the whole effects of random dielectric constant on the second moment. Furthermore, (22) can be expressed approximately by the following equation (See Appendix A):

$$\sigma_n^2 \simeq \frac{1}{2}\int_0^z \mathrm{d}z_1 \int_0^{\infty} \mathrm{d}z_2 (z - z_1)^{2-n} b_0(z_1, z_2). \tag{23}$$

Finally, the substitution of (15) into (21) yields the second moment of Gaussian beam wave:

$$\begin{aligned} M_{11}(\mathbf{r}_+, \mathbf{r}_-, z) \;=\; & \int \mathrm{d}\mathbf{r}' \frac{2A}{\pi w^2} \exp\left\{ -\frac{2}{w^2}(\mathbf{r}_+ - \mathbf{r}')^2 \right. \\ & \left. + j\frac{2p}{w^2}\left[(\mathbf{r}_+ - \mathbf{r}')\cdot \mathbf{r}_-\right] - \frac{r_-^2}{2w^2} \right\} \\ & \cdot \frac{1}{2\pi\sigma_0^2} \exp\left(-\frac{r'^2}{2\sigma_0^2}\right) \exp\left(jk\frac{\sigma_1^2}{\sigma_0^2}\mathbf{r}_- \cdot \mathbf{r}'\right) \\ & \cdot \exp\left[k^2 r_-^2 \left(\frac{\sigma_1^4}{2\sigma_0^2} - \frac{\sigma_2^2}{2}\right)\right]. \end{aligned} \tag{24}$$

We deduce (23) by using two type of the correlation function which are the Gaussian and the Kolmogorov models.

2.3.1. Gaussian Model

In many practical situations, a random medium may be approximated by the Gaussian correlation function:

$$B(\mathbf{r}_-, z_+, z_-) = B(z_+) \exp\left[-\frac{r_-^2 + z_-^2}{l^2(z_+)}\right], \tag{25}$$

where $B(z_+)$ and $l(z_+)$ are the local intensity and the correlation length of the random medium, respectively. We then obtain

$$\sigma_n^2 = \frac{\sqrt{\pi}}{2} \int_0^z \mathrm{d}z_1 (z - z_1)^{2-n} \frac{B(z_1)}{l(z_1)}, \quad n = 0,\, 1,\, 2. \tag{26}$$

2.3.2. Kolmogorov Model

The Kolmogorov model is known to be a good approximation for atmospheric turbulence. Here we use the von Karman spectrum which is the modified model of the Kolmogorov spectrum. The von Karman spectrum is defined by the following equation [9]:

$$\begin{aligned} \Phi_n(\kappa, z_+) \;&=\; 0.033 C_n^2(z_+) \frac{\exp\left[-\kappa^2/\kappa_m^2(z_+)\right]}{\left[\kappa^2 + 1/L_0^2(z_+)\right]^{11/6}}, \; 0 \le \kappa < \infty \\ \kappa_m^2(z_+) \;&=\; 5.92/l_0(z_+) \end{aligned} \tag{27}$$

where $C_n^2(z_+)$ is the refractive index structure constant, $L_0(z_+)$ is the outer scale of turbulence and $l_0(z_+)$ is the inner scale of turbulence, whose scales are here assumed to be functions of the altitude.

Under the assumption of a statistically homogeneous and isotropic atmosphere, the spectrum is related to the correlation function of random refractive index $B_n(\mathbf{r}_-, z_+, z_-)$ by the following Fourier transform:

$$B_n(\mathbf{r}_-, z_+, z_-) = 4\pi \int_0^\infty \frac{\sin \kappa\sqrt{r_-^2 + z_-^2}}{\sqrt{r_-^2 + z_-^2}} \kappa\Phi_n(\kappa, z_+)\mathrm{d}\kappa. \tag{28}$$

Furthermore, when $B(\mathbf{r}_-, z_+, z_-) \simeq 4B_n(\mathbf{r}_-, z_+, z_-)$ is assumed, the correlation function of random dielectric constant is given by

$$B(\mathbf{r}_-, z_+, z_-) = 16\pi \int_0^\infty \frac{\sin \kappa\sqrt{r_-^2 + z_-^2}}{\sqrt{r_-^2 + z_-^2}} \kappa\Phi_n(\kappa, z_+)\mathrm{d}\kappa. \tag{29}$$

Using (27) and (29), we obtain

$$\begin{aligned} \sigma_n^2 \ &= \ 0.033\pi^2 \int_0^z \mathrm{d}z_1(z - z_1)^{2-n} C_n^2(z_1) \left(\frac{1}{L_0(z_1)}\right)^{1/3} \\ &\quad \cdot U\left(2; \frac{7}{6}; \frac{1}{L_0^2(z_1)\kappa_m^2(z_1)}\right), \end{aligned} \tag{30}$$

where $U(a, b, z)$ is the confluent hypergeometric function of the second kind [10].

