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RESONANT LENGTH CALCULATION AND RADIATION PATTERN SYNTHESIS OF LONGITUDINAL SLOT ANTENNA IN RECTANGULAR WAVEGUIDE

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Abstract—Main intension is to calculate admittance and radiation pattern of longitudinal shunt slot in rectangular waveguide using Method of Moments (MoM) technique. Resonant length calculation of the slot is a critical parameter in the design of waveguide slot array antenna. All computed results are compared with simulated results. CST Microwave studio is used for the simulation and is totally based on FIT techniques. For computation purpose MATLAB 7.0 is used. The numerical data on transmission and reflection coefficient are evaluated. Method of moment solution is used to calculate resonant length versus slot offset for given waveguide dimension and frequency. E and H field radiation pattern are calculated for different offset in different frequencies.

1. INTRODUCTION

Waveguide slots are extensively used as radiators and couplers. For such problem an equivalent circuit model for waveguide and the slot, can make the design process easier and more and more feasible. In this paper analysis of a single longitudinal slot in the broad wall of rectangular waveguide is presented. The equivalent shunt admittance of the slot is obtained and the solution also provides the resonant length of the slot. The analysis of waveguide longitudinal slot has been carried out by number of workers include the work of Oliner [1], Stevenson [2] and Khac [3]. They work for the effect of the offset of the slot from the centerline of the waveguide. A sinusoidal electrical field distribution along the slot length was assumed. Stegen [5] developed the first theory for longitudinal slots which would permit calculation of the susceptance as well as the conductance. The integral equation for the electric field is solved by the Method of Moments, choosing pulse basis function and point matching technique. Sangster and Lyon [4] used an entire basis with sinusoidal function and concluded that two basis functions are sufficient for the power calculations. Stren and Elliot studied moment method solution for the resonant length. With the help of Stevenson paper in which he established the internal green function for a rectangular waveguide and used those function to analyze scattering from a longitudinal slot excited by an incident TE_{10} mode. For design of slotted waveguide array the resonant length must be known very accurately. In the present work attention has been paid to evaluate the electrical characteristic in a rectangular waveguide. The relation between vector potential and electric field [6, 11, 12], radiation pattern is calculated. The analysis is carried out using moment method techniques. The application of aperture field methods permits use of entire domain sinusoidal basis function [14] which gives faster convergence than other current based method. The application of Galerkin techniques leads to symmetric moment matrix which reduces the computation time appreciably. Computed resonant length compare very well with measured results.

2. COMPUTATION OF S PARAMETER OF LONGITUDINAL SLOT ANTENNA

To analyze the structure, the following assumptions are made:

- The slot ground plane is of infinite extent.
- There is no variation of electric field along the width of the slot.
- Only the transverse component of the magnetic field along the slot aperture is considered.
- The waveguide is excited by the dominant TE_{10} mode.

The electric field at the slot aperture is expressed as

$$\overrightarrow{E} = \overrightarrow{u_x} \sum_{p=1}^{M} E_p e_p$$

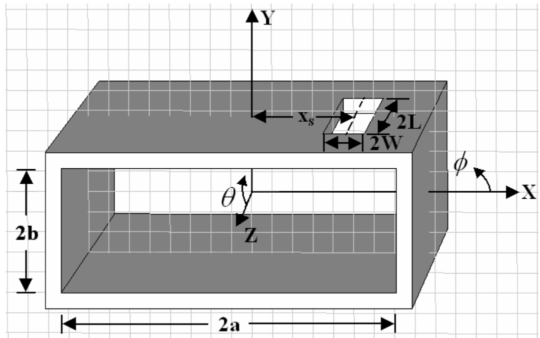


Figure 1. Longitudinal slot on the broad wall of a rectangular waveguide.

