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INFLUENCE OF MOTION ON THE EDGE-DIFFRACTION

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Abstract—The aim of the present paper is to reveal the effect of motion on the scattering by an edge. To this end one considers a canonical structure formed by a perfectly conducting half-plane illuminated by a time-harmonic and uniformly moving infinitely long line source. The relevant line source is located parallel to the edge and moves with a constant velocity which is also parallel to the half-plane. This is the dual of a previously studied problem in which the half-plane was moving uniformly. The present problem is first reduced into a Wiener-Hopf problem in the *sense of distribution* and then solved by an ad-hoc method. The edge-diffracted field is discussed in detail.

1. INTRODUCTION

Since the establishment of the electromagnetic theory in 1873, the scattering of electromagnetic waves has become more and more interesting for scientists and engineers. So far a great deal of problems have been formulated and investigated by considering various media and scatterers. In most of these problems the scatterers and sources were assumed to be at rest with respect to the observer. But the applications of the contemporary technology need the consideration of cases where the scatterers and/or the sources are in motion with respect to the observer. The communication satellites, guided missiles, modern swift vehicles and mobile antennas can be enumerated among simple examples.

An exact analysis for the case where the motion is accelerated can only be formulated in the framework of the General Theory of Relativity, which causes rather tough and complicated mathematical difficulties. Therefore, the works devoted to this kind of problems are very few in the open literature. However, when the motion is uniform, an exact analysis can sometimes be done with simple mathematical tools by using the concepts of the Special Theory of Relativity. So, in the open literature, there are many interesting works devoted to the case of uniformly moving scatterers. Among them one can enumerate, for example those which concern moving half-spaces or planes [1–5], cylinders [5–10], strips [11, 12], wedges [13–15], half-planes and edges [16] and bodies of arbitrary shapes [5, 17–24]. A recent analysis performed in [16], which considers the scattering of a plane wave by a uniformly moving half-plane reveals the effect of the motion on the reflection, refraction, aberration, frequency shift, etc. The planar structure of the incident wave considered in [16] permitted one to reduce the problem into the solution of Helmholtz equation which is much more simple as compared to the time-depending wave equation. If the wave is not planar, it is not possible to benefice of the advantages provided by the complex representation of the monochromatic waves. In such a case it seems better to assume that it is the source which is in motion. As far as we know, the investigations devoted to this case also is very few in the available literature. [25, 26] can be cited as significant examples on this subject.

The aim of the present paper is the investigation of the effect of motion on the edge diffraction of a cylindrical wave. To this end one considers the canonical structure formed by a perfectly conducting half-plane illuminated by a uniformly moving infinitely long line source. One assumes that the line source is located parallel to the edge of the half-plane and moves in a direction which is parallel to the half-plane.

In what follows, in Section 2 one formulates the problem in its most general form. In Sections 3 and 4 one considers the solution of the problem and analysis of the excited fields, respectively. Finally, in Section 5 one presents an illustrative example to see the variation of the edge-diffracted wave with respect to normalized time.

2. FORMULATION OF THE PROBLEM

Consider two cartesian co-ordinate systems $Oxyz$ and $O'x'y'z'$ which are denoted by K and K', respectively. In K one assumes that the half-plane $P \equiv \{x \in (-\infty, 0),\ y = 0,\ z \in (-\infty, \infty)\}$ consists of a perfectly conducting sheet. In K'an infinitely long line source $S \equiv \{x' = a,\ y' = b,\ z' \in (-\infty, \infty)\}$ emits a cylindrical wave of ω'. In the problem to be addressed in this work the line source S (and K') makes a *uniform rectilinear* motion in vacuum with respect to the half-plane P (and K) (see Fig. 1). Then the problem consists of the investigation of the scattered wave to be observed in K. To this end we first use the well-known Lorentz transformation formulas to transform the expressions of the charge and current densities given in K' into K, and then consider and solve the problem in K.