Conducting the limit of $B(\mathbf{r}_-, z_+, z_-)$ as $\mathbf{r}_-$ and z_- approach zero in (29), we have $C_n^2(z_+)$ related with the local intensity of the random media $B(z_+) = B(\mathbf{0}, z_+, 0)$ as follows:

$$\begin{aligned} C_n^2(z_+) \ &= \ \left[4\pi^{3/2} \cdot 0.033 \left(\frac{1}{L_0(z_+)}\right)^{-2/3} \right. \\ &\quad \left. \cdot U\left(\frac{3}{2}; \frac{2}{3}; \frac{1}{L_0^2(z_+)\kappa_m^2(z_+)}\right)\right]^{-1} B(z_+). \end{aligned} \tag{31}$$

Therefore, (30) can be described in terms of the local intensity of the random media $B(z_1)$:

$$\sigma_n^2 \ = \ \frac{\sqrt{\pi}}{4} \int_0^z \mathrm{d}z_1(z - z_1)^{2-n} \frac{B(z_1)}{L_0(z_1)} \frac{U\left(2; \frac{7}{6}; \frac{1}{L_0^2(z_1)\kappa_m^2(z_1)}\right)}{U\left(\frac{3}{2}; \frac{2}{3}; \frac{1}{L_0^2(z_1)\kappa_m^2(z_1)}\right)},$$

$$n \ = \ 0, 1, 2. \tag{32}$$

2.4. Complex Degree of Coherence of Received Waves

We examine the loss of spatial coherence of received waves on the aperture of a receiving antenna by the complex degree of coherence (DOC) defined by the second moment [11]:

$$\begin{aligned}\mathrm{DOC}(r,\, z) \;&=\; \frac{M_{11}(\mathbf{0},\, \mathbf{r},\, z)}{[M_{11}(\mathbf{r}/2,\, \mathbf{0},\, z)M_{11}(-\mathbf{r}/2,\, \mathbf{0},\, z)]^{1/2}} \\ &=\; \exp\left(\left\{\frac{2}{w^2+4\sigma_0^2}\left[-\frac{\sigma_0^2}{w^2}(1+p^2)\right.\right.\right. \\ &\qquad \left.\left.\left. +k\sigma_1^2(p+k\sigma_1^2)\right] - \frac{k^2\sigma_2^2}{2}\right\} r^2\right), \end{aligned} \tag{33}$$

where r is the separation distance between received wave fields at two points on the aperture.

2.5. BER Derived from the Average Intensity

We define the BER derived from the average intensity received by an aperture antenna, whose derivation is the same as Reference [5].

We define the average intensity in free space on the receiving antenna with a Gaussian distribution of attenuation across the aperture as follows:

$$\begin{aligned} I_{\mathrm{in}}(z) \;&=\; \int_{S_{\mathrm{a}}} u_{\mathrm{in}}(\mathbf{r},\, z)g(\mathbf{r})\,[u_{\mathrm{in}}(\mathbf{r},\, z)g(\mathbf{r})]^*\,\mathrm{d}\mathbf{r} \\ &=\; \int_{S_{\mathrm{a}}} M_{11}^{\mathrm{in}}(\mathbf{r},\, \mathbf{0},\, z)\,[g(\mathbf{r})]^2\,\mathrm{d}\mathbf{r}, \end{aligned} \tag{34}$$

where

$$g(\mathbf{r}) = \exp\left(-\frac{r^2}{\sigma_{\mathrm{a}}^2}\right), \quad \sigma_{\mathrm{a}}^2 = 2a^2 \tag{35}$$

and S_{a} is the circular area with the aperture radius a. Similarly, the average intensity of the received wave through the inhomogeneous random medium is defined by

$$\langle I(z)\rangle = \int_{S_{\mathrm{a}}} M_{11}(\mathbf{r},\, \mathbf{0},\, z)\,[g(\mathbf{r})]^2\,\mathrm{d}\mathbf{r}. \tag{36}$$

We define the energy per bit E_{b} as the products of the intensity and the bit time T_{b}; then, E_{b} in free space is given by

$$E_{\mathrm{b}} = T_{\mathrm{b}} \cdot I_{\mathrm{in}}(z) = T_{\mathrm{b}} \cdot \int_{S_{\mathrm{a}}} M_{11}^{\mathrm{in}}(\mathbf{r},\, \mathbf{0},\, z)\,[g(\mathbf{r})]^2\,\mathrm{d}\mathbf{r}. \tag{37}$$

Similarly, E_b in the inhomogeneous random medium is defined by:

$$\langle E_\mathrm{b}\rangle = T_\mathrm{b}\cdot\langle I(z)\rangle = T_\mathrm{b}\cdot\int_{S_\mathrm{a}} M_{11}(\mathbf{r},\,\mathbf{0},\,z)\,[g(\mathbf{r})]^2\,\mathrm{d}\mathbf{r}. \tag{38}$$

We consider QPSK modulation which is very popular among satellite communication. It is known that BER in QPSK modulation is defined by:

$$PE = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_\mathrm{b}}{N_0}}\right), \tag{39}$$

where $\mathrm{erfc}(x)$ is the complementary error function. We define the BER derived from the average intensity on a receiving antenna in order to evaluate the influence of atmospheric turbulence as follows:

$$PE_\mathrm{I} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{\langle E_\mathrm{b}\rangle}{N_0}}\right). \tag{40}$$

