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TRANSMISSION LINE ANALYSIS OF APERTURE-COUPLED REFLECTARRAYS

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Abstract—A fast analysis of aperture-coupled reflectarrays is presented in this work in terms of transmission line model. The circuital approach is adopted to derive the phase design curve as a function of the current flowing on the equivalent impedance of the single radiating element. Computational costs are drastically reduced with respect to standard full-wave methods. Numerical and experimental validations are discussed on slot-coupled reflectarray configurations working at different operating frequencies.

1. INTRODUCTION

A fundamental task in microstrip reflectarrays design is the determination of an appropriate phase distribution on the radiating elements giving a field pattern with prescribed features. The optimum phase delay can be produced by properly choosing each array element through selection on a curve relating the phase of the reradiated field to one or more geometrical parameters of the individual radiator [1, 2]. The phase design curve is usually obtained as a function of the element resonant size or using stubs with different lengths [3, 4]. Others reflectarray configurations have been developed and applied to design efficient radiating structures for different applications, such as multilayer reflectarrays with variable size elements [5, 6], reflectarrays based on microstrip elements with variable angular rotations [7], compound-cross-elements [8] patches aperture-coupled to microstrip lines of different lengths [9–11], or reflectarray elements integrated with active components [12]. For any reflectarray configuration, the calculation of the phase design curve is a time consuming procedure requiring to solve the scattering problem [13–15] inherent to the reflectarray unit cell for each dimension of the phase tuning parameter.

The analysis is usually based on full-wave [16] methods, such the Moment Method [17–19], by considering either an isolated element [20] or an infinite array environment [4]. However, the full-wave approach is characterized by relatively high computational costs, depending on the complexity of the radiating structure.

A simplified approach in terms of transmission line model is addressed in this work to the analysis of an isolated reflectarray element aperture-coupled [21] to a microstrip line of variable length. The choice of this configuration is fully justified by its versatile features, primarily related to the wideband behavior [11, 22]. The basic idea is to adopt the well-known transmission line model [23] for the analysis of patch antenna and to develop an equivalent circuit modelling the radiating element. This is considered to derive the field scattered by the plane-wave illuminated patch, with the specular reflection from the ground plane taken into account by the physical optics theory [20]. The proposed approach can be considered valid enough when assuming a negligible mutual coupling between patches. A strong advantage is to give a fast evaluation procedure for the phase design curves, thus offering an efficient tool to perform a parametric analysis with respect to geometrical and electrical features of the reflectarray unit cell. Numerical validations are presented on the phase curves computed for different slot-coupled reflectarrays operating at 10 GHz and 20 GHz, respectively. Results are compared with Moment Method analysis and measurements.

2. FORMULATION

The analysis is addressed to the structure in Fig. 1(a), where a rectangular patch slot-coupled to a phase tuning microstrip line is used as single reflectarray element. The phase delay introduced by the element is directly related to the length of the microstrip line, which is ideally composed by two sections with respect to the aperture center. The first section of the line is characterized by a variable length L_m and represents the effective phase tuning parameter, while the second section of fixed size L_S is used to realize the matching between the line itself and the slot-coupled patch. This multi-layer structure significantly reduces interference effects on the scattered field due to phase tuning elements usually located on the patches side. As a matter of fact, the prescribed field is obtained without changing the geometry of the reflecting surface. The aperture-coupled reflectarray configuration offers an appealing solution able to provide good performances in terms of bandwidth, which can be optimized by accurately choosing the dielectric constant and the thickness of the two different substrates or by properly acting on the inter-elements spacings, as demonstrated in [11, 22].

