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ON INITIALIZATION OF ML DOA COST FUNCTION FOR UCA

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Abstract—Maximum likelihood (ML) direction-of-arrival (DOA) estimation is essentially an optimization of multivariable nonlinear cost function. Since the final estimate is highly dependent on the initial estimate, an initialization is critical in nonlinear optimization. Alternating Projection (AP) initialization has been proposed as computationally efficient method for the initialization of the ML DOA cost function. In this paper, we propose a multi-dimensional (M-D) search scheme of uniform exhaustive search and improved exhaustive search. Improved exhaustive search is used to reduce the computational load of uniform exhaustive search. In the improved exhaustive search algorithm, the two-step procedure is applied to reduce the computational load of the uniform exhaustive search initialization scheme. In numerical results, it is shown that the performance of the proposed scheme is better than that of AP initialization.

1. INTRODUCTION

Direction-of-arrival (DOA) estimation [1–6] is an important task for wireless communications and radar [7–30]. The maximum likelihood (ML) algorithm [31] gives a superior performance compared to other methods. However, the likelihood function is multi-variate and highly nonlinear, and therefore requires much computational load.

ML DOA estimation is essentially optimization of multivariable nonlinear cost function. The global convergence of the problem is highly dependent on the initial estimate. AP (Alternating Projection) has been proposed as a computationally efficient initialization scheme [32]. AP algorithm consists of successive 1-D search.

To improve the performance of initialization in the viewpoint of RMSE (Root-mean-square error) of an initial estimate, we propose exhaustive M-D (multi-dimensional) search.

The paper is organized as follows. In Section 2, the data model for DOA in uniform circular array (UCA) is formulated and maximum-likelihood estimation is briefly described. Next, we propose uniform exhaustive search method and improved exhaustive search method in Section 3. In Section 4, simulation results are given to compare the performance of three different algorithms. Our concluding remarks are given in Section 5.

2. PROBLEM FORMULATION

2.1. Array Signal Model

It is assumed that L narrowband sources from far fields in the directions of $\mathbf{\Theta} = [\theta_1, \ldots, \theta_L]$ are impinging on the array consisting of M antennas. The received signals can be modeled as

$$\mathbf{x}(t) = \mathbf{A}(\mathbf{\Theta})\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{A}(\mathbf{\Theta}) = [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_L)]$ is array manifold, $\mathbf{s}(t) = [s_1(t), \ldots, s_L(t)]^T$ is the L source signals at time t, and $\mathbf{n}(t)$ is the additive white Gaussian noise which is not correlated to the signals.

2.2. Uniform Circular Array

We assume a uniform circular array (UCA) with identical antennas and uniform spacing d as shown in Fig. 1 where θ is the azimuth angle, r is

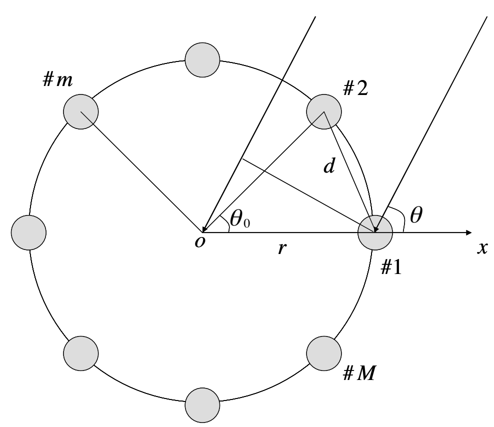


Figure 1. Geometry of the UCA with M elements.

the radius of the array, and $\theta_0 = 2\pi/M$ is the angle between adjacent elements. ϕ is elevation angle measured from the z-axis. The response of the mth sensor to the ith signal is given by

$$a_m(\theta_i) = e^{j\Psi_m(\theta_i)} \tag{2}$$

where ψ_m is given by

$$\psi_m = 2\pi \frac{r}{\lambda} \cos(\theta - (m-1)\theta_0) \tag{3}$$

and θ_i lies in $[0,\, 2\pi)$.