And then, using E_b in free space obtained by (37), the BER derived from the average intensity is expressed by:

$$PE_\mathrm{I} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{S_\mathrm{I}\cdot\frac{E_\mathrm{b}}{N_0}}\right), \tag{41}$$

where

$$\begin{aligned}
S_\mathrm{I} &= \frac{\langle E_\mathrm{b}\rangle}{E_\mathrm{b}} = \frac{T_\mathrm{b}\cdot\langle I(z)\rangle}{T_\mathrm{b}\cdot I_\mathrm{in}(z)} = \frac{\langle I(z)\rangle}{I_\mathrm{in}(z)} = \frac{\displaystyle\int_{S_\mathrm{a}} M_{11}(\mathbf{r},\,\mathbf{0},\,z)\,[g(\mathbf{r})]^2\,\mathrm{d}\mathbf{r}}{\displaystyle\int_{S_\mathrm{a}} M_{11}^{\mathrm{in}}(\mathbf{r},\,\mathbf{0},\,z)\,[g(\mathbf{r})]^2\,\mathrm{d}\mathbf{r}} \\
&= \frac{1}{1+(2\sigma_0/w)^2}\left(\frac{1}{1+(2\sigma_0/w)^2}\frac{1}{w^2}+\frac{1}{\sigma_\mathrm{a}^2}\right)^{-1} \\
&\quad\cdot\left\{1-\exp\left[-2\left(\frac{1}{1+(2\sigma_0/w)^2}\frac{1}{w^2}+\frac{1}{\sigma_\mathrm{a}^2}\right)a^2\right]\right\} \\
&\quad\cdot\left(\frac{1}{w^2}+\frac{1}{\sigma_\mathrm{a}^2}\right)\left\{1-\exp\left[-2\left(\frac{1}{w^2}+\frac{1}{\sigma_\mathrm{a}^2}\right)a^2\right]\right\}^{-1}.
\end{aligned} \tag{42}$$

2.6. BER Derived from the Average Received Power

We define the BER derived from the average received power using the mutual coherence function of received wave fields on a receiving antenna.

Here we assume a parabolic antenna as a receiving antenna. When a point detector is placed at the focus of a parabolic concentrator, the instantaneous response in the receiving antenna is proportional to the electric field strength averaged over the area of the reflector. When the aperture size is large relative to the electromagnetic wavelength, the electric field strength averaged over the area of the reflector can be described [12] by

$$\overline{u_{\rm in}} = \frac{1}{S_{\rm a}} \int_{S_{\rm a}} u_{\rm in}(\mathbf{r},\, z) g(\mathbf{r}) {\rm d}\mathbf{r}, \tag{43}$$

where $S_{\rm a}$ is the circular area of a reflector with a radius a and $g(\mathbf{r})$ defined by (35) is the field distribution of attenuation across the reflector. Then, the power received by the antenna is given by

$$\begin{aligned} P_{\rm in}(z) &= S_{\rm a} \cdot \frac{{\rm Re}[\overline{u_{\rm in}} \cdot \overline{u_{\rm in}}^*]}{Z_0} \\ &= \frac{1}{S_{\rm a} Z_0} \cdot {\rm Re}\left[\int_{S_{\rm a}} \int_{S_{\rm a}} u_{\rm in}(\mathbf{r}_1,\, z) g(\mathbf{r}_1) \left[u_{\rm in}(\mathbf{r}_2,\, z) g(\mathbf{r}_2)\right]^* {\rm d}\mathbf{r}_1 {\rm d}\mathbf{r}_2\right] \\ &= \frac{1}{S_{\rm a} Z_0} \cdot {\rm Re}\left[\int_{S_{\rm a}} \int_{S_{\rm a}} M_{11}^{\rm in}(\mathbf{r}_+,\, \mathbf{r}_-,\, z) g(\mathbf{r}_1) g(\mathbf{r}_2) {\rm d}\mathbf{r}_1 {\rm d}\mathbf{r}_2\right], \end{aligned} \tag{44}$$

where ${\rm Re}[x]$ denotes the real part of x and Z_0 is the characteristic impedance. Similarly, the average received power in the inhomogeneous random medium is given by

$$\langle P(z) \rangle = \frac{1}{S_{\rm a} Z_0} \cdot {\rm Re}\left[\int_{S_{\rm a}} \int_{S_{\rm a}} M_{11}(\mathbf{r}_+,\, \mathbf{r}_-,\, z) g(\mathbf{r}_1) g(\mathbf{r}_2) {\rm d}\mathbf{r}_1 {\rm d}\mathbf{r}_2\right]. \tag{45}$$

We define $E_{\rm b}$ as the products of the average received power and the bit time $T_{\rm b}$; then, $E_{\rm b}$ in free space is given by

$$E_{\rm b} = T_{\rm b} \cdot P_{\rm in}(z) = \frac{T_{\rm b}}{S_{\rm a} Z_0} \cdot {\rm Re}\left[\int_{S_{\rm a}} \int_{S_{\rm a}} M_{11}^{\rm in}(\mathbf{r}_+, \mathbf{r}_-, z) g(\mathbf{r}_1) g(\mathbf{r}_2) {\rm d}\mathbf{r}_1 {\rm d}\mathbf{r}_2\right] \tag{46}$$