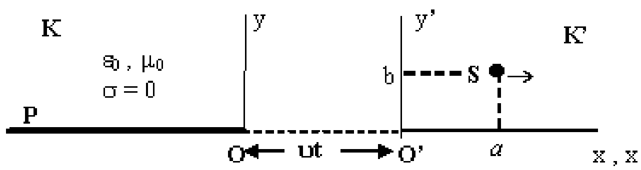


Figure 1. A source moving with velocity $\boldsymbol{v} = v\boldsymbol{e}_x$.

Let the clocks in K and K' are so adjusted that $O \equiv O'$ when $t = t' = 0$. Here t and t' show the times measured in K and K', respectively. We assume that the incident wave is generated by the current and charge densities given as follows:

$$\rho' = 0, \tag{1a}$$

$$\vec{J'} = I\cos(\omega' t' - \varphi)\delta(x' - a)\delta(y' - b)\vec{e}_z. \tag{1b}$$

Here $\vec{e}_z$ shows the unit co-ordinate vector in the direction of $O'z'$-axis while $\delta(.)$ is the usual Dirac distribution. In accordance with the Special Theory of Relativity one writes then (see [27] or [28])

$$x' = \gamma(x - vt), \quad y' = y, \quad z' = z, \quad t' = \gamma(t - vx/c^2) \tag{2}$$

$$\vec{J} = \vec{J'} - [1 - \gamma](\vec{J'} \cdot \vec{v})\frac{\vec{v}}{v^2} + \gamma\rho'\vec{v} \tag{3a}$$

and

$$\rho = \gamma(\rho' + \vec{J'} \cdot \vec{v}/c^2). \tag{3b}$$

Here γ stands for

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \tag{4}$$

From (1a)–(3b) one gets

$$\rho = 0, \tag{5a}$$

$$\vec{J} = I\cos(\omega t - \omega v x/c^2 - \varphi)\delta(x\gamma - a - \gamma v t)\delta(y - b)\vec{e}_z. \tag{5b}$$

In the expressions given above $\vec{e}_x$ is the unit co-ordinate vector parallel to Ox-axis, $\vec{v} = v\vec{e}_x$ is the velocity of the source S, c is the velocity of the wave in the free-space while ω denotes

$$\omega = \gamma\omega'. \tag{5c}$$

From the expressions of the source densities given by (5a), (5b) one concludes that the electric fields of both the incident and the scattered waves are parallel to the edge. If the total electric field is written as

$$\vec{E}(x, y, t) = u(x, y, t)\vec{e}_z, \tag{6}$$

then the function $u(x, y, t)$ satisfies the non-homogeneous wave equation

$$\begin{aligned} \Delta u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} &= \mu\frac{\partial(\vec{J}\cdot\vec{e}_z)}{\partial t} \\ &= -\mu I[\omega\sin(\omega t - \omega v x/c^2 - \varphi)\delta(x\gamma - a - \gamma v t) \\ &\quad +\gamma v\cos(\omega t - \omega v x/c^2 - \varphi)\delta'(x\gamma - a - \gamma v t)]\delta(y - b) \end{aligned} \tag{7a}$$

in the sense of distribution outside the half-plane P under the continuity, boundary and radiation conditions given as follows:

$$u(x, +0, t) = u(x, -0, t); \quad x,\ t \in (-\infty, \infty) \tag{7b}$$

$$\frac{\partial}{\partial y}u(x, +0, t) = \frac{\partial}{\partial y}u(x, -0, t); \quad x > 0,\ t \in (-\infty, \infty) \tag{7c}$$

$$u(x, +0, t) = u(x, -0, t) = 0; \quad x < 0,\ t \in (-\infty, \infty) \tag{7d}$$

$$u(x, y, t) \sim \text{outgoing wave for } r = \sqrt{x^2 + y^2} \to \infty, t \in (-\infty, \infty). \tag{7e}$$

In (7a) δ' denotes the derivative of delta Dirac function with respect to its argument.

