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THE ELECTROMAGNETIC ION-CYCLOTRON INSTABILITY IN THE PRESENCE OF A.C. ELECTRIC FIELD FOR LORENTZIAN KAPPA

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Abstract—Electromagnetic ion cyclotron (EMIC) waves have been studied in the presence of AC electric field perpendicular to ambient magnetic field in the ionosphere with observed superthermal electrons. The presence of 4 eV–50 eV superthermal electrons have been recently seen by Indian SROSS-C2 satellite, Lorentzian Kappa distribution has been used to derive dispersion relation and growth rate using method of characteristics and kinetic approach. The free energy source like anisotropy in temperature, AC electric field and presence of superthermal electrons affect the growth rate. Lorentzian kappa distribution plays important role in giving the wide spectrum range of emitted frequencies.

1. INTRODUCTION

There is strong evidence that the high latitude ionosphere is an important source of magnetospheric ions, which is confirmed by the observations of the upward flowing accelerated ionospheric ions in the auroral zone. These up flowing ions can be characterized as either a beam like distribution of particles or conic like distribution depending upon their pitch angles [1].

In the last decade some authors have reported the presence of upward and downward flow of ions in the ionosphere [2–4]. Evidence for ion heating exist in the F-region where an intense super thermal ion beam was reported in confirmation with electrostatic oxygen cyclotron wave observed near an auroral arc and was interpreted in terms of ion heating. Moreover, different satellite observations have confirmed the presence of energetic (kev) up streaming ion beams and electrostatic waves in the magnetospheric plasma also [5–7]. Penetration of ion beam in background plasma has been observed by various satellites and space probes. The ion accelerated towards the Earth's magneto tail far away from the Earth are reported to be of ionospheric origin.

Recent observations by AMPTE/CCE spacecraft show that electromagnetic ion cyclotron (EMIC) waves are observed most frequently in the outer magnetosphere beyond $L = 7$. The wave spectral properties and frequency of occurrence have pronounced dependence on magnetic local time [8]. Most theoretical studies of EMIC wave excitation has concentrated on convective instabilities [9, 10]. However conditions in the outer magnetosphere where the plasma beta ($\beta = 2\mu_0 NT/\beta_0^2$) is high and where the energetic ion anisotropy ($A_{HT} = T_\perp/T_{||} - 1$) can be enhanced by solar wind compression [11] may permit the onset of absolute or non-convective instabilities [12, 13] considered the importance of non-convective instabilities to interpret EMIC waves observed by GEOS 1 and 2 near geostationary orbit. They found that the additional presence of H_e^+ did not significantly affect the threshold for non-convective instabilities. However they pointed out that inhomogeneities in the medium would probably prevent absolute instabilities from forming. There is a class of waves above Ω_{He}^+ that reflect at high latitudes and are geomagnetically trapped. Since the signal transit gain may be small, Roux et al. [13] suggested that such waves might be amplified each time they cross the magnetic equator causing a laser like growth to observable levels. Inpast Horn and Thorne [14] demonstrated that unless special density gradients exist the laser like effect can not occur in the outer magnetosphere since the waves return to the favored equatorial growth region with large wave normal angles. Wave growth to observable levels must therefore occur on the first equatorial pass [14]. Absolute instabilities may never be realized in natural plasma because of other physical processes. Nevertheless, the pronounced reduction in wave group speed for conditions close to the outset or non-convective instabilities can dramatically enhance the path integrated wave gain and thus allow waves to grow rapidly to observable levels. The EMIC waves were observed near the start of the main phase of a geomagnetic storm. This study strengthens the suggestion that relativistic electron scattering by EMIC waves can compete with the effect as a mechanism of decreasing relativistic electron fluxes from the outer zone during magnetic storms [15].

The observation of AC electric field at ionospheric, magnetospheric and shock regions have also been reported along and perpendicular to ambient magnetic field [16, 17]. Earlier GEOS-I and ISEE mission reported electric field observations and related wave particle observations [18–20].