2.3. Maximum Likelihood Estimate

Assuming that the signals $\mathbf{s}(t)$ are deterministic and unknown sequence, the maximum likelihood estimate of the DOA vector $\mathbf{\Theta} = [\theta_1, \ldots, \theta_L]$ is given by [8]

$$\hat{\mathbf{\Theta}} = \arg\max_{\mathbf{\Theta}} tr \left\{ \mathbf{P}_{\mathbf{A}(\mathbf{\Theta})} \hat{\mathbf{R}} \right\} \tag{4}$$

where $\mathbf{P}_{\mathbf{A}(\mathbf{\Theta})} = \mathbf{A}(\mathbf{\Theta})(\mathbf{A}(\mathbf{\Theta})^H \mathbf{A}(\mathbf{\Theta}))^{-1} \mathbf{A}(\mathbf{\Theta})^H$ is the projection operator onto the space spanned by the columns of the matrix $\mathbf{A}(\mathbf{\Theta})$, $\hat{\mathbf{R}} = (1/N) \sum_{t=1}^{N} \mathbf{x}(t) \mathbf{x}^H(t)$ is the sample covariance matrix and N denotes the number of data snapshots.

3. INITIALIZATION ALGORITHM

Once initial estimates of L-DOA's are available, (4) is a nonlinear L-variable optimization problem. It can be solved via gradient based-Newton iteration.

3.1. Alternating Projection Initialization

AP (Alternating Projection) algorithm is one-dimensional (1-D) optimization. AP initialization started on solving (4) for the first DOA

$$\hat{\theta}_1 = \arg\max_{\theta_1} tr \left\{ \mathbf{P}_{\mathbf{a}(\theta_1)} \hat{\mathbf{R}} \right\}. \tag{5}$$

Next, it is solved for the second source with $\theta_1 = \hat{\theta}_1$:

$$\hat{\theta}_2 = \arg\max_{\theta_2} tr \left\{ \mathbf{P}_{\left[\mathbf{a}\left(\hat{\theta}_1\right),\, \mathbf{a}(\theta_2)\right]} \hat{\mathbf{R}} \right\} \tag{6}$$

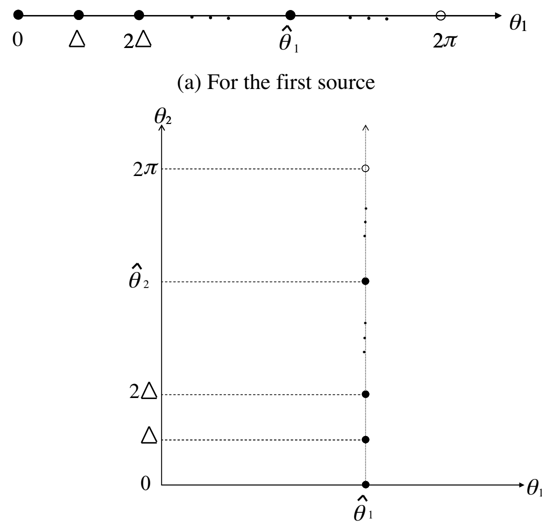


Figure 2. The search range and the search increment of the alternating projection for two signals.

The same procedure is repeated for all L sources :

$$\hat{\theta}_l = \arg\max_{\theta_l} tr\left\{\mathbf{P}_{\left[\mathbf{a}(\hat{\theta}_1),\mathbf{a}(\hat{\theta}_2),\ldots,\mathbf{a}(\hat{\theta}_{l-1}),\mathbf{a}(\theta_l)\right]}\hat{\mathbf{R}}\right\} \quad l = 1,\ldots,L \quad (7)$$

We get all the initial estimates for all L sources $\hat{\mathbf{\Theta}} = [\hat{\theta}_1,\ldots,\hat{\theta}_L]$. Fig. 2 shows the search increment and the search range of AP algorithm for two signals ($L = 2$). In this paper, the computational load K_{AP} denotes the number of cost function evaluations for AP initialization, which is given by

$$K_{AP} = L \cdot \left\lceil \frac{2\pi}{\Delta} \right\rceil. \quad (8)$$

where $\lceil\rceil$ rounds an argument towards plus infinity.