3. SOLUTION OF THE PROBLEM

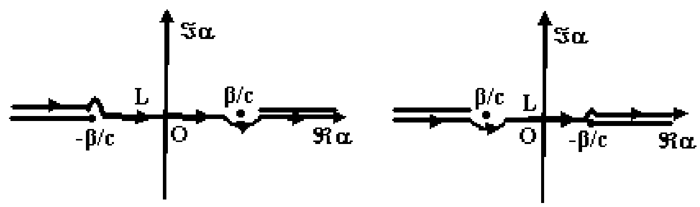
Let the double Fourier transform of the function $u(x, y, t)$ with respect to x and t, say $\hat{u}(\alpha, y, \beta)$, be defined via

$$\hat{u}(\alpha, y, \beta) = \int_{x=-\infty}^{\infty}\int_{t=-\infty}^{\infty} u(x, y, t)e^{i\beta t}e^{i\alpha x}dtdx, \quad \beta \in (-\infty, \infty),\ \alpha \in L. \tag{8a}$$

The inverse transform is then

$$u(x,y,t) = \frac{1}{(2\pi)^2} \int_{\beta=-\infty}^{\infty} \int_{L} \hat{u}(\alpha,y,\beta) e^{-i\alpha x} e^{-i\beta t} d\alpha d\beta. \tag{8b}$$

Here L denotes the real axis indented above $(-\beta/c)$ and below (β/c) as shown in Fig. 2.



(a) The case when $\beta > 0$ (b) The case when $\beta < 0$

Figure 2. The integration line L in the complex α-plane.

The application of the double transformation (8a) to (7a) yields

$$\frac{d^2\hat{u}}{dy^2} - \lambda^2(\alpha,\beta)\hat{u} = K(\alpha,\beta)\delta(y-b), \quad \beta \in (-\infty,\infty),\ \alpha \in L, \tag{9}$$

where $\lambda(\alpha,\beta)$ stands for the square-root function

$$\lambda(\alpha,\beta) = \sqrt{\alpha^2 - \beta^2/c^2} \tag{10}$$

defined in the complex α-plane cut as shown in Fig. 2 with the condition $\lambda(0,\beta) = -i\beta/c$ while $K(\alpha,\beta)$ is given by

$$\begin{aligned} K(\alpha,\beta) \ = \ & \frac{i\pi\mu I}{\gamma} e^{i\alpha a/\gamma} \left[\left(\frac{\omega}{\gamma^2} + \alpha v\right) e^{-i\left(\frac{\omega v a}{\gamma c^2} + \varphi\right)} \delta\left(\beta + \frac{\omega}{\gamma^2} + \alpha v\right) \right. \\ & \left. - \left(\frac{\omega}{\gamma^2} - \alpha v\right) e^{i\left(\frac{\omega v a}{\gamma c^2} + \varphi\right)} \delta\left(\beta - \frac{\omega}{\gamma^2} + \alpha v\right) \right]. \end{aligned} \tag{11}$$

The solution of (9), which satisfies the radiation condition (7e) is given by

$$\hat{u}(\alpha,y,\beta) = \begin{cases} A(\alpha,\beta) e^{-\lambda(\alpha,\beta)y} & ; \quad y > b \\ B(\alpha,\beta) e^{-\lambda(\alpha,\beta)y} + C(\alpha,\beta) e^{\lambda(\alpha,\beta)y} & ; \quad 0 < y < b \\ D(\alpha,\beta) e^{\lambda(\alpha,\beta)y} & ; \quad y < 0. \end{cases} \tag{12}$$

The expressions of the coefficients $A(\alpha,\beta), B(\alpha,\beta), C(\alpha,\beta)$ and $D(\alpha,\beta)$ can be obtained through the relations required by (9) and (7b)–(7d), which yield

$$A(\alpha,\beta)e^{-\lambda(\alpha,\beta)b} - B(\alpha,\beta)e^{-\lambda(\alpha,\beta)b} - C(\alpha,\beta)e^{\lambda(\alpha,\beta)b} = 0 \tag{13a}$$

$$A(\alpha,\beta)e^{-\lambda(\alpha,\beta)b} - B(\alpha,\beta)e^{-\lambda(\alpha,\beta)b} + C(\alpha,\beta)e^{\lambda(\alpha,\beta)b} = -\frac{K(\alpha,\beta)}{\lambda(\alpha,\beta)} \tag{13b}$$

$$B(\alpha,\beta) + C(\alpha,\beta) - D(\alpha,\beta) = 0 \tag{13c}$$

$$D(\alpha,\beta) = \Phi^+(\alpha,\beta) \tag{13d}$$

$$B(\alpha,\beta) - C(\alpha,\beta) + D(\alpha,\beta) = -\frac{\Phi^-(\alpha,\beta)}{\lambda(\alpha,\beta)}. \tag{13e}$$