In the past there has been great interest in monitoring enhanced emission from the thermosphere in connection with high power, high frequency (HF) radio wave modification of the F-region ionosphere. These emissions are considered, as evidence that the H-F modified electron distribution function is non- Maxwellian because a significant flux of 5–6 eV electrons is required to produce the airglow. The superthermal tail is believed to develop as a result of non-linear plasma processes.

This study may be useful in general to examine the effect of streaming electron and ion- beams in situations where they may exist, for example in the auroral region of the ionosphere, as the parallel potential drop along the auroral field lines may lead to the down coming electron and up flowing ion-beams [1, 21] and also in the ionosphere, where the presence of upward and down flow of ions are reported.

In this paper the left hand temporal electromagnetic ion cyclotron instability in the presence of perpendicular AC electric field, temperature anisotropy and superthermal electrons has been studied for bi Lorentzian Kappa distribution in the outer ionosphere using modified dispersion function and method of characteristic solutions. The growth rate and dispersion relation have been calculated for ionospheric plasma.

2. DISPERSION RELATION

A spatially homogeneous anisotropic, collision less plasma subjected to an external magnetic field $B_0 = B_0\hat{e}_z$ and an electric field $E_0 = (E_0 \sin vt\hat{e}_x)$ has been considered in order to obtain the dispersion relation .In this case, the Vlasov-Maxwell equations are linearized. The linearized equations obtained after neglecting the higher order terms and separating the equilibrium and non equilibrium parts, following the techniques of [22] are given as:

$$v.\left(\delta f_{s0}/\delta r\right) + \left(e_s/m_s\right)\left[E_0 \mathrm{sin} vt + (v \times B_0)\right]\left(\delta f_{s0}/\delta v\right) = 0 \tag{1}$$

$$\left(\delta f_{s1}/\delta t\right) + v.\left(\delta f_{s1}/\delta r\right) + \left(F/m_s\right)\left(\delta f_{s1}/\delta v\right) = S(r, v, t), \tag{2}$$

where force is defined as $F = mdv/dt$.

$$F = e_s\left[E_0 \sin vt + (v \times B_0)\right] \tag{3}$$

The particle trajectories are obtained by solving the equation of motion defined in Eq. (3) and $S(r,v,t)$ is defined as

$$s(r,v,t) = (-e_s/m_s)[E_1 + (v \times B_1)](\delta f_{s0}/\delta v) \tag{4}$$

where s denotes species and E_1, B_1 and f_{s1} are perturbed quantities and are assumed to have harmonic dependence in f_{s1}, B_1 and $E_1 \cong \exp i(k, r - \omega \mathrm{t})$.

The method of characteristic solution is used to determine the perturbed distribution function. f_{s1}, which is obtained from Eq. (2) by

$$f_{s1}(r,v,t) = \int_0^\infty s\left\{r_0(r,v,t'), v_0(r,v,t'), t-t'\right\} dt'. \tag{5}$$

The phase space coordinate system has been transformed from $(r,\ v,\ t)$ to $(r_0,\ v_0,\ t-t')$. The particle trajectories which have been obtained by solving Eq. (3) for the given external field configuration and wave propagation, $k = \left[k_\perp \hat{e}_x, 0, k_{||}\hat{e}_z\right]$ are

$$\begin{aligned} x_0 &= x + \left(\frac{v_y}{\omega_{cs}}\right) + \left(\frac{1}{\omega_{cs}}\right)\left[v_x \sin\omega_{cs}t' - v_y \cos\omega_{cs}t'\right] \\ &\quad + \left(\frac{\Gamma_x}{\omega_{cs}}\right)\left[\frac{\omega_{cs}\sin \upsilon t' - \upsilon \sin\omega_{cs}t'}{\omega_{cs}^2 - \upsilon^2}\right] \end{aligned}$$