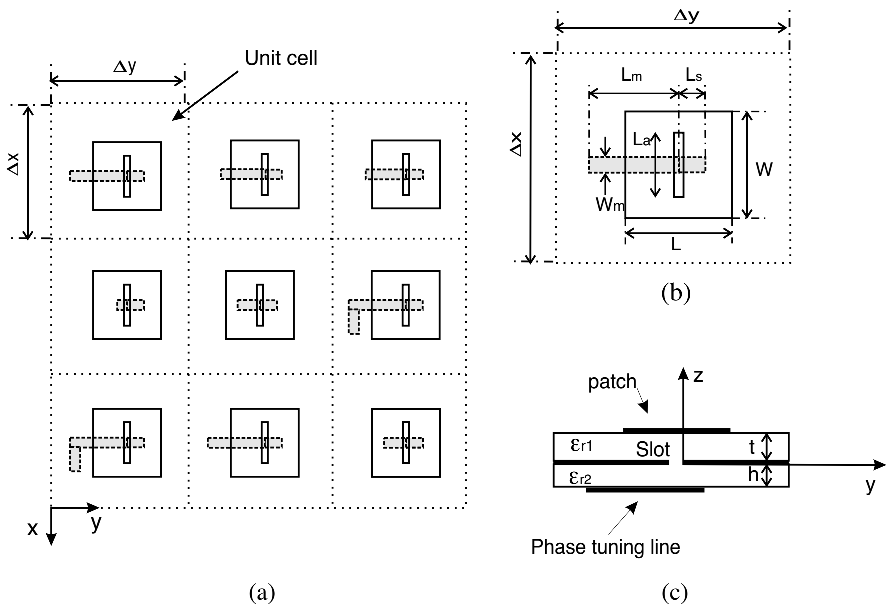

Figure 1. Aperture-coupled reflectarray: (a) array grid and (b) geometry layout of the unit cell ((b) top view and (c) side view).

The single slot-coupled reflectarray element of Fig. 1(b) is analyzed in this work by a transmission line approach [23–25], in order to have a fast and efficient procedure for computing the phase design curve.

Let us define as elementary unit cell an isolated aperture-coupled element printed on a finite grounded dielectric slab having dimensions according to the element grid spacing (Fig. 1). The total field scattered from the unit cell can be written as:

$$\underline{E}^S_{tot} = \underline{E}^S_{patch} + \underline{E}^S_{grp} \tag{1}$$

where $\underline{E}^S_{patch}$ and $\underline{E}^S_{grp}$ give the fields scattered by the isolated microstrip patch and the finite ground plane, respectively. These two contributions can be evaluated separately. In this work, the term $\underline{E}^S_{patch}$ is derived from a proper transmission line model of the single radiator, while the term $\underline{E}^S_{grp}$ is computed with the Physical Optics theory [20].

A fundamental TEM mode propagation is assumed for the patch, as given by the upper resonating transmission line of length L in Fig. 2. This is terminated at both ends by a complex admittance Y_S modelling the radiation and fringing effect of the radiating edges of length W. The slot in the ground plane is represented by an admittance Y_a including the stored energy near the aperture. It is evaluated as the parallel admittance of two sections of a shorted slotline (Fig. 2(b)) with characteristic impedance Z_{oa} and propagation constant k_{oa}, which can be computed by using the Cohn's method [26] or other more recent approximated techniques [27].

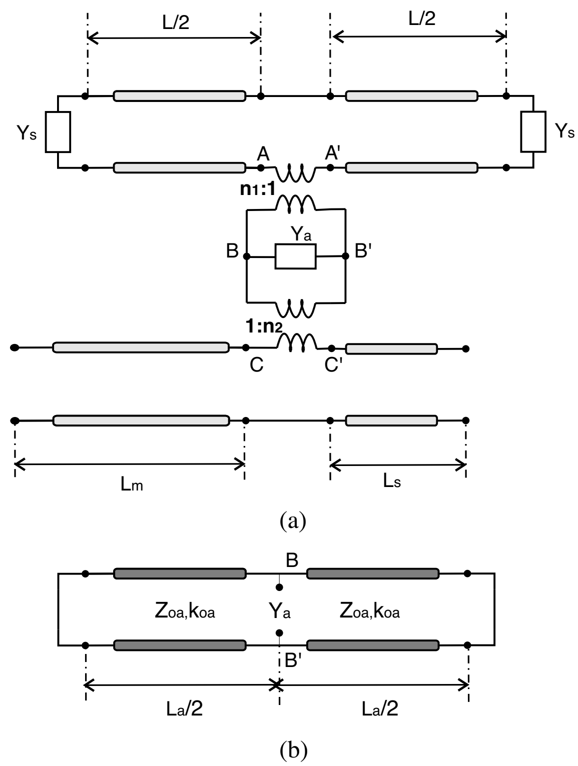

Figure 2. Transmission line model for (a) the aperture-coupled patch and (b) the slot.