The functions $\Phi^+(\alpha,\beta)$ and $\Phi^-(\alpha,\beta)$ appearing in (13d)–(13e) are yet unknown and defined by

$$\Phi^+(\alpha,\beta) = \int_0^\infty \left(\int_{-\infty}^{\infty} u(x,0,t)e^{i\beta t}dt \right) e^{i\alpha x}dx \tag{14a}$$

and

$$\Phi^-(\alpha,\beta) = \int_{-\infty}^{0} \left(\int_{-\infty}^{\infty} \left[\frac{\partial}{\partial y}u(x,+0,t) - \frac{\partial}{\partial y}u(x,-0,t) \right] e^{i\beta t}dt \right) e^{i\alpha x}dx. \tag{14b}$$

From (14a)–(14b) it is obvious that the functions $\Phi^+(\alpha,\beta)$ and $\Phi^-(\alpha,\beta)$ are regular in the upper half-plane $\Im\alpha > 0$ and in the lower half-plane $\Im\alpha < 0$, respectively. Moreover, since $u(x,0,t) = O(1)$ when $x \to +0$ while $\frac{\partial}{\partial y}u(x,\pm 0,t) = O(x^{-1/2})$ when $x \to -0$ [29], the functions $\Phi^\pm(\alpha,\beta)$ have also the following asymptotic behaviors when $\alpha \to \infty$ [30]:

$$\Phi^+(\alpha,\beta) = O(1/\alpha) \quad \text{as} \quad \alpha \to \infty \quad \text{in } \Im\alpha > 0 \tag{15a}$$

$$\Phi^-(\alpha,\beta) = O(\alpha^{-1/2}) \quad \text{as} \quad \alpha \to \infty \quad \text{in } \Im\alpha < 0. \tag{15b}$$

From (13a)–(13b) one gets

$$C(\alpha,\beta) = -\frac{K(\alpha,\beta)}{2\lambda(\alpha,\beta)}e^{-\lambda(\alpha,\beta)b} \tag{16a}$$

and

$$A(\alpha,\beta) = B(\alpha,\beta) - \frac{K(\alpha,\beta)}{2\lambda(\alpha,\beta)}e^{\lambda(\alpha,\beta)b}. \tag{16b}$$

This shows that the coefficient $C(\alpha,\beta)$ is known beforehand and $A(\alpha,\beta)$ is obtained in terms of $B(\alpha,\beta)$ which can be expressed by the use of (13c)–(13e) as

$$B(\alpha,\beta) = \Phi^+(\alpha,\beta) + \frac{K(\alpha,\beta)}{2\lambda(\alpha,\beta)} e^{-\lambda(\alpha,\beta)b} \tag{17a}$$

and

$$B(\alpha,\beta) = -\Phi^+(\alpha,\beta) - \frac{\Phi^-(\alpha,\beta)}{\lambda(\alpha,\beta)} - \frac{K(\alpha,\beta)}{2\lambda(\alpha,\beta)} e^{-\lambda(\alpha,\beta)b}. \tag{17b}$$

To determine $\Phi^+(\alpha,\beta)$ and $\Phi^-(\alpha,\beta)$ appearing in (17a)–(17b) one has to eliminate $B(\alpha,\beta)$ through these equations. Thus, one gets a functional equation of the Wiener-Hopf type satisfied by $\Phi^+(\alpha,\beta)$ and $\Phi^-(\alpha,\beta)$, namely:

$$\frac{\Phi^-(\alpha,\beta)}{\lambda(\alpha,\beta)} + 2\Phi^+(\alpha,\beta) = -\frac{K(\alpha,\beta)}{\lambda(\alpha,\beta)} e^{-\lambda(\alpha,\beta)b}, \quad \alpha \in L. \tag{18}$$

