# DESIGN METHOD OF THE RING-FOCUS ANTENNA WITH A VARIABLE FOCAL DISTANCE FOR FORMING AN ELLIPTIC BEAM 

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#### Abstract

In this paper, equations are derived for solving the important geometrical parameters of the ring-focus antenna with a variable focal distance for forming an elliptic beam, a simple and efficient method for this antenna is presented, and measured and calculated patterns are given. This antenna can form a high-efficiency elliptic beam.


## 1. INTRODUCTION

Antennas for forming elliptic beam are especially suitable for some special radio system where the space for loading antenna is limited and the radiating aperture is commonly rectangular. It is obvious that the elliptic aperture antenna with the long and short axis of the ellipse as the long and short side length is an ideal choice.

Previously, antennas for forming the elliptic beam were of form of cut parabolic reflector with low efficiency. The paper presents a configuration with a variable focal distance which can convert the cone beam due to the feed into an elliptic beam due to the main reflector with a constant illumination angle from the feed to the sub-reflector edge and also a constant illumination taper at the main reflector edge. As a result, high efficiency can be obtained [1-5].

## 2. THE DESIGN COURSE OF THE ANTENNA

### 2.1. The Method of Solving the Antenna

The reference frame for solving the ring-focus antenna with a variable focal distance for forming an elliptic beam is shown in Fig. 1, in which

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Figure 1. Reference frame sketch map.
$x, y$ and $z$ form a right-angle reference frame, $r, \theta$ and $\varphi$ form a sphere reference frame, $O$ is the origin of the two reference frames, the plane $x O y$ is the reference plane of equivalent ray path condition, the phase center of the feed is at $O, 1$ is the main reflector and 2 is the subreflector [6-14].

In order to obtain the main reflector with the desired elliptic aperture, firstly the parameters of the main and sub reflectors of $\varphi=0^{\circ}$ and $\varphi=90^{\circ}$ plane need be determined, when the curve of sub-reflector when $\varphi=0^{\circ}$ is corresponding to short axis of the main reflector aperture, and the one when $\varphi=90^{\circ}$ is corresponding to long one. After determining the curve parameters of the sub-reflector in the plane when $\varphi=0^{\circ}$ and $\varphi=90^{\circ}$, the whole sub-reflector can be obtained supposing the sub-reflector curve parameters in the plane when $\varphi=$ degree between $0^{\circ}$ and $90^{\circ}$ are gradual changed according to some function. Finally according to the law of reflection and equivalent ray path condition, the equation of whole main reflector is obtained.

### 2.2. Determining the Sub-Reflector

Firstly, the parameters of the $\varphi=90^{\circ}$ plane are determined, such as the diameter of the main reflector $D$, the diameter of the sub-reflector $D s$, the ratio between focal distance and diameter $\tau$, the distance between the vertex of sub-reflector curve and the phase center of the feed. The else parameters, such as the open angle from the phase center of the feeder to focal point $\beta$, the focal distance $2 c$ and long axis $2 a$ of the ellipse in the sub-reflector can be educed by geometry relation.

The equivalent ray path condition in the plane when $\varphi=90^{\circ}$ is:

$$
\begin{equation*}
C_{k}=2\left(\frac{c_{2}}{e_{2}}+f_{2}\right)-2 c_{2} \cos \beta_{2} \tag{1}
\end{equation*}
$$

where $f_{2}, \beta_{2}, 2 c_{2}, e_{2}$ are respectively focal distance, open angle, focal


Figure 2. Solving the parameter of the $\varphi=0^{\circ}$ plane.
distance of ellipse in sub-reflector (in the paper, 1 denotes the parameters in the plane when $\varphi=0^{\circ}, 2$ denotes the parameters in the plane when $\varphi=90^{\circ}$ ).