$$\begin{aligned} y_0 &= y + \left(\frac{v_x}{\omega_{cs}}\right) - \left(\frac{1}{\omega_{cs}}\right)\left[v_x \cos\omega_{cs}t' - v_y \sin\omega_{cs}t'\right] \\ &\quad - \left(\frac{\Gamma_x}{\upsilon\omega_{cs}}\right)\left[1 + \frac{\upsilon^2\cos\omega_{cs}t' - \omega_{cs}^2\cos\upsilon t'}{\omega_{cs}^2 - \upsilon^2}\right] \end{aligned}$$

$$z_0 = z - v_z t' \tag{6}$$

and the velocities are

$$v_{x0} = v_x \cos\omega_{cs}t' - v_y \sin\omega_{cs}t' + \left\{\frac{\upsilon\Gamma_x(\cos\upsilon t' - \cos\omega_{cs}t')}{\omega_{cs}^2 - \upsilon^2}\right\}$$

$$v_{y0} = v_x \sin\omega_{cs}t' + v_y \cos\omega_{cs}t' - \left\{\frac{\Gamma_x}{(\omega_{cs}\sin\upsilon t' - \upsilon\sin\omega_{cs}t')}\omega_{cs}^2 - \upsilon^2\right\}$$

$$v_{z0} = v_z \tag{7}$$

where $\omega_{cs} = \frac{e_s B_0}{m_s}$ is the cyclotron frequency of species s and $\Gamma_x = \frac{e_s E_0}{m_s}$ and a.c. electric field is varying as $E = E_{0x}\sin \upsilon t$, υ being the angular a-c frequency. After some lengthy algebraic simplifications and carrying out the integration, the perturbed distribution function is given as

$$f_{s1}(r,v,t) = -\frac{e_s}{m_s\omega}\sum_{m,n,p,q}^{\infty}\frac{J_p(\lambda_2)J_m(\lambda_1)J_q(\lambda_3)e^{i(k.r-\omega t)}}{\left\{\omega-k_{\|}v_{\|}-(n+q)\omega_{cs}+p\upsilon\right\}}$$
$$\left[E_{1x}J_nJ_p\left\{\left(\frac{n}{\lambda_1}\right)U^*+D_1\left(\frac{p}{\lambda_2}\right)\right\}\right.$$
$$\left.-iE_{1y}\left\{J_n'J_pC_1+J_nJ_p'D_2\right\}+E_{1z}J_nJ_pW^*\right]$$

where the Bessel identity

$$e^{i\lambda/\sin\theta}=\sum_{k=-\infty}^{\infty}J_k(\lambda)e^{ik\theta}$$

has been used, the arguments of the functions are

$$\lambda_1=(k_\perp v_\perp)/(\omega_{cs}),\ \lambda_2=(k_\perp\Gamma_x\upsilon)/\left(\omega_{cs}^2-\upsilon^2\right),\ \lambda_3=(k_\perp\Gamma_x\upsilon)/\left(\omega_{cs}^2-\upsilon^2\right)$$

$$C_1=(1/v_\perp)\left(\delta f_0/\delta v_\perp\right)\left(\omega-k_{||}.v_{||}\right)+(\delta f_0/\delta v_{||})k_{||}$$
$$U*=C_1\left[v_\perp-\left\{\upsilon\Gamma_x/\left(\omega_{cs}^2-\upsilon^2\right)\right\}\right],$$
$$w* = \left[(n\omega_{cs}v_{||}/v_\perp)(\delta f_0/\delta v_\perp)-n\omega_{cs}(\delta f_0/\delta v_\perp)\right]$$
$$+\left[1+\left\{k_\perp\Gamma_x\upsilon/\left(\omega_{cs}^2-\upsilon^2\right)\right\}\left\{(p/\lambda_2)-(n/\lambda_1)\right\}\right],$$
$$D_1=C_1\left\{\upsilon\Gamma_x/\left(\omega_{cs}^2-\upsilon^2\right)\right\},\quad D_2=C_1\left\{\omega_{cs}\Gamma_x/\left(\omega_{cs}^2-\upsilon^2\right)\right\},$$
$$J'_n=\left\{dJ_n(\lambda_1)\right\}/d\lambda_1\quad\text{and}\quad J'_p=\{dJ_p(\lambda_2)\}/d\lambda_2. \tag{8}$$