By substituting the expression of $\lambda(\alpha,\beta)$ and $K(\alpha,\beta)$ given by (10) and (11) into (18) and multiplying both sides of this equation by $\sqrt{\alpha+\beta/c}$ one gets

$$\frac{\Phi^-(\alpha,\beta)}{\sqrt{\alpha-\beta/c}} + 2\Phi^+(\alpha,\beta)\sqrt{\alpha+\beta/c} = f_1(\beta)\delta(\alpha-\beta_1) + f_2(\beta)\delta(\alpha-\beta_2) \tag{19}$$

where β_1 and β_2 are given by

$$\beta_1 = -\frac{1}{v}(\beta+\omega/\gamma^2), \tag{20a}$$

$$\beta_2 = -\frac{1}{v}(\beta-\omega/\gamma^2). \tag{20b}$$

In (19) the functions $f_1(\beta)$ and $f_2(\beta)$ refer to

$$f_1(\beta) = \frac{i\pi\mu I}{\gamma|v|} e^{-i\left(\frac{\omega va}{\gamma c^2}+\varphi\right)} \frac{\beta e^{-\lambda_1(\beta)b} e^{ia\beta_1/\gamma}}{\sqrt{\beta_1-\beta/c}} \tag{21a}$$

and

$$f_2(\beta) = \frac{i\pi\mu I}{\gamma|v|} e^{i\left(\frac{\omega va}{\gamma c^2}+\varphi\right)} \frac{\beta e^{-\lambda_2(\beta)b} e^{ia\beta_2/\gamma}}{\sqrt{\beta_2-\beta/c}} \tag{21b}$$

with

$$\lambda_1(\beta) = \lambda(\beta_1,\beta) = \sqrt{\left(\frac{\beta}{v}+\frac{\omega}{v\gamma^2}\right)^2 - \frac{\beta^2}{c^2}} \tag{22a}$$

and

$$\lambda_2(\beta) = \lambda(\beta_2, \beta) = \sqrt{\left(\frac{\beta}{v} - \frac{\omega}{v\gamma^2}\right)^2 - \frac{\beta^2}{c^2}}. \tag{22b}$$

Here it is worthwhile to remark that in order to obtain the expressions given by (20a), (20b) and (21a), (21b), the following identities related to Dirac distribution have also been considered:

$$\delta(v\alpha + \zeta) = \frac{1}{|v|}\delta\left(\alpha + \frac{\zeta}{v}\right), \tag{23a}$$

$$f(\alpha)\delta(\alpha - \zeta) = f(\zeta)\delta(\alpha - \zeta). \tag{23b}$$

It is obvious that the expression given by (19) constitutes a Wiener-Hopf equation which involves Dirac distributions. To solve this kind of an equation with respect to the parameter α one can use the well-known Plemelj-Sokhotski formulas [31] which yields

$$\delta(\alpha) = \frac{1}{2\pi i}\left[\frac{1}{\alpha - i0} - \frac{1}{\alpha + i0}\right] = \frac{1}{2\pi i}\lim_{\varepsilon\to+0}\left[\frac{1}{\alpha - i\varepsilon} - \frac{1}{\alpha + i\varepsilon}\right]. \tag{24}$$

