Progress In Electromagnetics Research B, Vol. 7, 299–307, 2008

PERIODICITY DEFECT INFLUENCE ON THE ELECTROMAGNETIC PROPERTIES OF A SEQUENCE WITH BI-ISOTROPIC LAYERS

V. Tuz and V. Kazanskiy

Department of Theoretical Radio Physics
Kharkov National University
Svobody Sq. 4, Kharkov, Ukraine, 61077

Abstract—A series of N identical periods of pairs of isotropic and bi-isotropic layers with defect in j-th basic element is investigated. The universal method that simultaneously allows to taking into account different types of defects in the structure is proposed. The problem is solved using the circuit theory and the transfer matrix methods. The analysis of the dynamic of electromagnetic properties of the investigated structure was carried out for different types of defects.

1. INTRODUCTION

One-dimensional periodic structures have found particularly wide application as filters and lasers. They are the basis of distributed Bragg reflection lasers. Introducing a quarter-wavelength defect in the middle of a periodic one-dimensional sample produces a photonic mode in the center of the reflection band. Such a defect is used to produce high-Q laser cavities in vertical cavity surface-emitting lasers and may be the basis of large-area coherent laser emission in thin films [1–5].

In the last time a lot of investigations are pointed on the periodical structures that include layers which are made from natural (cholesteric liquid crystal) or artificial material with the spatial dispersion (bi-anisotropic, reciprocal (chiral) and nonreciprocal bi-isotropic media) [6–13]. Such structures realize the polarization transforming of the waves in additional to their spatial and frequency selections.

A rich variety of defects introduced into the perfect chiral structure has been proposed. A defect can be created by modifying the refractive index, thickness, rotating helical axis of existing layers or inserting additional layer into the sequence [14–20]. Defect modes in chiral structures have received much attention and may be used for realizing low threshold lasers, narrow bandpass circular-polarization filters.

The other aspect is the possibility of the periodicity defect appearing in the structure when it has been made. Therefore investigations of the sequence with defect are important to provide the quality assurance of the bandpass filters manufacturing and allow determining both the disturbance type and the defect element position in the structure.

In the present work is proposed the universal method that simultaneously allows to taking into account different types of defects in the structure. It is based on the circuit theory and the transfer matrix method [21–26].

2. PROBLEM FORMULATION

As a perfect (without defects) structure, the periodic in the z-axis direction, with period L, structure of N identical basic elements (periods) is investigated (Fig. 1). Each of periods includes a homogeneous magnetodielectric (with permittivities ε_1, μ_1) and bi-isotropic (with $\varepsilon_2, \mu_2, \xi, \zeta$) layers with thicknesses d_1 and d_2 ($L = d_1 + d_2$), respectively. Here the parameters ξ and ζ define the degree of electric-magnetic coupling [6, 7]. In general, the parameters $\varepsilon_1, \mu_1, \varepsilon_2, \mu_2, \xi, \zeta$ are frequency dependent and complex. The outer half-spaces $z \leq 0$ and $z \geq NL$ are homogeneous, isotropic and have permittivities ε_0, μ_0 and ε_3, μ_3, respectively.

As the excitation fields, the plane (in the XOZ plane)

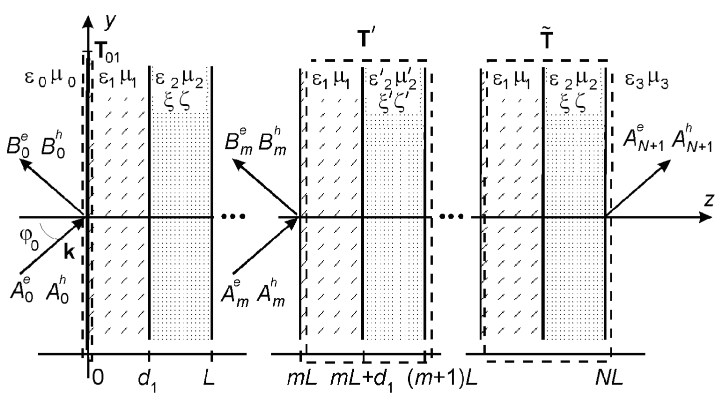


Figure 1. The bounded periodical sequence of isotropic and bi-isotropic layers with defect element.