Following [22], the conductivity tensor $||\sigma||$ is written as

$$||\sigma|| = -\sum(e_s^2/m_s\omega)\sum_{m_1n_1p_1q_1=\infty}^{\infty}\int d^3v$$
$$\left[\left\{Jq(\lambda_3)S_{ij}\right\}/\left\{\omega-k_{||}v_{||}-(n+q)\omega_{cs}+p\upsilon\right\}\right] \tag{9}$$

where

$$S_{ij}=\left|\begin{array}{ccc} v_\perp J_n^2J_p(n/\lambda_1)A & iv_\perp J_nB & v_\perp(n/\lambda_1)J_n^2J_pW* \\ v_\perp J'_nJ_nJ_pA & v_\perp J'_nB & iv_\perp J'_nJ_nJ_pW* \\ v_{||}J_n^2J_pA & v_{||}J_nB & v_{||}J_n^2J_pW* \end{array}\right| \tag{10}$$

$$A = \{(n/\lambda_1)U* + (p/\lambda_1)D_1\}, \quad B = \{J'_n J_p C_1 + J_n J'_n D_2\} \tag{11}$$

From $J = ||\sigma||E_1$ and two Maxwell's curl equations for the perturbed quantities, the wave equation can be obtained as

$$\left[k^2 - k.k - \left(\omega^2/c^2\right) \in (k,\omega)\right] E_1 = 0 \tag{12}$$

where

$$|| \in (k,\omega)|| = 1 - (4\pi/i\omega)||\sigma(k,\omega)|| \tag{13}$$

is a dielectric tensor. After using Eq. (9), Eq. (13) becomes

$$\begin{aligned} \in_{ij}(k,\omega) \;=\; & 1 + \sum_s \left\{\left(4\pi e_s^2\right)/\left(m_s\omega^2\right)\right\} \sum_n \sum_p J_p(\lambda_2) J_q(\lambda_3) \\ & \int \left\{\left(d^3 v S_{ij}\right)/\left(\omega - k_{||}v_{||} - (n+q)\omega_{cs} + pv\right)\right\} \end{aligned} \tag{14}$$

The unperturbed bi-Lorentzian distribution function is

$$F_{0\kappa} = \frac{n_0}{\pi^{3/2}\theta_1^2\theta_{||}\kappa^{3/2}} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa+1/2)} \left[1 + \frac{V_{||}^2}{\kappa\theta_{||}^2} + \frac{V_\perp^2}{\kappa\theta_\perp^2}\right]^{-(\kappa+1)} \tag{15}$$

and associated parallel and perpendicular effective thermal speeds.

$$\theta_{||} = \left[\frac{(2\kappa-3)}{\kappa}\right]^{1/2} \left(\frac{T_{||}}{m_s}\right)^{1/2}, \quad \theta_\perp = \left[\frac{(2\kappa-3)}{\kappa}\right]^{1/2} \left(\frac{T_\perp}{m_s}\right)^{1/2}$$

Applying the approximation in ion-cyclotron range of frequencies. In this case the electron temperature are assumed $T_{es} = T_{e||} = T_e$ and assumed to be magnetized with $|\omega_r + i\cdot\gamma| \ll \omega_{cs}$ while ions are assumed to have $T_{\perp i} > T_{||i}$ and $|k_{||}\alpha_{||}| \ll |\omega_r \pm \omega_{cs} + i\cdot\gamma|$ and using Eq. (15) becomes.