With the use of (24), the functional equation (19) can be rewritten as

$$\frac{\Phi^-(\alpha,\beta)}{\sqrt{\alpha - \beta/c}} + 2\Phi^+(\alpha,\beta)\sqrt{\alpha + \beta/c} = f^+(\alpha) + f^-(\alpha), \tag{25a}$$

where the functions $f^+(\alpha)$ and $f^-(\alpha)$ which are regular in the upper half-plane $\Im\alpha > 0$ and in the lower half-plane $\Im\alpha < 0$, respectively, are defined by

$$f^+(\alpha) = -\frac{1}{2\pi i}\lim_{\varepsilon\to+0}\left[\frac{f_1(\beta)}{\alpha - \beta_1 + i\varepsilon} + \frac{f_2(\beta)}{\alpha - \beta_2 + i\varepsilon}\right] \tag{25b}$$

and

$$f^-(\alpha) = \frac{1}{2\pi i}\lim_{\varepsilon\to+0}\left[\frac{f_1(\beta)}{\alpha - \beta_1 - i\varepsilon} + \frac{f_2(\beta)}{\alpha - \beta_2 - i\varepsilon}\right]. \tag{25c}$$

Thus functions $\Phi^+(\alpha,\beta)$ and $\Phi^-(\alpha,\beta)$ are determined immediately through the classical Wiener-Hopf technique [30], namely:

$$\Phi^+(\alpha,\beta) = \frac{i}{4\pi\sqrt{\alpha + \beta/c}}\left[\frac{f_1(\beta)}{\alpha - \beta_1 + i0} + \frac{f_2(\beta)}{\alpha - \beta_2 + i0}\right] \tag{26a}$$

and

$$\Phi^-(\alpha,\beta) = \frac{\sqrt{\alpha - \beta/c}}{2\pi i}\left[\frac{f_1(\beta)}{\alpha - \beta_1 - i0} + \frac{f_2(\beta)}{\alpha - \beta_2 - i0}\right]. \tag{26b}$$

Finally, by substituting (26a), (26b) into (13d) and (17a), (17b), and considering also (16a,b), all the spectral coefficients taking place in (12) become determined.

4. ANALYSIS OF THE FIELD

By inserting first the expressions for the coefficients $A(\alpha,\beta)$, $B(\alpha,\beta)$, $C(\alpha,\beta)$ and $D(\alpha,\beta)$, which were obtained in the previous section, into (12) and then considering (8b) one obtains the expression of the total field in the whole space. In what follows we will consider the regions $y > b$, $y \in (0, b)$ and $y < 0$ separately.

4.1. Waves in the Region $y > b$

By inserting (16b), (17a) and (12) in (8b) one gets the total field in the region $y > b$ as follows:

$$u(x,y,t) = u^{inc}(x,y,t) + u^{ref}(x,y,t) + u^{s}(x,y,t). \tag{27a}$$

Here we put

$$u^{inc}(x,y,t) = -\frac{1}{8\pi^2}\int_{\beta=-\infty}^{\infty}\int_{L}\frac{K(\alpha,\beta)}{\lambda(\alpha,\beta)}e^{-\lambda(\alpha,\beta)(y-b)-i\alpha x-i\beta t}d\alpha d\beta \tag{27b}$$

$$u^{ref}(x,y,t) = \frac{1}{8\pi^2}\int_{\beta=-\infty}^{\infty}\int_{L}\frac{K(\alpha,\beta)}{\lambda(\alpha,\beta)}e^{-\lambda(\alpha,\beta)(y+b)-i\alpha x-i\beta t}d\alpha d\beta \tag{27c}$$

and

$$u^{s}(x,y,t) = \frac{1}{4\pi^2}\int_{\beta=-\infty}^{\infty}\int_{L}\Phi^{+}(\alpha,\beta)e^{-\lambda(\alpha,\beta)y-i\alpha x-i\beta t}d\alpha d\beta. \tag{27d}$$

If one inserts the expression of $K(\alpha,\beta)$ given by (11) into the integrand of (27b) and evaluates the resulting integral on L, then one gets

$$\begin{aligned}u^{inc}(x,y,t) =& \frac{i\mu I}{8\pi\gamma|v|}e^{-i\left(\frac{\omega v a}{\gamma c^2}+\varphi\right)}\int_{\beta=-\infty}^{\infty}\frac{\beta e^{-\lambda_1(\beta)(y-b)+i\left(\frac{\beta}{v}+\frac{\omega}{v\gamma^2}\right)\left(x-\frac{a}{\gamma}\right)-i\beta t}}{\lambda_1(\beta)}d\beta\\ &+\frac{i\mu I}{8\pi\gamma|v|}e^{i\left(\frac{\omega v a}{\gamma c^2}+\varphi\right)}\int_{\beta=-\infty}^{\infty}\frac{\beta e^{-\lambda_2(\beta)(y-b)+i\left(\frac{\beta}{v}-\frac{\omega}{v\gamma^2}\right)\left(x-\frac{a}{\gamma}\right)-i\beta t}}{\lambda_2(\beta)}d\beta.\end{aligned} \tag{28}$$