monochromatic waves ($\exp(-i\omega t)$) with perpendicular ($\boldsymbol{E}^e \| \boldsymbol{x}_0,\ H_x^e = 0$) or parallel ($\boldsymbol{H}^h \| \boldsymbol{x}_0,\ E_x^h = 0$) polarizations are selected (e- and h-waves). They are obliquely incident under angle φ_0 to the z-axis from the region $z \leq 0$.

The structure under study can be considered as a consecutive connection of the eight-poles which are equivalent to the illuminated boundary ($\boldsymbol{T}_{01}$), repeated heterogeneity ($\boldsymbol{T} = \boldsymbol{T}_1\boldsymbol{T}_2$) and the last element which is loaded on the waveguide channel having the admittance Y_3^s ($\tilde{\boldsymbol{T}}$) (Fig. 1). The equations coupling the field amplitudes at the structure input ($A_0^s, B_0^s, B_0^{s'}$) and output ($A_{N+1}^s, A_{N+1}^{s'}$) for the incident fields of e-type ($A_0^h = 0$) and h-type ($A_0^e = 0$) are obtained as [13, 23]

$$\begin{pmatrix} A_0^s \\ B_0^s \\ 0 \\ B_0^{s'} \end{pmatrix} = \boldsymbol{T}(0) \begin{pmatrix} A_{N+1}^s \\ 0 \\ A_{N+1}^{s'} \\ 0 \end{pmatrix}, \quad \boldsymbol{T}(0) = \boldsymbol{T}_{01}\boldsymbol{T}^{N-1}\tilde{\boldsymbol{T}} = \boldsymbol{T}_{01}\boldsymbol{T}^N\boldsymbol{T}_{13}. \tag{1}$$

In the block representation (2×2) the transfer matrices are:

$$\begin{aligned} \boldsymbol{T}_{pv} &= \begin{pmatrix} \left(\boldsymbol{T}_{pv}^s\right) & 0 \\ 0 & \left(\boldsymbol{T}_{pv}^{s'}\right) \end{pmatrix}, \\ \boldsymbol{T}_{pv}^s &= \frac{1}{2\sqrt{Y_p^s Y_v^s}} \begin{pmatrix} Y_p^s + Y_v^s & \pm\left(Y_p^s - Y_v^s\right) \\ \pm\left(Y_p^s - Y_v^s\right) & Y_p^s + Y_v^s \end{pmatrix}, \\ \boldsymbol{T}_1 &= \begin{pmatrix} \left(\boldsymbol{T}_{1+}^{ss}\right) & \left(\boldsymbol{T}_{1-}^{ss}\right) \\ \left(\boldsymbol{T}_{1+}^{ss'}\right) & \left(\boldsymbol{T}_{1-}^{ss'}\right) \end{pmatrix}, \ \boldsymbol{T}_2 = \begin{pmatrix} \left(\boldsymbol{T}_{2+}^{ss}\right) & \left(\boldsymbol{T}_{2+}^{ss'}\right) \\ \left(\boldsymbol{T}_{2-}^{ss'}\right) & \left(\boldsymbol{T}_{2-}^{ss'}\right) \end{pmatrix} \end{aligned} \tag{2}$$

where $\boldsymbol{T}_{pv}$ corresponds to the matrices $\boldsymbol{T}_{01}$ and $\boldsymbol{T}_{13}$, the upper sign relates to $s = h$, and the lower sign relates to $s = e$ in terms of the wave types. The elements of the transfer matrices $\boldsymbol{T}_1$ and $\boldsymbol{T}_2$ are determined from solving the boundary-value problem and are shown in [13].