$$\begin{aligned} D(\kappa, \omega_r + i\cdot\gamma) \;=\; & 1 - \frac{k^2c^2}{\omega_r + i\cdot\gamma} + \sum \frac{J_p(\lambda_2) J_q(\lambda_3)}{\alpha_{\perp s}^2} \\ & \left[\left\{\frac{\omega_e^2}{\omega_{ci}^2} - \frac{\omega_{pe}^2}{(\omega_r + i\cdot\gamma)(\pm\omega_{ei})}\right\}\right. \\ & \left\{X_{le}\frac{\omega_{pe}^2}{(\omega_r + i\cdot\gamma)^2}\right\} X1i\frac{\omega_r + i\cdot\gamma}{k||\theta||i}\left(\frac{\kappa-1}{\kappa}\right)1/2 \\ & \left(\frac{\kappa-1}{\kappa-3/2}\right) Z\kappa - 1\left(\left(\frac{\kappa-1}{\kappa}\right)1/2\xi i\right) \\ & A_T\left\{1 + \xi_i\left(\left(\frac{\kappa-1}{\kappa}\right)^{1/2}\left(\frac{\kappa-1}{\kappa-3/2}\right) Z_{\kappa-1}^*\right.\right. \end{aligned}$$

$$\left(\left(\left(\frac{\kappa-1}{\kappa}\right)^{1/2}\xi_i\right)\right)\Bigg\}\Bigg] \tag{16}$$

where $X_{le} = \theta_{\perp e}^2 - \frac{v\Gamma_{xe}}{\omega_{ce}^2 - v^2}\frac{\theta_{\perp e}}{2}\sqrt{\pi}$

$$X_{li} = \theta_{\perp i}^2 - \frac{v\Gamma_{xi}}{\omega_{ci}^2 - v^2}\frac{\theta_{\perp i}}{2}\sqrt{\pi}$$

$$A_T = \frac{\theta_{\perp i}^2}{\theta_{||i}^2} - 1$$

Applying conditions $\frac{k^2c^2}{\omega^2} \gg 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}$ using $p = 1$, $n = 1$, $q = 0$ the growth rate and real frequency becomes.

$$\gamma/\omega_{ci} = \frac{\frac{\sqrt{\pi}}{\overline{k}}\left(\frac{(\kappa-1)\,!\kappa^{\kappa-1/2}}{\kappa-3/2}\right)(A_T - K_4)\,K_3^3\left(\frac{k^3}{\overline{k}}\right)^{-2k}}{1+X_4+\left(\frac{\overline{k}^2\,(1+X_4)}{2K_3^2} - \frac{\overline{k}^2}{K_3}\,(A_T - K_4)\left(\frac{\kappa}{\kappa-3/2}\right)\right) - \frac{X_{ie}}{X_{1i}}K_3^2} \tag{17}$$

$$X_3 = \frac{\omega_r}{\omega_{ci}} = \frac{\overline{k}^2}{\beta}\left[\frac{X_{1i}\,(1+X_4)}{X_{1i} - X_{1e}(1+X_4)} + \frac{A_T\beta\,(X_{1i})}{2\,(1+X_4)(X_{1i} - X_{1e}(1+X_4))}\right] \tag{18}$$

where

$$K_3 = 1 - X_3 + X_4, \quad K_4 = \frac{X_3}{1 - X_3 + X_4}, \quad \overline{k} = \frac{k_{\|}\,\alpha_{\|}}{\omega_{cs}}$$

$$\beta = \frac{K_B T_{||}\mu_0 n_0}{B_0^2}, \quad \Gamma_{xs} = \frac{eE_0}{m_s}, \quad X_4 = \frac{-\upsilon}{\omega_{ci}}$$

3. RESULT AND DISCUSSION

The growth rate and the real frequency for EMIC wave have been evaluated for bi-Lorentzian Kappa distribution in the presence of perpendicular AC electric field for ionospheric plasma parameters considered as $B_0 = 2 \times 10^{-7}\,\mathrm{T}$, $n_0 = 1 \times 10^9\,\mathrm{m}^{-3}$, $k_B T_{||} = 1\,\mathrm{keV}$, υ between zero and 18 Hz. $E_0 = 4 \times 10^{-3}\,\mathrm{V/m}$, κ between 2 to 4 and A_T between 1 to 5.