Similarly, $E_{\rm b}$ in the inhomogeneous random medium is defined by:

$$\begin{aligned} \langle E_{\rm b} \rangle &= T_{\rm b} \cdot \langle P(z) \rangle = \frac{T_{\rm b}}{S_{\rm a} Z_0} \\ &\quad \cdot {\rm Re}\left[\int_{S_{\rm a}} \int_{S_{\rm a}} M_{11}(\mathbf{r}_+, \mathbf{r}_-, z) g(\mathbf{r}_1) g(\mathbf{r}_2) {\rm d}\mathbf{r}_1 {\rm d}\mathbf{r}_2\right] \end{aligned} \tag{47}$$

From the above, the BER derived from the average received power is obtained as same as the BER shown in the previous section.

$$PE_{\mathrm{P}} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{\langle E_{\mathrm{b}}\rangle}{N_0}}\right) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{S_{\mathrm{P}}\cdot\frac{E_{\mathrm{b}}}{N_0}}\right), \tag{48}$$

where

$$\begin{aligned}
S_{\mathrm{P}} &= \frac{\langle E_{\mathrm{b}}\rangle}{E_{\mathrm{b}}} = \frac{T_{\mathrm{b}}\cdot\langle P(z)\rangle}{T_{\mathrm{b}}\cdot P_{\mathrm{in}}(z)} = \frac{\langle P(z)\rangle}{P_{\mathrm{in}}(z)} \\
&= \frac{\mathrm{Re}\left[\int_{S_{\mathrm{a}}}\int_{S_{\mathrm{a}}} M_{11}(\mathbf{r}_+,\,\mathbf{r}_-,\,z)g(\mathbf{r}_1)g(\mathbf{r}_2)\mathrm{d}\mathbf{r}_1\mathrm{d}\mathbf{r}_2\right]}{\mathrm{Re}\left[\int_{S_{\mathrm{a}}}\int_{S_{\mathrm{a}}} M_{11}^{\mathrm{in}}(\mathbf{r}_+,\,\mathbf{r}_-,\,z)g(\mathbf{r}_1)g(\mathbf{r}_2)\mathrm{d}\mathbf{r}_1\mathrm{d}\mathbf{r}_2\right]} \\
&= C_0\int_0^a \mathrm{d}r_1\int_0^a \mathrm{d}r_2 r_1 r_2 \exp\left[C_1(r_1^2+r_2^2)\right] \\
&\quad \cdot\cos\left[C_2(r_1^2-r_2^2)\right] I_0(C_3 r_1 r_2)
\end{aligned} \tag{49}$$

$$\begin{aligned}
C_0 &= \frac{4\left[(w^2+\sigma_{\mathrm{a}}^2)^2+p^2\sigma_{\mathrm{a}}^4\right]}{w^2\sigma_{\mathrm{a}}^2(4\sigma_{\mathrm{a}}^2+w^2)}\left\{1+\exp\left[-2\left(\frac{1}{w^2}+\frac{1}{\sigma_{\mathrm{a}}^2}\right)a^2\right]\right. \\
&\quad \left. -2\exp\left[-\left(\frac{1}{w^2}+\frac{1}{\sigma_{\mathrm{a}}^2}\right)a^2\right]\cos\left[\frac{p}{w^2}\left(\frac{1}{w^2}+\frac{1}{\sigma_{\mathrm{a}}^2}\right)a^2\right]\right\} \\
C_1 &= -\frac{1}{4\sigma_0^2+w^2}\left(1+\frac{2\sigma_0^2}{w_0^2}\right)+\frac{2k\sigma_1^2(k\sigma_1^2+p)}{4\sigma_0^2+w^2}-\frac{k^2\sigma_2^2}{2}-\frac{1}{\sigma_{\mathrm{a}}^2} \\
C_2 &= \frac{2k\sigma_1^2+p}{4\sigma_0^2+w^2} \\
C_3 &= \frac{1}{4\sigma_0^2+w^2}\frac{4\sigma_0^2}{w_0^2}-\frac{k\sigma_1^2(k\sigma_1^2+p)}{4\sigma_0^2+w^2}+k^2\sigma_2^2,
\end{aligned}$$

and $I_0(x)$ is the modified Bessel function of the first kind.

In case that the spatial coherence of received waves on a receiving antenna keeps constant; therefore, it is satisfied that $M_{11}(\mathbf{r}_+,\,\mathbf{r}_-,\,z)g(\mathbf{r}_1)g(\mathbf{r}_2) = M_{11}(\mathbf{r}_1,\,\mathbf{0},\,z)g(\mathbf{r}_1)g(\mathbf{r}_1)$, PE_{P} shown in (48) is equal to PE_{I} derived from the average intensity shown in (41).