In order to investigate the structure with a large number of periods ($N \gg 1$), the direct product of matrices in (1) is computationally demanding. Therefore we use the algorithm from the matrix polynomial theory [24] for raising the matrix $\boldsymbol{T}$ to the power N

$$\boldsymbol{T}^N = \sum_{n=1}^{4} \lambda_n^N \boldsymbol{F}_n, \qquad \boldsymbol{F}_n = \boldsymbol{P}\boldsymbol{I}_n\boldsymbol{P}^{-1}. \tag{3}$$

Where λ_n are the eigenvalues of the transfer matrix $\boldsymbol{T}, \boldsymbol{P}$ is the matrix which columns are the set of independent eigenvectors of $\boldsymbol{T}, \boldsymbol{I}_n$ is the matrix with a 1 in the (n, n) location and zeros elsewhere.

Let's consider, that the defect of the periodicity is determined by inserting into the sequence of the m-th element, which parameters (X_δ) and the transfer matrix $\boldsymbol{T}'$ differ from the basic element parameters $(X_\delta^0, \boldsymbol{T})$. Instead of (1) we have

$$\boldsymbol{T}(\Delta X_s) = \boldsymbol{T}_{01}\left(\sum_{n=1}^{4}\lambda_n^{m-1}\boldsymbol{F}_n\right)\boldsymbol{T}'\left(\sum_{n=1}^{4}\lambda_n^{N-m}\boldsymbol{F}_n\right)\boldsymbol{T}_{13} \tag{4}$$

Follow [25], evaluate the matrix $\boldsymbol{T}'$ and its coefficients for small deviations $|\Delta X_\delta| = |X_\delta - X_\delta^0| \ll |X_\delta^0|$, via the linear function

$$\boldsymbol{T}' = \boldsymbol{T} + \sum_{\delta=1}^{M}(\partial\boldsymbol{T}/\partial X_\delta)\cdot\Delta X_\delta \equiv \boldsymbol{T} + \sum_{\delta=1}^{M}\boldsymbol{T}^{(\delta)}\cdot\Delta X_\delta; \tag{5}$$

$$t'_{jk} = t_{jk} + \sum_{\delta=1}^{M}(\partial t_{jk}/\partial X_\delta)\cdot\Delta X_\delta = t_{jk} + \sum_{\delta=1}^{M}t_{jk}^{\delta}\cdot\Delta X_\delta. \tag{6}$$

Here the operator $\boldsymbol{T}^{(\delta)}\cdot\Delta X_\delta$ is named as the transfer matrix differential of the defect element with X_δ parameter disturbing, M is the total number of the disturbing parameters of the defect element. Taking into account these relations, the transfer matrix of the structure with both M and single defects are defined via next sums, respectively