3.2. Uniform Exhaustive Search Initialization

Exhaustive search initialization is proposed to improve the AP-based initial estimates. AP algorithm is computationally efficient 1-D search algorithm, but the performance degrades since all the DOA's are not optimized simultaneously. To alleviate this problem, we propose a uniform exhaustive search method which performs L-Dimensional search:

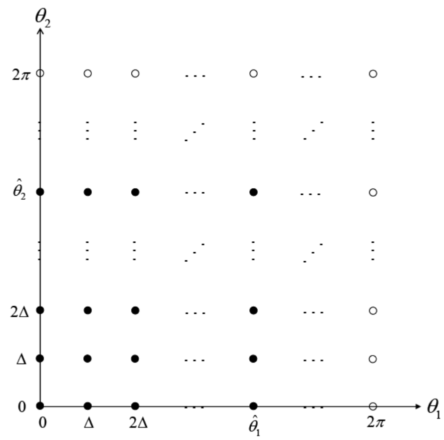


Figure 3. The search range and the search increment of the uniform exhaustive search for two signals.

$$\left[\hat{\theta}_1,\, \ldots,\, \hat{\theta}_L\right] = \arg\max_{\theta_1,\, \ldots,\, \theta_L} tr\left\{\mathbf{P}_{[\mathbf{a}(\theta_1),\, \ldots,\, \mathbf{a}(\theta_L)]}\hat{\mathbf{R}}\right\} \tag{9}$$

where the search range for each source is $[0,\, 2\pi)$, and the search increment is Δ. To be more specific, it is L-Dimensional search, whose increment between adjacent search angle is Δ. The computational load of the scheme rapidly increases. At each search point of L-D space, we evaluate cost function, and select the angle which maximizes the cost function. Fig. 3 shows the search increment and the search range of the uniform exhaustive search method for two signals ($L = 2$). The number of cost function evaluation, K, is given by

$$K_{UES} = \left\lceil \frac{2\pi}{\Delta} \right\rceil^L. \tag{10}$$

3.3. Improved Exhaustive Search Initialization

In this section, we explain the improved exhaustive search algorithm. We describe how to reduce the computational load of the uniform exhaustive search initialization scheme. The proposed scheme is based on two-step procedure of coarse search and fine search.

The coarse search can be formulated as follows:

$$\begin{aligned}\left[\hat{\theta}_1^{Coarse},\, \ldots,\, \hat{\theta}_L^{Coarse}\right] \;=\; &\arg\max_{\left[\theta_1^{Coarse},\, \ldots,\, \theta_L^{Coarse}\right]} \\ &\qquad tr\left\{\mathbf{P}_{\left[\mathbf{a}\left(\theta_1^{Coarse}\right),\, \ldots,\, \mathbf{a}\left(\theta_L^{Coarse}\right)\right]}\hat{\mathbf{R}}\right\}\end{aligned} \tag{11}$$

where the search range is $[0,\, 2\pi)$, and the search increment is $k\Delta$ which is k times search increment of the uniform exhaustive search, Δ. k is a constant that sets the coarse search increment. The second step, called the fine search, is a refinement procedure;

$$\left[\hat{\theta}_1^{Fine},\, \ldots,\, \hat{\theta}_L^{Fine}\right] = \arg \max_{\left[\theta_1^{Fine},\, \ldots,\, \theta_L^{Fine}\right]} tr \left\{ \mathbf{P}_{\left[\mathbf{a}\left(\theta_1^{Fine}\right),\, \ldots,\, \mathbf{a}\left(\theta_L^{Fine}\right)\right]} \hat{\mathbf{R}} \right\} \quad (12)$$