3. RESULT

3.1. Model of Numerical Analysis

3.1.1. Atmospheric Turbulence

We assume a profile model of the local intensity of atmospheric turbulence as shown in Figure 1. In satellite communication in Ka-band, it is known that atmospheric turbulence mainly affects propagation characteristics of electromagnetic waves and the ionospheric turbulence can be neglected [5]. Therefore, we consider only atmospheric turbulence here. We assume the local intensity of atmospheric turbulence as a function of altitude h from the ground as follows:

$$\begin{aligned} B(h) &= B_\mathrm{a}\left[1-\left(\frac{h}{d_1}\right)^2\right], \quad 0 \le h \le h_1 \\ &= 0, \quad \text{elsewhere}, \end{aligned} \tag{50}$$

where B_a is the maximum value of $B(h)$, h_1 is the altitude of the atmosphere from the ground and d_1 is the decay length of the atmosphere.

When the elevation angle is θ, then (50) is given as a function of z for the uplink communication:

$$\begin{aligned} B(z) &= B_\mathrm{a}\left[1-\left(\frac{\sqrt{z^2+R^2+2zR\sin\theta}-R}{d_1}\right)^2\right], \\ &\qquad 0 \le z \le \sqrt{(h_1+R)^2-(R\cos\theta)^2}-R\sin\theta \\ &= 0, \quad \text{elsewhere}. \end{aligned} \tag{51}$$

Similarly, the local intensity for the downlink communication is given by

$$\begin{aligned} B(z) &= B_\mathrm{a}\left[1-\left(\frac{\sqrt{z^2+R^2+2zR\sin\theta}-R}{d_1}\right)^2\right], \\ &\qquad \sqrt{(R+L)^2-(R\cos\theta)^2}-\sqrt{(h_1+R)^2-(R\sin\theta)^2} \le z \\ &= 0, \quad \text{elsewhere}. \end{aligned} \tag{52}$$

We assume parameters of atmospheric turbulence as shown in Table 1. Here we assume that the correlation length, the outer and the inner scale of turbulence are constant for simplicity.

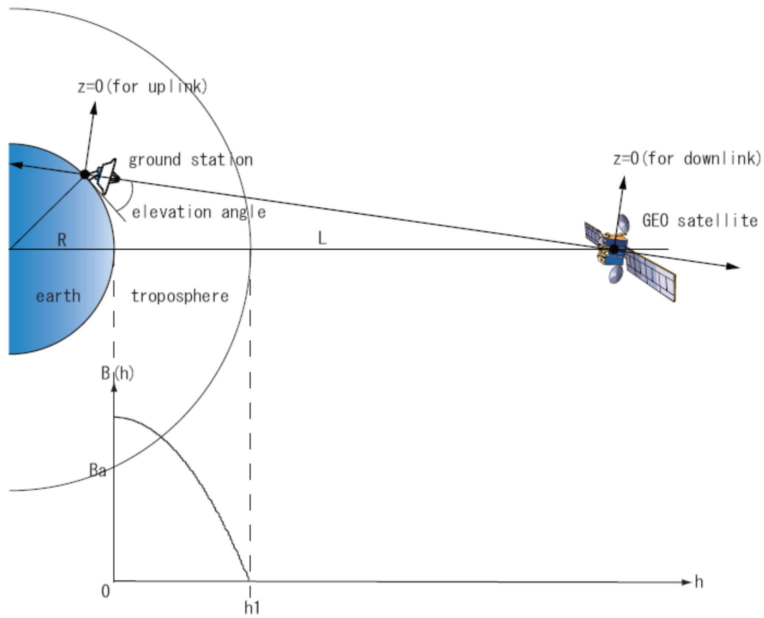

Figure 1. Model of atmospheric turbulence.

Table 1. Model of atmospheric turbulence.

ITEM	VALUE
Maximum value of $B(z)$: B_a	10^{-12} (strong turbulence) 10^{-14} (only use in analysis of DOC) 10^{-16} (only use in analysis of DOC)
Height of random media: h_1	20 [km]
Decay length of random media: d_1	20[km]
Correlation length in Gaussian model: $l(z)$	10[m] [constant]
Outer scale of turbulence in Kolmogorov model: $L_0(z)$	10 [m] [constant]
Inner scale of turbulence in Kolmogorov model: $l_0(z)$	0.001 [m] [constant]

Table 2. Model of a satellite and a ground station.

ITEM	VALUE
Frequency (Uplink/Downlink): f	30.0/20.0 [GHz]
Elevation angle: θ	5.0 [degree]
Minimum spot size of the transmitting antenna: w_0 ($z_0 = 0$)	1.2 [m]
Aperture radius of the receiving antenna: a	1.2 [m] 2.5 [m] (only use Fig. 10 & 11) 5.0 [m] (only use Fig. 12 & 13)
Earth radius: R	6,378 [km]
Height of GEO satellite: L	35,786 [km]

3.1.2. Satellite and Ground Station

We assume the GEO satellite communication in Ka-band in the present analysis. The frequencies for the uplink and the downlink communications, the elevation angle from the ground station to the satellite, and parameters about a transmitting and a receiving antenna are shown in Table 2.

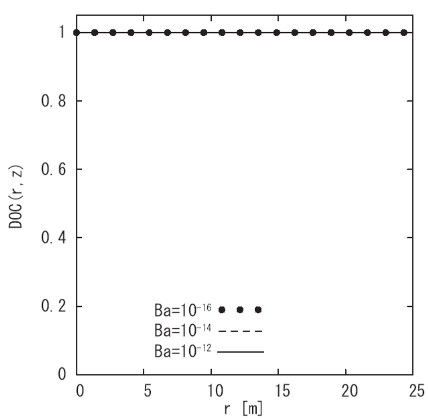

Figure 2. DOC as a function of the separation distance r for the uplink GEO satellite communication using the Gaussian model.