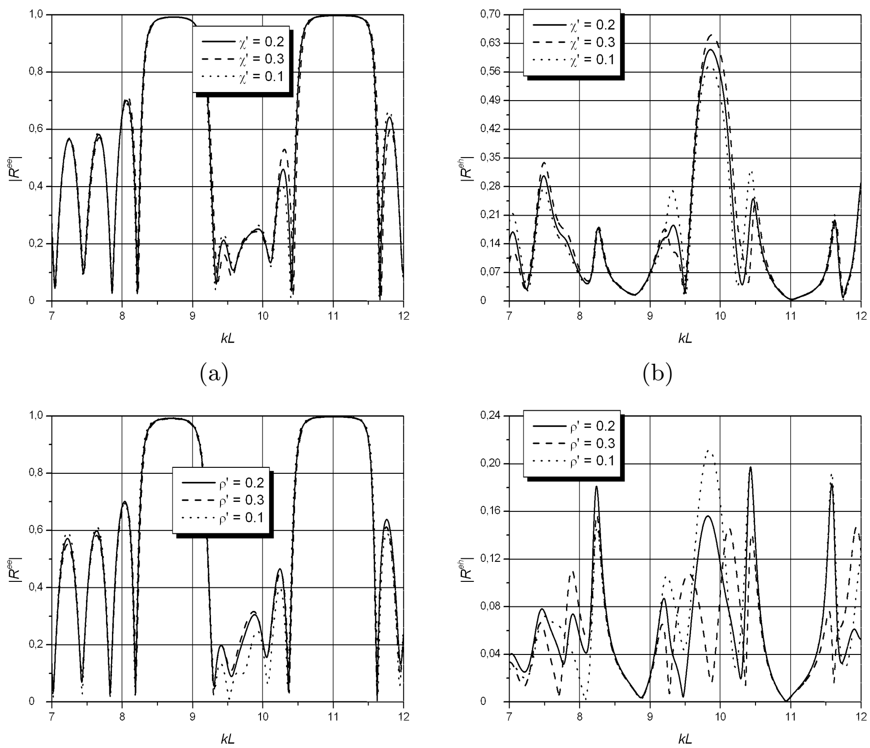
$$\begin{aligned}\boldsymbol{T}(\Delta X_\delta) &= \boldsymbol{T}(0) + \boldsymbol{T}_{01}\left(\sum_{n=1}^{4}\lambda_n^{m-1}\boldsymbol{F}_n\right)\\ &\quad\cdot\left(\sum_{\delta=1}^{M}\boldsymbol{T}^{(\delta)}\cdot\Delta X_\delta\right)\left(\sum_{n=1}^{4}\lambda_n^{N-m}\boldsymbol{F}_n\right)\boldsymbol{T}_{13},\\ \boldsymbol{T}(\Delta X_\delta) &= \boldsymbol{T}(0) + \Delta X_\delta\cdot\boldsymbol{T}_{01}\left(\sum_{n=1}^{4}\lambda_n^{m-1}\boldsymbol{F}_n\right)\\ &\quad\cdot\boldsymbol{T}^{(\delta)}\left(\sum_{n=1}^{4}\lambda_n^{N-m}\boldsymbol{F}_n\right)\boldsymbol{T}_{13}.\end{aligned} \tag{7}$$

Since the last terms are presented via the non-commutative operator (the matrix production), the amplitudes of the scattered fields are dependent on both the disturbance value and the defect element position in the structure.

We will consider the following values of disturbances in bi-isotropic layer parameters: $X_1^0 = \varepsilon_2,\ X_2^0 = \mu_2,\ X_3^0 = \chi,\ X_4^0 = \rho$, i.e., $M = 4$.

3. SCATTERING AND TRANSFORMATION FIELDS OF STRUCTURE WITH DEFECT

On the Fig. 2 the solid line presents of the frequency dependences of the reflection (Figs. 2a,c) and transformation (Figs. 2b,d) coefficient magnitudes of the plane monochromatic wave from the perfect structure [13]. They have interleaved areas with the high (the quasi-stop bands) and low (the quasi-pass bands) average level of the reflection. The interference of the reflected wave from outside boundaries gives $N-1$ small-scale oscillations in the pass bands. The maximum of the reflection coefficient magnitude for the cross-



(a) (b)

(c) (d)

Figure 2. The frequency dependences of the reflection coefficient magnitude of the sequence with bi-isotropic and isotropic layers and with disturbing of the nonreciprocality χ (a, b) and chirality ρ (c, d) parameters in the $m=3$ period: $\varepsilon_j=\mu_j=1,\ j\neq 2,\ \varepsilon_2=4,\ \mu_2=1,\ d_1/L=d_2/L=0.5,\ \rho=0.2,\ \chi=0.2,\ \varphi_0=25°$.

polar wave corresponds to the minimum of the reflection coefficient magnitude for the co-polar wave.

The impedance parameters $\eta^{\pm}$ of a bi-isotropic layer do not depend on the chirality parameter ρ at all and when $\chi \neq 0$ are complex even for lossless media because they depend from complex value of $\varepsilon^{\pm}$ and $\mu^{\pm}$ (see the parameter definitions in [7, 13]). The propagation constants $\gamma^{\pm}$ in a bi-isotropic medium depend on both chirality ρ and nonreciprocal χ parameters [7]. These features of the bi-isotropic medium determine the character of the frequency dependences of the scattering and transformation coefficients of the periodical structure with the defect element (Fig. 2).