Two transformers are used for modelling the coupling of the aperture to the patch and the microstrip line, respectively. The procedure to evaluate the turn ratios n_1 and n_2, based on the reciprocity theorem [28] and the spectral-domain immittance approach [29], is detailed in [25]. It requires a proper approximation of the induced electric field across the slot and the knowledge of the magnetic-field eigenvectors on both sides of the ground plane. As reported in [24], the transformation ratios can be computed by assuming the following expression of the normalized electric field $\underline{e}_a$ on the aperture:

$$\underline{e}_a = \widehat{y} \cdot \frac{1}{\pi\sqrt{\left(\frac{W_a}{2}\right)^2 - y^2}} \cos\left(\frac{\pi}{L_a} x\right) \tag{2}$$

where W_a and L_a give the width and the length of the aperture (Fig. 1), respectively.

The magnetic-field components on the ground plane are derived from the Fourier transforms of the surface current densities on the feed and the patch conductor [24], respectively. These current densities are approximated by the following closed form expression [30], accurate enough for both narrow and wide microstrip lines:

$$J_y^i(x) = A^i \left\{1 + B^i \left[M^i(x) - 1\right]\right\} \tag{3}$$

where:

$$A^i = \frac{1}{\sqrt{Z_o^i}} \cdot \frac{1}{W^i \left[1 + B^i \left(\frac{\pi}{2} - 1\right)\right]} \tag{4}$$

$$B^i = 10 \cdot \frac{1 - \frac{2x_c}{W^i}}{M^i(x_c) - 1} \tag{5}$$

$$M^i(x) = \frac{\frac{W^i}{2}}{\sqrt{\left(\frac{W^i}{2}\right)^2 - x^2}} \tag{6}$$

The apex i into Equations (3)–(6) can assume two distinct values, namely $i = 0$ and $i = 1$. The case $i = 0$ is associated to the values $Z_o^i = Z_{of}$ and $W^i = W_m$, where Z_{of} and W_m represent the characteristic impedance and the width of the feeding line, respectively. Analogously, the case $i = 1$ is associated to the values $Z_o^i = Z_{op}$ and $W^i = W$, where Z_{op} and W give the characteristic impedance and the width of the patch. The dependence of the term $\frac{2x_c}{W^i}$ into Equation (5) versus the ratio of the length W^i with respect to the relative height substrate is reported in [30].

Once computed the circuital parameters of the transmission line model in Fig. 2, the field scattered by the patch is derived from the current flowing through the equivalent Thevenin circuit of Fig. 3. This models the aperture-coupled element illuminated by an incident plane wave which is represented by a voltage source $\widehat{V}_{eq}$.

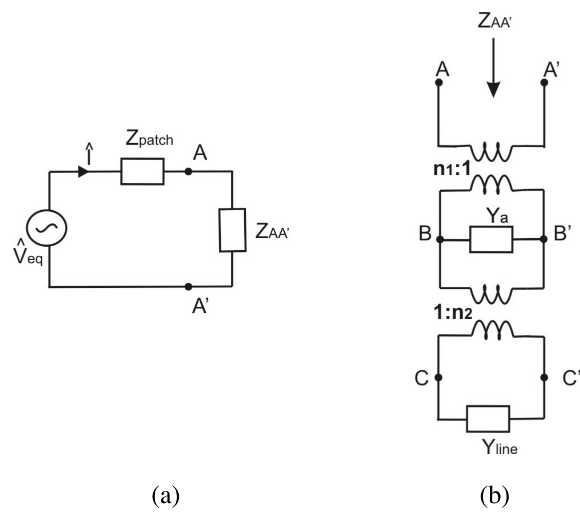

Figure 3. (a) Thevenin equivalent network of the aperture-coupled reflectarray and (b) circuit for the computation of impedance $Z_{AA'}$.