where the search range is around $[\hat{\theta}_1^{Coarse},\, \ldots,\, \hat{\theta}_L^{Coarse}]$, and the search increment is Δ. Fig. 4 shows the search increment and the search range of the improved exhaustive search method for two signals ($L = 2$). The number of cost function evaluation for the improved exhaustive search, K, is given by

$$K_{IES} = \left\lceil \frac{2\pi}{k\Delta} \right\rceil^L + (2k+1)^L \quad (13)$$

k is dependent on Δ, therefore, k is determined based on the following criterion:

$$k = \arg \min_k K_{IES} \quad (14)$$

In Fig. 5, we compare the computational load of the improved exhaustive search with that of the AP and the uniform exhaustive search with respect to Δ. The optimum k value is determined on the basis of (13) and (14), and is dependent on Δ. The improved exhaustive search is much more efficient than the uniform exhaustive search in terms of computational load. The computational load of the improved exhaustive search is approximately two times that of the AP algorithm.

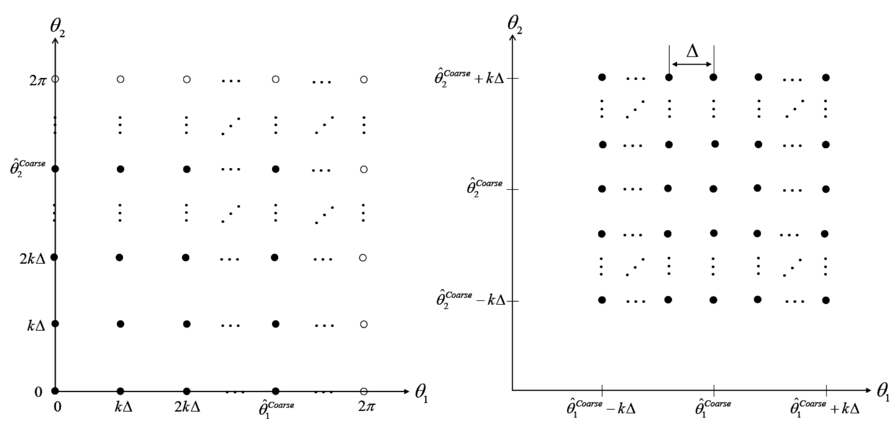


Figure 4. The search range and the search increment of the improved exhaustive search for two signals.

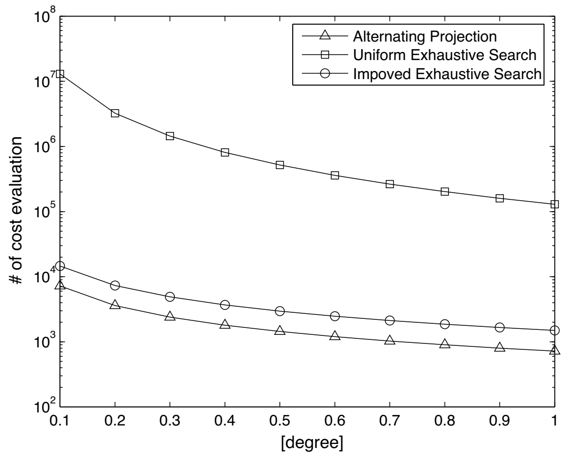


Figure 5. Comparison of the computational load with respect to Δ.