From the continuity of tangential component of magnetic field at the slot aperture plane, the boundary condition is obtained as follows

$$H_z^{inc} + H_z^{\mathrm{int}} = H_z^{ext}$$

where H_z^{inc} is the z-component incident magnetic field at the slot aperture and for the TE_{10} mode it can be written as

$$H_z^{inc} = -j \sin\left(\frac{\pi x}{2a}\right) e^{-j\beta z}$$

β is the propagation constant of TE_{10} mode. H_z^{ext} is the z-component of the externally scattered magnetic field and by using the plane wave spectrum approach, it is obtained as follows

$$\begin{aligned} H_z^{ext} = & \frac{LW}{\pi^2 k\eta} \sum_{p=1}^{M} E_p \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{(k^2 - k_z^2)}{(k^2 - k_z^2 - k_x^2)^{1/2}} \sin c\,(k_x W) \\ & \frac{\left\{\begin{array}{l} j\sin n\,(k_z L)\ldots p\ldots even \\ \cos(k_z L)\ldots p\ldots odd \end{array}\right\}}{\frac{p\pi}{2}\left\{1 - \left(\frac{2Lk_z}{p\pi}\right)^2\right\}} \\ & \times e^{-jk_x x_S} e^{j\{k_Z z + k_X x\}} dk_x dk_z \end{aligned}$$

Galerkin specialization of Method of Moments (MoM) has been used to transform the integral equation into matrix equation. The weighting function is given by

$$w_q(x, y, z) = \sin\left\{\frac{q\pi}{2L}(z+L)\right\}\delta(y-b)\ldots x_s - W \le x \le x_s + W, -L \le z \le L$$

$$= 0 \quad \text{elsewhere}$$

Following the procedure of MoM we obtain

$$[E_p] = \{[L^{ext}] - [L^{\text{int}}]\}^{-1}[L^{inc}]$$

where $[E_p]$ is the unknown vector corresponding to the basis coefficients. Waveguide wall thickness has been introduced by treating the slot as stub waveguide.

3. EQUIVALENT CIRCUIT REPRESENTATION

The scattering of a longitudinal slot in the broad wall of a rectangular waveguide is usually modeled by means of equivalent shunt admittance from the measured or computed reflection coefficients Γ. For the fundamental $\text{TE}_{(1,0)}$ mode the equivalent normalize slot admittance is obtained as

$$Y/Y_0 = -2\Gamma/(1+\Gamma)$$

The reflection coefficient can be measured quite accurately. It provides a useful basis for comparison with theoretical data. The parameter derived from the reflection coefficient are the normalized admittance and resonant length. The simple shunt model implies that the scattering from the slot is symmetrical, i.e., the back scattering equals the forward scattering. But this is not exactly true for all real slots.

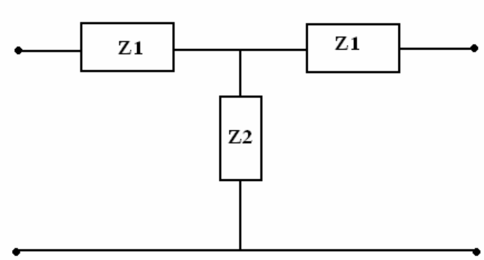


Figure 2. Equivalent circuit of a single longitudinal slot antenna.

Considering an equivalent T-network of the slot, the series (Z_1) and shunt (Z_2) arm.

Impedances can be determined from the following relations [6], defined by moment method formulations.

$$Z_1 = \frac{1+\Gamma-T}{1-\Gamma+T}$$

$$Z_2 = \frac{2T}{(1-\Gamma+T)(1-\Gamma-T)}$$

MATLAB codes are written to evaluate the S-parameters of the slotted waveguide.

Computation on the basis of the equation of Z_1 and Z_2 shows that Z_1 is negligibly small around resonant frequency of the slot. Therefore, the broad wall longitudinal slot can be represented by shunt admittance. Fig. 3 and Fig. 4 show the variation of resonant conductance with the offset of the slot having length $2L = 16\,\text{mm}$ and width $2W = 1\,\text{mm}$. The waveguide is a standard WR90 waveguide ($2a = 22.86$, $2b = 10.16$ and wall thickness $t = 1.27\,\text{mm}$). Measured result shows that the resonant conductance increases with the increase of offset value. Computed aperture field on the basis of diagonal matrix and FIT based simulation, shows that the strength of the filed increases with the increase of offset of the slot. As the slot is displaced more and more from the waveguide center line it produces greater interruption to the conduction current and the equivalent magnetic current responsible for radiation into the outer space increases. The variation of resonant frequency of the slot with the change in offset is shown in figures. The figure shows that the resonant frequency of a slot increases with the increase of offset value.