The impedances Z_{patch} and $Z_{AA'}$ are computed as the impedances seen from the upper and the lower side of the circuit at the section AA' (Fig. 2(a)), respectively. The term Z_{patch} gives the input impedance of the patch, while $Z_{AA'}$ is the load impedance taking into account the effect of the slot and the phasing line. It can be computed from the circuit of Fig. 3(b) by the expression:

$$Z_{AA'} = \frac{n_1^2}{n_2^2 Y_{line} + Y_a} \tag{7}$$

3. NUMERICAL AND EXPERIMENTAL VALIDATIONS

The circuital analysis approach is numerically validated by computing the current on the impedance Z_{patch} for some test cases. All the simulated structures have the same substrate stratification, which is characterized by two layers of Diclad 870 dielectric with $\epsilon_r = 2.33$ and thickness $t = h = 0.762\,\text{mm}$. A unit cell of dimensions $\Delta x = \Delta y = 0.6\lambda$ is assumed for all test cases, λ being the wavelength at the resonating frequency. The first reported example is related to a 10 GHz patch of dimensions $L = 8.4\,\mathrm{mm}$ and $W = 9.5\,\mathrm{mm}$, aperture-coupled through a rectangular slot with $La = 4.6\,\mathrm{mm}$ and $Wa = 0.5\,\mathrm{mm}$ to a microstrip line of variable length $0.6\,\mathrm{mm} \leq L_m \leq 11\,\mathrm{mm}$ and width $W_m = 1.16\,\mathrm{mm}$. The phase curve obtained with the proposed transmission line approach is reported under Fig. 4 and successfully compared with the result coming from the full-wave Moment Method analysis of the isolated element.

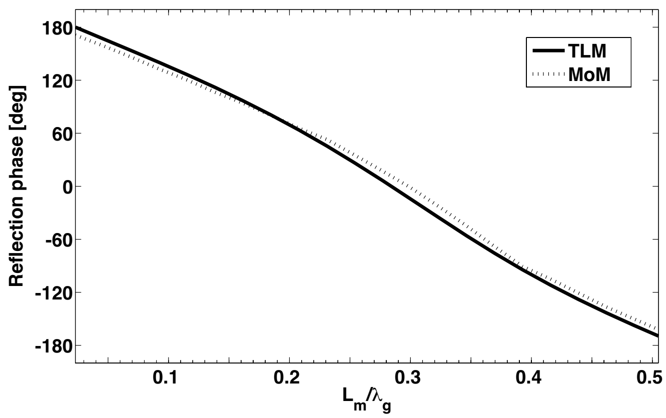

Figure 4. Phase design curve of a 10 GHz reflectarray with asymmetrical delay lines: comparison between transmission line model (TLM) and Moment Method (MoM).

The second discussed example is related to a 20 GHz aperture-coupled patch of dimensions $L = 3.57\,\mathrm{mm}$ and $W = 4.9\,\mathrm{mm}$. The slot has length $L_a = 3.2\,\mathrm{mm}$ and width $W_a = 0.4\,\mathrm{mm}$, while the tuning line is characterized by width $W_m = 0.65\,\mathrm{mm}$ and length L_m variable in the range $[0.25 \div 5.5]\,\mathrm{mm}$. Again, the reflection phase computed with the proposed approach results to be in good agreement with the curve evaluated by the Moment Method, as illustrated in Fig. 5.

As a further validation, the proposed approach is applied to the analysis of a reflectarray element aperture-coupled with a phasing line composed by two sections of variable length L_m. As illustrated in Fig. 6, one section is open ended while the other has a shorted termination. In this case, the phase design curve is obtained by symmetrically varying the length of the coupled microstrip line. Fig. 7 illustrates the phase curve computed with the described approach for a square 10 GHz patch of side length $L = 8.2\,\mathrm{mm}$. It is coupled through a rectangular slot of length $L_a = 5\,\mathrm{mm}$ to a microstrip line of variable length $0.6\,\mathrm{mm} \leq L_m \leq 8\,\mathrm{mm}$ and width $W_m = 1.16\,\mathrm{mm}$. The