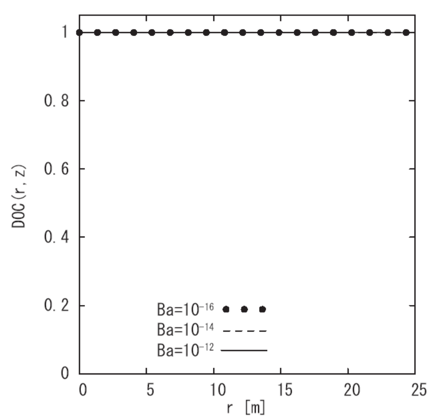

Figure 3. Same as Figure 2 except using the Kolmogorov model.

3.2. Numerical Analysis

3.2.1. Complex Degree of Coherence of Received Waves

Figures 2 and 3 show DOC given by (33) in the uplink communication through atmospheric turbulence which only exists near the transmitting antenna. These are analyzed by using the Gaussian and the Kolmogorov models, respectively. It is shown that DOC is nearly equal to 1; therefore, the spatial coherence radius is much larger than a radius of a receiving antenna of the GEO satellite.

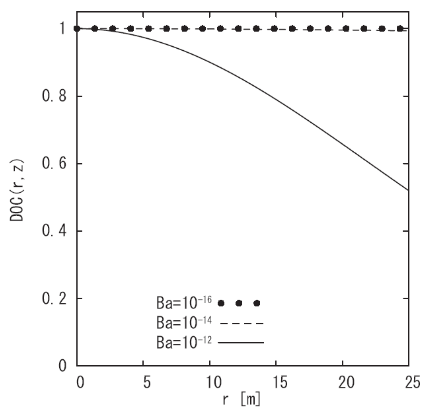

Figure 4. DOC as a function of the separation distance r for the downlink GEO satellite communication using the Gaussian model.

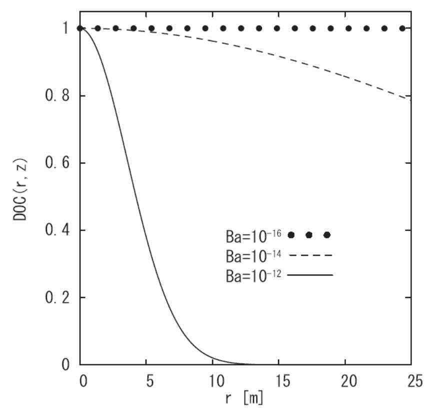

Figure 5. Same as Figure 4 except using the Kolmogorov model.

Figures 4 and 5 show DOC in the downlink communication through atmospheric turbulence which only exists near the receiving antenna. It is shown that DOC decreases; therefore, the spatial coherence radius is not much larger than a radius of a receiving antenna of the ground station. Moreover, it is shown that DOC using the Kolmogorov model decreases much more than DOC using the Gaussian model.

3.2.2. Bit Error Rate

Figures 6 and 7 show BER for the uplink communication through the strong atmospheric turbulence ($B_{\mathrm{a}} = 10^{-12}$) using the Gaussian and the Kolmogorov models, respectively. The dotted line shows the BER (PE_{I}) derived from the average intensity given by (41). The solid line shows the BER (PE_{P}) derived from the average received power given

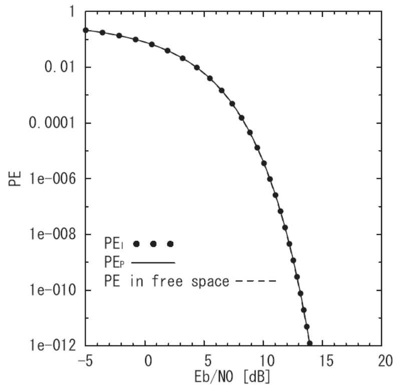

Figure 6. BER in strong atmospheric turbulence ($B_\mathrm{a} = 10^{-12}$) for the uplink GEO satellite communication using the Gaussian model.

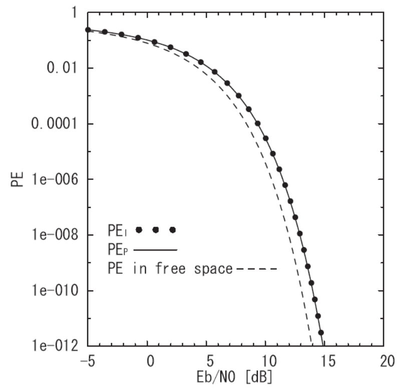

Figure 7. Same as Figure 6 except using the Kolmogorov model.

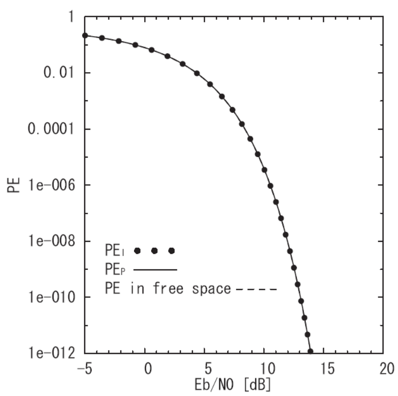

Figure 8. BER in strong atmospheric turbulence ($B_\mathrm{a} = 10^{-12}$) for the downlink GEO satellite communication using the Gaussian model.