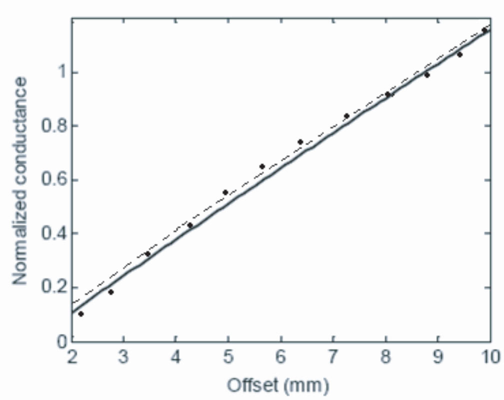


Figure 3. Normalized conductance as a function of slot offset. Solid line: computed, dashed line: simulated, dotted line: measured results. Dimensions are: $2a = 22.86$, $2b = 10.16$, $t = 1.27$, $2L = 16$, $2w = 1\,\text{mm}$.

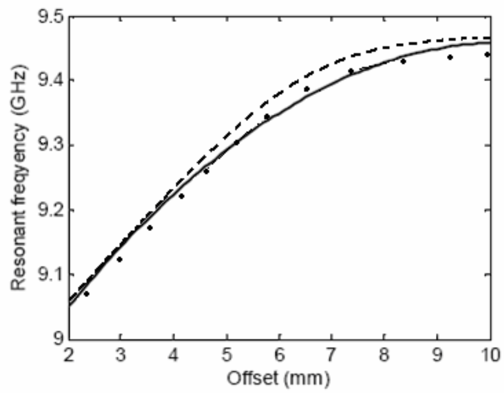


Figure 4. Resonant frequency as a function of slot offset. Solid line: computed, dashed line: simulated, dotted line: measured results. Dimensions are: $2a = 22.86$, $2b = 10.16$, $t = 1.27$, $2L = 16$, $2w = 1\,\mathrm{mm}$.

4. RADIATION PATTERN

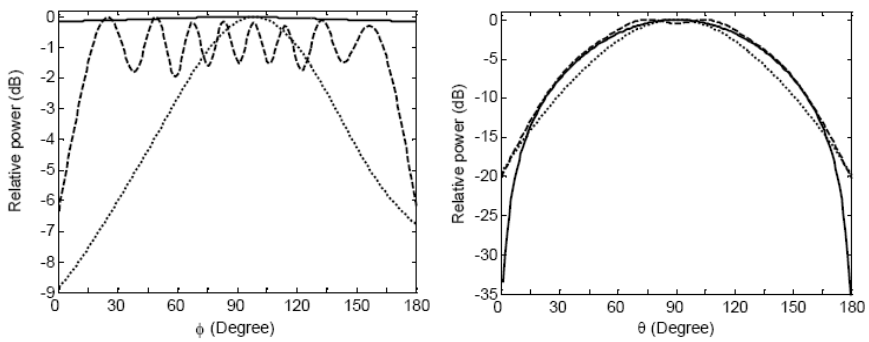
The radiated electric field pattern of the slot can be obtained from [7].

$$\vec{E} = -\vec{\nabla} \times \vec{F}$$

where F is the electric vector potential at any point in the space. In spherical coordinate system the electric field