As criterions of the defect influences we will consider the changing of the quasi-pass band width, the magnitude and the average level of the small-scale oscillations, the resonant frequency shifting in a comparison with properties of the perfect periodical structure.

The medium nonreciprocality parameter influences on the wave admittance of the bi-isotropic layer $\eta_2^{\pm}$. Therefore the increasing (decreasing) of the χ value slightly changes the average level of the reflection coefficient in the quasi-pass band. It is not depend from the type of the excitation field and the defect element position in the structure (Figs. 2a,b). The dependence of $\gamma^{\pm}$ from the medium nonreciprocality parameter χ influences on the structure phase properties too, that yields the shifting of the high-frequency boundary of the quasi-stop band. It is the most obvious for the reflection coefficient of the cross-polar wave (Fig. 2b).

The chiral parameter ρ defines the phase properties of the structure. Its changing gives the proportional shifting the resonant frequency. Such disturbance essentially influences on the character of the high-frequency (small-scale) oscillations in the quasi-pass bands. This effect is determined by the composition of the eigen modes of the identical transmission lines which are placed before and after the defect element, and by the additional eigen modes appearing as a result of the wave transformation. Their frequency and phase difference explain both the magnitude modulation of the high-frequency oscillations in quasi-pass band and dependence of the modulation period from the position of the defect element in the structure (Figs. 2c,d). There is an interesting peculiarity of the behavior of the reflection coefficient magnitude for the cross-polar wave when $\rho' = 0.3$. Here is observed the reflection minimum for the defect structure in contrast to the maximum for the perfect structure (as example, nearly $kL \approx 9.8$).

4. CONCLUSIONS

In this paper, we have applied the circuit theory and the transfer matrix methods to solving the plane wave diffraction problem for the DBR-like bounded sequence of pairs of bi-isotropic and magnetodielectric layers with the periodicity defect. The analysis of its electromagnetic properties was carried out. The nature, disturbance value, defect element position, period composition influence on the properties of the structure. The disturbance of nonreciprocal and chiral parameters of a layer yields shift in the maximum transparency frequency and the magnitude of the small-scale oscillations modulation. It is explained by the composition of the eigen modes of the identical transmission lines which are placed before and after the defect element. Furthermore the nonreciprocal parameter disturbance slightly changes the average level of the reflection coefficient in the quasi-pass band.

REFERENCES

1. Yokoyama, H. and K. Ujihara, *Spontaneous Emission and Laser Oscillation in Microcavities*, CRC, Boca Raton, Fla., 1995.
2. Awasthi, S. K., U. Malaviya, S. P. Ojha, N. K. Mishra, and B. Singh, "Design of a tunable polarizer using a one-dimensional nano sized photonic bandgap structure," *Progress In Electromagnetics Research B*, Vol. 5, 133–152, 2008.
3. Srivastava, R., K. B. Thapa, S. Pati, and S. P. Ojha, "Design of photonic band gap filter," *Progress In Electromagnetics Research*, PIER 81, 225–235, 2008.
4. Smith, D. R., R. Dalichaouch, N. Kroll, et al., "Photonic band structure and defects in one and two dimensions," *J. Opt. Soc. Am. B.*, Vol. 10, No. 2, 314–321, 1993.
5. Leung, K. M., "Defect modes in photonic band structures: a Green's function approach using vector Wannier functions," *J. Opt. Soc. Am. B.*, Vol. 10, No. 2, 303–306, 1993.
6. Lakhtakia, A., V. K. Varadan, and V. V. Varadan, *Time-Harmonic Electromagnetic Fields in Chiral Media, Lecture Notes in Physics*, Springer-Verlag, 1989.
7. Lindell, I. V., A. H. Sihvola, S. A. Tretyakov, A. J. Viitanen, *Electromagnetic Waves in Chiral and Bi-Isotropic Media*, Artech House, Boston, London, 1994.
8. Chadrasekhar, S., *Liquid Crystals*, Cambridge University Press, 2004.
9. Hussain, A. and Q. A. Naqvi, "Fractional curl operator in chiral

medium and fractional non-symmetric transmission line," *Progress In Electromagnetics Research*, PIER 59, 199–213, 2006.