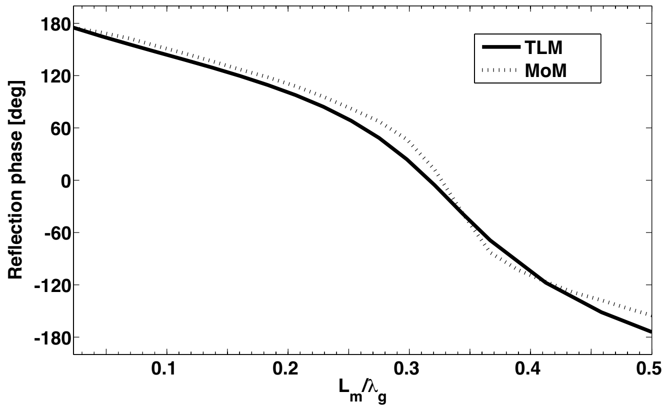

Figure 5. Phase design curve of a 20 GHz reflectarray with asymmetrical delay lines: comparison between transmission line model (TLM) and Moment Method (MoM).

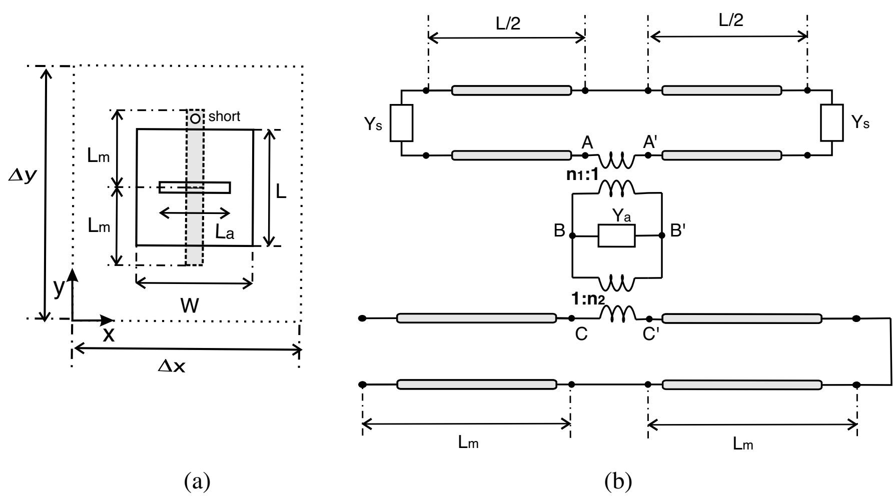

Figure 6. Reflectarray element aperture-coupled with symmetrical delay line of variable length $2L_m$: (a) geometry layout and (b) equivalent transmission line network.

computed phase design curve is successfully compared in Fig. 7 with experimental data obtained from measurements which are performed as described in [31] into the anechoic chamber of Microwave Laboratory at University of Calabria.

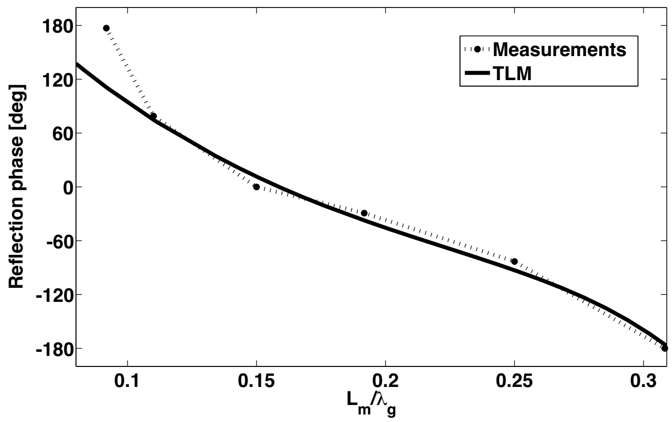

Figure 7. Phase design curve for a 10 GHz reflectarray with symmetrical delay lines: comparison between transmission line model (TLM) and measurements.

4. CONCLUSIONS

A circuital approach is adopted in this work to fast compute the phase design curve of aperture-coupled reflectarrays. A transmission line model is formulated for deriving the field scattered by the single unit cell, with the Physical Optics approximation adopted to separately compute the field contribution from the finite grounded plane. When compared to standard full-wave methods usually adopted in literature, the proposed approach significantly reduces the computation time, so providing an accurate procedure to perform efficient parametric analysis on the reflectarray unit cell. Numerical results are discussed on aperture-coupled reflectarray configurations working at different operating frequencies. Validations are presented with both the Moment Method results and experimental data.

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