From the geometry relation in the figure, we can obtain:

$$
\begin{equation*}
\tan \frac{\Psi_{01}}{2}=\frac{D_{1}}{D_{2}} \tan \frac{\Psi_{02}}{2} \tag{2}
\end{equation*}
$$

Then the open angle $\Psi_{01}$ of main reflector in the plane when $\varphi=0^{\circ}$ can be solved, and the else parameters can be solved.

$$
\begin{align*}
r_{01} & =\frac{a_{0}}{\sin \theta_{m}} \frac{2 \tan \frac{\theta_{m}}{2}}{1-\tan \frac{\Psi_{01}}{2} \tan \frac{\theta_{m}}{2}}  \tag{3}\\
s_{1} & =r_{01} \sin \theta_{m} \tag{4}
\end{align*}
$$

The following question is how to determine the parameters in the any $\varphi$ plane (as Figure 3). The main thought is that the parameters in the sub-reflector are translated gradually from $\varphi=0^{\circ}$ to $\varphi=90^{\circ}$. Let sub-reflector radius s be the function with $\varphi$, then $s(\varphi)$ in any plane can be denoted as:

$$
\begin{equation*}
s(\varphi)=f\left(\varphi, s_{1}, s_{2}\right) \tag{5}
\end{equation*}
$$

In (5), s( $\varphi$ ) is the function with $\varphi$, which is from $s_{1}$ the $\varphi=0^{\circ}$ plane to $s_{2}$ in the $\varphi=90^{\circ}$ plane. There are some choices to the translation function $s(\varphi)$, but the criterion is that the forming main


Figure 3. Solving the parameters in any $\varphi$ plane.
reflector aperture is similar to the wanted ellipse aperture, and main reflector has less odd part. The translation function should meet the following four conditions.

$$
\left\{\begin{array}{l}
s(0)=s_{1}  \tag{6}\\
s(\pi / 2)=s_{2} \\
s^{\prime}(0)=0 \\
s^{\prime}(\pi / 2)=0
\end{array}\right.
$$

Then,

$$
\begin{equation*}
r_{0}(\varphi)=s(\varphi) / \sin \theta_{m} \tag{7}
\end{equation*}
$$

and

$$
\begin{equation*}
\Psi_{0}(\varphi)=2 \arctan \frac{r_{0}(\varphi)\left(1+\cos \theta_{m}\right)-2 a_{0}}{r_{0}(\varphi) \sin \theta_{m}} \tag{8}
\end{equation*}
$$

Also

$$
\begin{align*}
\beta(\varphi) & =\arctan \frac{s(\varphi) \tan \Psi_{0}(\varphi)}{a_{0} \tan \Psi_{0}(\varphi)-s(\varphi)}  \tag{9}\\
c(\varphi) & =\frac{s(\varphi)}{2 \sin \beta(\varphi)}  \tag{10}\\
e(\varphi) & =\frac{2 c(\varphi)}{2 a(\varphi)}=\frac{2 c(\varphi) \sin \Psi_{0}(\varphi)}{a_{0} \sin \Psi_{0}(\varphi)+s(\varphi)} \tag{11}
\end{align*}
$$

Now, the parameters of the curve in the $\varphi$ plane are determined. The coordinate of any point in the ( plane can be derived as following.

$$
\begin{equation*}
r(\theta)=a_{0} \frac{1-e(\varphi) \cos \beta(\varphi)}{1-e(\varphi) \cos (\beta(\varphi)-\theta)} \tag{12}
\end{equation*}
$$

Now, any point $r(\theta)$ in the sub-reflector is decided by $\theta$ and $\varphi$, and the shape of the sub-reflector is known entirely. The rectangular coordinate format of (13) is

$$
\left\{\begin{array}{l}
x_{s}=r(\theta) \sin \theta \cos \varphi  \tag{13}\\
y_{s}=r(\theta) \sin \theta \sin \varphi \\
z_{s}=r(\theta) \cos \theta
\end{array}\right.
$$

### 2.3. Determining the Main Reflector

After determining the equation of sub-reflector, the law vector of any point in the sub-reflector can be solved, and the corresponding point in the main reflector can be obtained according to the law of reflection and the equivalent ray path condition.