In Fig. 1 ions temperature anisotropy variation have been shown for ionosphere. The growth rate increases with increasing the temperature anisotropy and also the bandwidth increases with anisotropy. In cases of ionospheric plasma the density and magnetic fields are more and thus affect of the real frequency. Temperature anisotropy remains the prime source of sledding the free energy to plasma.

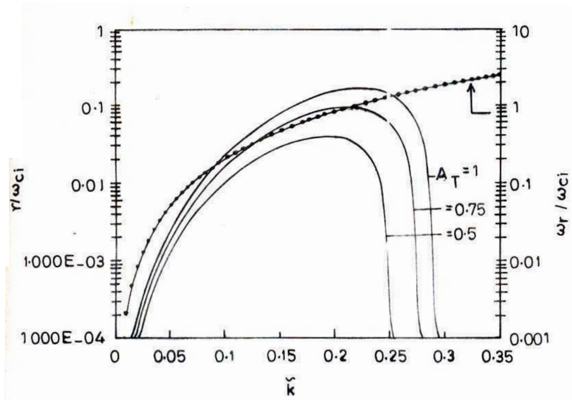

Figure 1. Variation of growth rate and real frequency with respect to $\bar{k}$ for various values of temperature anisotropy A_T at other fixed plasma parameters.

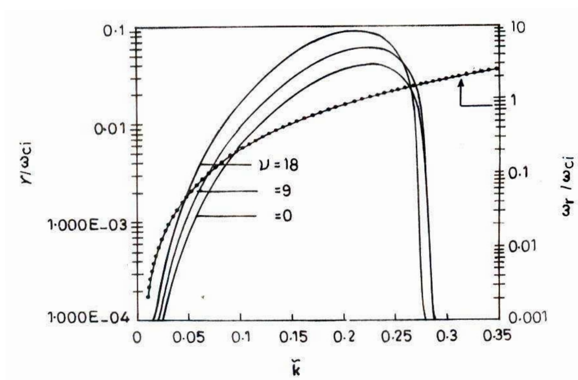

Figure 2. Variation of growth rate and real frequency with respect to $\bar{k}$ for various values of AC field frequency at other fixed plasma parameters.

The effect of AC field frequency is shown in Fig. 2 the growth rate increases with increase in AC frequency, Increase of AC frequency have significant effect in ionosphere, maxima shifts towards higher values of k as the AC frequency changes from 0 to 18 Hz. The real frequency is also affected significantly modifying the resonance frequency. The magnitude of the wave gain and frequency range for this instability is strongly influenced by the adopted parameters for the energetic proton component.

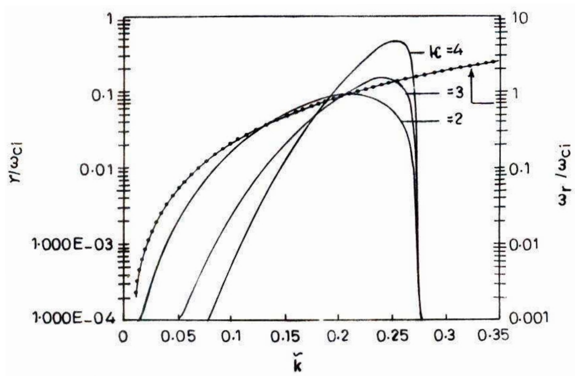

Figure 3. Variation of growth rate and real frequency with respect to $\bar{k}$ for various values of Kappa distribution index κ at other fixed plasma parameters.

Figure 3 shows the variation of distribution index kappa for ionospheric plasma. Larger the value of kappa the growth rate increases indicting that Maxwellian distribution function yields higher growth rate. The smaller the the value of κ the greater the population and the harder the spectrum of superthermal particles and vice-versa. The generalized distribution function for κ greater than 4 starts approaching Maxwellian distribution ($\kappa \to \infty$).

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