10. Panin, S. B., P. D. Smith, and A. Y. Poyedinchuk, "Elliptical to linear polarization transformation by a grating on a chiral medium," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 13, 1885–1899, 2007.
11. Kuzu, L., V. Demir, A. Z. Elsherbeni, and E. Arvas, "Electromagnetic scattering from arbitrarily shaped chiral objects using the finite difference frequency domain method," *Progress In Electromagnetics Research*, PIER 67, 1–24, 2007.
12. Hussain, A., M. Faryad, and Q. A. Naqvi, "Fractional curl operator and fractional chiro-waveguide," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 8, 1119–1129, 2007.
13. Tuz, V., "Three-dimensional Gaussian beam scattering from a periodic sequence of bi-isotropic and material layers," *Progress In Electromagnetics Research B*, Vol. 7, 53–73, 2008.
14. Gilles, L. and P. Tran, "Optical switching in nonlinear chiral distributed Bragg reflectors with defect layers," *J. Opt. Soc. Am. B.*, Vol. 19, No. 4, 630–639, 2002.
15. Kopp, V. I. and A. Z. Genack, "Twist defect in chiral photonic structures," *Phys. Rev. (Lett.)*, Vol. 89, No. 3, 2002.
16. Kopp, V. I., R. Bose, and A. Z. Genack, "Transmission through chiral twist defects in anisotropic periodic structures," *Opt. Lett.*, Vol. 28, No. 5. 349–351, 2003.
17. Chen, J.-Y. and L.-W. Chen, "Polarization-dependent filters based on chiral photonic structures with defects," *J. Opt. A. Opt.*, Vol. 7, 558–566, 2005.
18. Li, J., L. Jin, L. Li, and C. Li, "Bandgap separation and optical switching in nonlinear chiral photonic crystal with layered structure," *IEEE Phot. Techn. Lett.*, Vol. 18, No. 11, 1261–1263, 2006.
19. Ha, N. Y., Y. Takanishi, K. Ishikawa, and H. Takezoe, "Simultaneous RGB reflections from single-pitched cholesteric liquid crystal films with Fibonaccian defects," *Opt. Exp.*, Vol. 15, No. 3, 2007.
20. Yoshida, H., C. H. Lee, Y. Miura, A. Fujii, and M. Ozaki, "Optical tuning and switching of photonic defect modes in cholesteric liquid crystals," *Appl. Phys. Lett.*, Vol. 90, 071107, 2007.
21. Born, M. and E. Wolf, *Principle of Optics*, Oxford, Pergamon Press, 1968.

22. Kim, G. and E. Garmire, "Comparison between the matrix method and the coupled-wave method in the analysis of Bragg reflector structures," *J. Opt. Soc. Am. A*, Vol. 9, 132–137, 1992.
23. Kazanskiy, V. B., V. R. Tuz, and V. V. Khardikov, "Quasiperiodic metal-dielectric structure as a multifunctional control system," *Radioelectronics and Communications Systems*, Vol. 45, No. 7, 38–46, 2002.
24. Dickey, L. J., "High powers of matrices," *ACM SIGAPL APL Quote Quad.*, Vol. 18, No. 2. 96–99, 1987.
25. Khardikov, V. V., V. R. Tuz, and V. B. Kazanskiy, "Influence of defects on electrodynamic properties of the finite series of identical metal-dielectric resonators," *Proc. X-th Int. Conf. Math. Methods Electromagn. Theory*, 130–132, Dniepropetrovsk, Ukraine, 2004.
26. Zhang, Y. J., A. Bauer, and E. P. Li, "T-matrix analysis of multiple scattering from parallel semi-circular channels filled with chiral media in a conducting plane," *Progress In Electromagnetics Research*, PIER 53, 299–318, 2005.