The derivative of radius vector with $\theta$ and $\varphi$ is:

$$
\begin{align*}
& \boldsymbol{r}_{\theta}=\frac{\partial \boldsymbol{r}(\theta)}{\partial \theta}=\frac{\partial x_{s}}{\partial \theta} \hat{\boldsymbol{x}}+\frac{\partial y_{s}}{\partial \theta} \hat{\boldsymbol{y}}+\frac{\partial z_{s}}{\partial \theta} \hat{\boldsymbol{z}}  \tag{14}\\
& \boldsymbol{r}_{\varphi}=\frac{\partial \boldsymbol{r}(\theta)}{\partial \varphi}=\frac{\partial x_{s}}{\partial \varphi} \hat{\boldsymbol{x}}+\frac{\partial y_{s}}{\partial \varphi} \hat{\boldsymbol{y}}+\frac{\partial z_{s}}{\partial \varphi} \hat{\boldsymbol{z}} \tag{15}
\end{align*}
$$

The normal vector in any point in the sub-reflector is:

$$
\begin{equation*}
\hat{\boldsymbol{n}}_{s}= \pm \frac{\boldsymbol{r}_{\theta} \times \boldsymbol{r}_{\varphi}}{\left|\boldsymbol{r}_{\theta} \times \boldsymbol{r}_{\varphi}\right|} \tag{16}
\end{equation*}
$$

From the law of reflection, the unit vector from the points in the subreflector to the corresponding ones in the main reflector is:

$$
\begin{equation*}
\hat{\boldsymbol{m}}=\hat{\boldsymbol{r}}_{s}-2\left(\hat{\boldsymbol{n}}_{s} \cdot \hat{\boldsymbol{r}}_{s}\right) \hat{\boldsymbol{n}}_{s} \tag{17}
\end{equation*}
$$

Let $c_{p}$ be a constant from focus to the reference plane, then we can obtain:

$$
\left\{\begin{array}{l}
\left|\boldsymbol{r}_{s}\right|+\frac{x_{m}-x_{s}}{m_{x}}\left(1-m_{z}\right)-z_{s}=c_{p}  \tag{18}\\
\left|\boldsymbol{r}_{s}\right|+\frac{y_{m}-y_{s}}{m_{y}}\left(1-m_{z}\right)-z_{s}=c_{p} \\
\left|\boldsymbol{r}_{s}\right|+\frac{z_{m}-z_{s}}{m_{z}}\left(1-m_{z}\right)-z_{s}=c_{p}
\end{array}\right.
$$

where,

$$
\begin{equation*}
\left|\boldsymbol{r}_{s}\right|=\sqrt{x_{s}^{2}+y_{s}^{2}+z_{s}^{2}} \tag{19}
\end{equation*}
$$

Solving the equation (18), we can determine the point $\left(x_{m}, y_{m}, z_{m}\right)$ in the main reflector corresponding to the sub-reflector $\left(x_{s}, y_{s}, z_{s}\right)$. Now the sub and main reflectors are both solved, so the design about the antenna has been accomplished.


Figure 4. The photo of the practical antenna.


Figure 5. Patterns through the azimuth plane.

## 3. MEASUREMENT RESULTS

The photo of a practical antenna is shown as Fig. 4. The practical measurement results have reached the anticipated aim and have been accord to the computed results, which validates the design method. Fig. 5 and Fig. 6 are the measured and computed patterns of the antenna.


Figure 6. Patterns through the elevation plane.

## 4. CONCLUSION

Though a simple and efficient design method, the ring-focus antenna with a variable focal distance for forming an elliptic beam with good performance is designed. By use of the loading space sufficiently, this antenna can form the wanted elliptic beam, and have high efficiency.

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