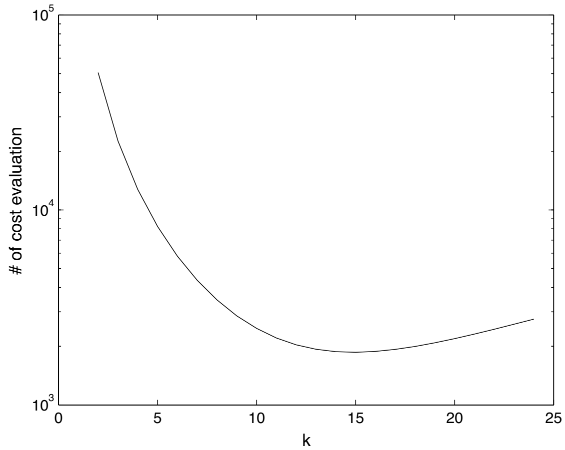
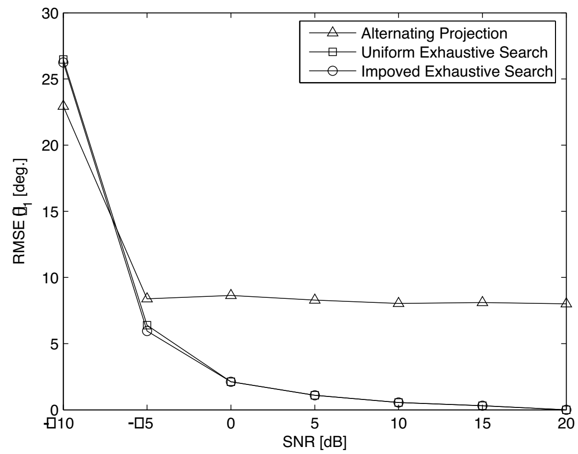


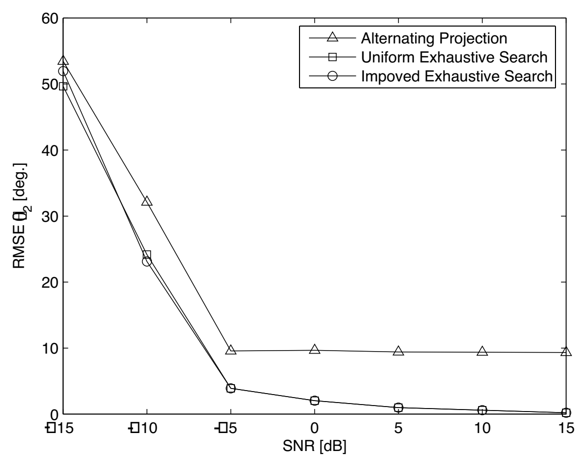
Figure 6. The computational load of improved exhaustive search method with respect to k.

4. SIMULATION RESULTS

In these simulations, three different initialization algorithms are compared in terms of computational load and estimation accuracy. Consider a uniform circular array composed of $M = 10$ isotropic sensors. The radius of the array is half a wavelength and the number of snapshots is 30. Two narrowband signals are incident from directions



(a) The first signal ($\theta_1 = 60^\circ$)



(b) The second signal ($\theta_2 = 120^\circ$)

Figure 7. Comparison of DOA estimates with respect to SNR (Improved exhaustive search with $k = 15$ and $\Delta = 0.8^\circ$).

of $\mathbf{\Theta} = [\theta_1, \theta_2] = [60^\circ, 120^\circ]$. It is assumed that two signals are fully correlated and that the power of the direct signal is greater than that of the reflected signal by 5 dB.

Figure 6 shows the computational load in improved exhaustive search with respect to k when the fine search increment, Δ, is 0.8 degree. From Fig. 6, we select $k = 15$ since the computational load increases at $k > 15$.

Figure 7 depicts the RMSE's for two signals obtained by AP, uniform exhaustive search and improved exhaustive search, versus the SNR. As shown in the figures, both exhaustive search methods show better performance than AP algorithm, and the performance of the improved exhaustive search and that of the uniform exhaustive search are similar in terms of the estimation accuracy. But, the computational burden of the uniform exhaustive search is much greater than that of the improved exhaustive search.

5. CONCLUSION

In this paper, we propose the improved exhaustive search to reduce the computational load for the initialization of the ML DOA algorithm and to improve the accuracy of the initial estimate of ML DOA algorithm. The computational load of the improved exhaustive search is much less than that of the uniform exhaustive search method. Moreover, the performance of the improved exhaustive search is better than AP algorithm and is similar to the performance of the uniform exhaustive search. There is a tradeoff between the accuracy of estimation and the computational load.

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