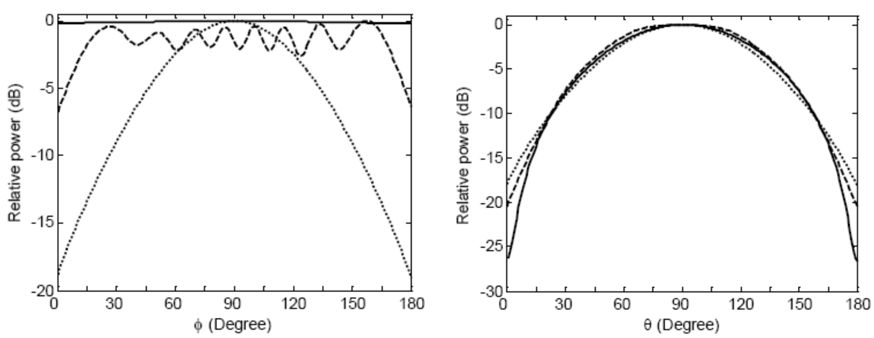
$$\begin{aligned}\vec{E} \; &= \; \frac{u_r}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(F_\theta \sin\theta\right) - \frac{\partial F_\theta}{\partial\phi}\right] + \frac{u_\theta}{r}\left[\frac{1}{\sin\theta}\frac{\partial F_r}{\partial\theta} - \frac{\partial}{\partial r}\left(rF_\theta\right)\right] \\ &\quad + \frac{u_\phi}{r}\left[\frac{\partial}{\partial r}(rF_\theta) - \frac{\partial F_r}{\partial\theta}\right]\end{aligned}$$

From that we can easily calculate E_r, E_θ, E_ϕ, terms. Radiation patterns of the slot are computed by using E fields and are shown in Fig. 5 for the offset 3 and 6 mm respectively at their resonant frequencies. It can be seen that the slot behaves as a magnetic dipole as it is assumed that the slot is embedded in an infinite ground plane. CST Microwave Studio simulated radiation patterns by considering the finite ground plane and the extended ground plane ($250\,\mathrm{mm} \times 150\,\mathrm{mm}$) are also shown in the Fig. 6. The E-plane radiation pattern changes significantly for the finite size of the ground plane. This is due to the diffraction effects from the edges of the finite size ground planes. The E-plane radiation patterns with extended ground planes show



(a) (b)

Figure 5. (a) E and (b) H-plane radiation patterns of a broad wall longitudinal slot for offset $= 3\,\mathrm{mm}$. Solid line: computed, dashed line: simulated with extended ground plane and dotted line: simulated with finite ground plane. Dimensions are: $2a = 22.86$, $2b = 10.16$, $t = 1.27$, $2L = 16$, $2W = 1\,\mathrm{m}$.



(a) (b)

Figure 6. (a) E and (b) H-plane radiation patterns of a broad wall longitudinal slot for offset $= 6\,\mathrm{mm}$. Solid line: computed, dashed line: simulated with extended ground plane and dotted line: simulated with finite ground plane. Dimensions are: $2a = 22.86$, $2b = 10.16$, $t = 1.27$, $2L = 16$, $2W = 1\,\mathrm{mm}$.

that the slot tends to become a magnetic dipole. Due to lack of some experimental facilities, the measured results are not provided.

In order to design a narrow beam antenna, the primary guide feeds cascaded sections of junctions. Synthesis of an array of such junction execited in accordance with the given amplitude distribution is presented. The Taylor distribution for a linear aperture has been considered to design a linear array antenna with specified SLL. The external mutual couplings between the array elements and the finite ground plane effect are considered. For slot coupling case the loaded apertures can be represented by circuit design model. Different procedure specially Fourier series expansion [15] or Fourier transform techniques [16] has been used for slot array calculations.

5. CONCLUSION

In this paper, a single longitudinal slot antenna and it's computed and simulated results are presented. Resonant frequency, conductance and radiation patterns of an isolated slot are computed assuming the slot ground plane as of infinite extent. It is seen that the strength of the slot aperture electric field and hence the radiation conductance of the longitudinal slot increases with the increase of displacement from the waveguide center line. The simulated radiation patterns of an isolated longitudinal slot shows that the diffraction effects from the edges of finite size ground plane only affects the E-plane radiation patterns of the slot. For liner array antenna, required Taylor array synthesis for getting desired side lobe level (SLL).

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