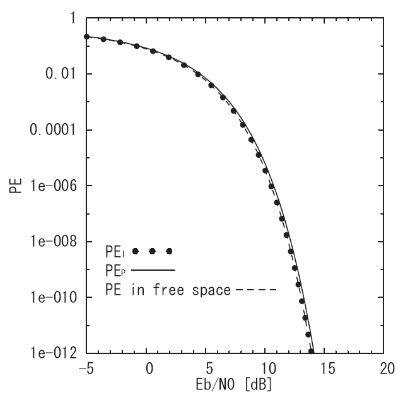

Figure 9. Same as Figure 8 except using the Kolmogorov model.

by (48). The broken line shows BER in free space as reference. It is assumed that $w_0 = a = 1.2$ [m]. For the Kolmogorov model, PE_I is increased as compared with BER in free space and PE_P is identical to PE_I. The result of $PE_P = PE_I$ was expected from Figure 3.

Figures 8 and 9 show BER for the downlink communication through the strong atmospheric turbulence. For the Kolmogorov

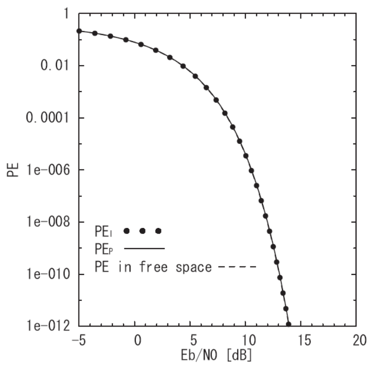

Figure 10. BER in strong atmospheric turbulence ($B_\mathrm{a} = 10^{-12}$) for the downlink GEO satellite communication in $a = 2.5$ [m] using the Gaussian model.

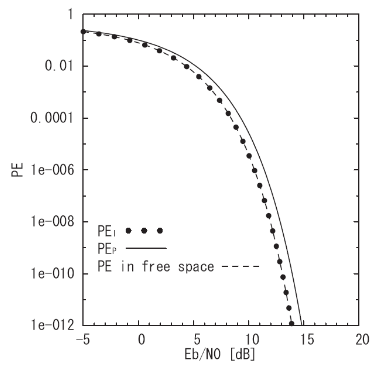

Figure 11. Same as Figure 10 except using the Kolmogorov model.

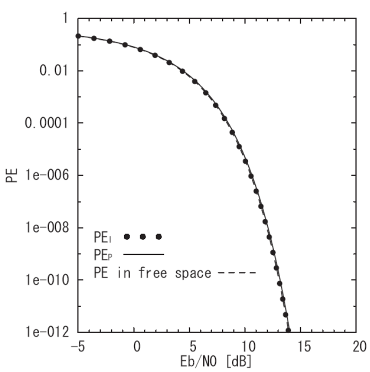

Figure 12. BER in strong atmospheric turbulence ($B_\mathrm{a} = 10^{-12}$) for the downlink GEO satellite communication in $a = 5.0$ [m] using the Gaussian model.

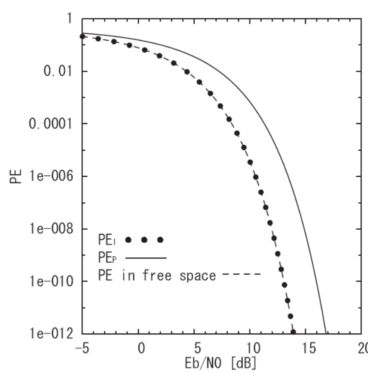

Figure 13. Same as Figure 12 except using the Kolmogorov model.

model, it is shown that PE_I is almost identical to BER in free space, but PE_P increases a little as compared with BER in free space.

Figures 10 to 13 show BER for the downlink communication when a radius of a receiving antenna is larger than the radius in Figures 8 and 9. Figures 10 and 11 show BER in $a = 2.5$ [m], and Figures 12 and 13 show BER in $a = 5.0$ [m].

It is shown for the Kolmogorov model that the larger a radius of a receiving antenna becomes, the bigger a difference between PE_{P} and PE_{I} becomes, because of the result shown in Figure 5.

On the other hand, for the Gaussian model, PE_{I} and PE_{P} are almost same as BER in free space for both the uplink and the downlink communication.

4. DISCUSSION

In the uplink communication using the Kolmogorov model, it is shown that PE_{I} derived from the average intensity increases in the strong turbulence compared with BER in free space by Figure 7. The increase in BER is caused by the decrease in the average received intensity due to spot dancing of received beam waves. On the other hand, the spatial coherence radius is much larger than a radius of a receiving antenna and then the spatial coherence of received waves keeps enough on a receiving antenna as shown in Figure 3. This indicates that there are little influences of the decrease in the spatial coherence on BER. For this reason, PE_{P} considering the spatial coherence of received waves is almost identical to PE_{I} derived from the average intensity in Figure 7. After all, we find that the decrease in the average received intensity due to spot dancing causes the increase in BER for the uplink communication.

In the downlink communication using the Kolmogorov model, PE_{I} derived from the average intensity increases little, as shown in Figures 9, 11 and 13. This indicates that the influence of spot dancing is very small. On the other hand, the spatial coherence of received waves decrease considerably in the strong turbulence and then the spatial coherence radius is not large enough relative to a radius of a receiving antenna, as shown in Figure 5. Because of the decrease in the spatial coherence of received waves, PE_{P} considering the spatial coherence of received waves is increased in Figures 9, 11 and 13. Furthermore, in case that a radius of a receiving antenna is larger, the spatial coherence radius becomes smaller relative to a radius of the antenna and then the spatial coherence of received waves decreases. Therefore, the larger a radius of a receiving antenna becomes, the more PE_{P} considering the spatial coherence of received waves increases. After all, we conclude that the decrease in the spatial coherence of received waves causes the increase in BER for the downlink communication.

5. CONCLUSION

In conclusion, we find the following influences of atmospheric turbulence on the GEO satellite communication in Ka-band on the assumption that the spatial coherence of received waves decreases and spot dancing only occurs.

(i) In the uplink communication, the decrease in the average intensity due to spot dancing causes the increase in BER, but the spatial coherence of received waves decreases little and there are little influences of this spatial coherence on BER.

(ii) In the downlink communication, the decrease in the spatial coherence of received waves by atmospheric turbulence causes the increase in BER, but spot dancing influences little on BER.

(iii) It is enough to estimate BER derived from the average intensity (PE_{I}) when the spatial coherence radius is much larger than a radius of a receiving antenna. But BER derived from the average power (PE_{P}) including with the influence of the spatial coherence of received waves has to be considered when the spatial coherence radius is not much larger than a radius of a receiving antenna.

Furthermore, we find that the decrease in DOC and the increase in BER becomes much more in the Kolmogorov model than in the Gaussian model; therefore, the effects of atmospheric turbulence is more sensitive in the Kolmogorov model than the Gaussian model.

In this paper, we do not consider effects of scintillation of received intensities. At the next stage, we will examine effects of both spot dancing and scintillation on satellite communication. We estimate BER in atmospheric turbulence by using the average intensity or the average received power. In order to make a more actual analysis, we have to consider the probability density function (PDF) about the bit error of satellite communication. The introduction of PDF is a future problem.

ACKNOWLEDGMENT

This research was partially supported by the Ministry of Education, Science, Sports and Culture, Grant-in-Aid for Scientific Research (C), 20560359, 2008.

APPENDIX A. APPROXIMATION OF EQUATION (22)

The approximation from (22) to (23) is shown as follows.

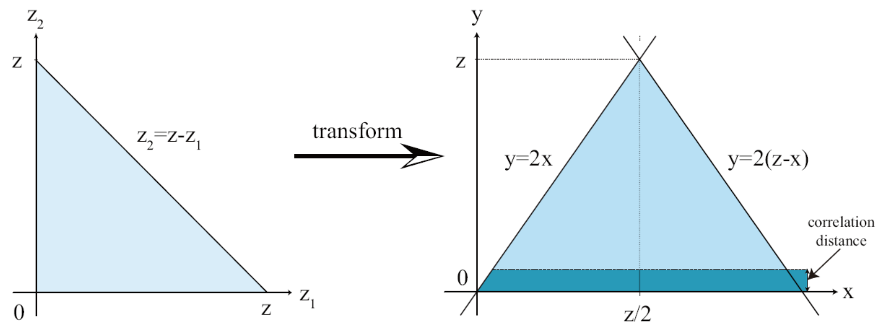

Figure A1. Range of integration for z_1 and z_2, and the corresponding range of integration for x and y.

We transform the integration of (22) with respect to z_1 and z_2 into the difference coordinate

$$x = z - z_2 - \frac{z_1}{2} \tag{A1}$$

$$y = z_1. \tag{A2}$$

The transformation changes the region of the integration as shown in Figure A1. Because the appreciable values of the function $b_0(x, y)$ exist only for y within the correlation distance, shown by the dark shaded region in Figure A1, it follows that the limits of integration on y can be extended from 0 to ∞ without significant error. In addition, z_2^{2-n} in the integrand can be approximated by

$$z_2^{2-n} = \left(z - x - \frac{y}{2}\right)^{2-n} \simeq (z-x)^{2-n}, \quad n = 0, 1, 2. \tag{A3}$$

From the above approximations, (22) can be represented by

$$\begin{aligned} \sigma_n^2 &= \frac{1}{2}\int_0^z \mathrm{d}z_1 \int_0^{z-z_1} \mathrm{d}z_2 z_2^{2-n} b_0\left(z - z_2 - \frac{z_1}{2}, z_1\right) \\ &\simeq \frac{1}{2}\int_0^z \mathrm{d}x \int_0^{\infty} \mathrm{d}y (z-x)^{2-n} b_0(x, y), \end{aligned} \tag{A4}$$

and (23) is obtained by replacing x, y by z_1, z_2 in (A4)

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