One can easily check that $u^{inc}(x,y,t)$ given by (28) is nothing but the solution to the equation (9) in the homogeneous space not involving the half-plane P. Therefore it is the wave emitted by the moving line source i.e., "the incident wave".

By repeating the same procedure which transformed (27b) to (28) for $u^{ref}(x,y,t)$ given by (27c) one obtains also

$$u^{ref}(x,y,t) = -\frac{i\mu I}{8\pi\gamma|v|}e^{-i\left(\frac{\omega va}{\gamma c^2}+\varphi\right)}\int_{\beta=-\infty}^{\infty}\frac{\beta e^{-\lambda_1(\beta)(y+b)+i\left(\frac{\beta}{v}+\frac{\omega}{v\gamma^2}\right)\left(x-\frac{a}{\gamma}\right)-i\beta t}}{\lambda_1(\beta)}d\beta$$
$$-\frac{i\mu I}{8\pi\gamma|v|}e^{i\left(\frac{\omega va}{\gamma c^2}+\varphi\right)}\int_{\beta=-\infty}^{\infty}\frac{\beta e^{-\lambda_2(\beta)(y+b)+i\left(\frac{\beta}{v}-\frac{\omega}{v\gamma^2}\right)\left(x-\frac{a}{\gamma}\right)-i\beta t}}{\lambda_2(\beta)}d\beta. \tag{29}$$

A comparison of (29) with (28) shows that $u^{ref}(x,y,t)$ is obtained from $u^{inc}(x,y,t)$ by making the substitution $I \to -I$ and $b \to -b$ in (28). Therefore $u^{ref}(x,y,t)$ can be interpreted as the field which is created by the image source. In other words, $u^{ref}(x,y,t)$ consists of the wave reflected from the conducting plane P. It is also important to emphasize that in some region of the space this reflected wave will be cancelled by a pole contribution which will appear during the evaluation of the integral in (27d).

As to the last term in (27a), from (27d) and (26a) one writes it as

$$u^s(x,y,t) = \frac{i}{16\pi^3}\int_{\beta=-\infty}^{\infty}\int_L\left[\frac{f_1(\beta)}{\alpha-\beta_1+i0}+\frac{f_2(\beta)}{\alpha-\beta_2+i0}\right]\frac{e^{-\lambda y-i\alpha x-i\beta t}}{\sqrt{\alpha+\beta/c}}d\alpha d\beta. \tag{30}$$

To recast the latter into more propitious form, we make the substitutions

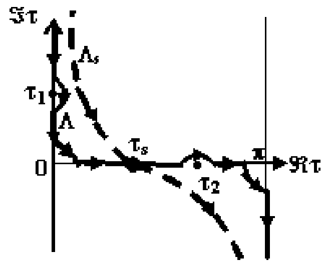
$$\alpha = -\frac{\beta}{c}\cos\tau, \quad \lambda(\alpha,\beta) = -i\frac{\beta}{c}\sin\tau, \quad d\alpha = \frac{\beta}{c}\sin\tau d\tau \tag{31a}$$

and

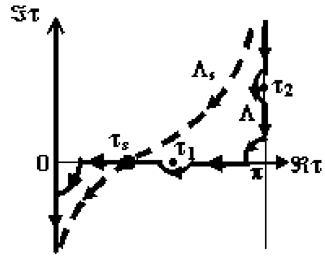
$$x = R\cos\phi, \quad y = R\sin\phi; \quad R = \sqrt{x^2+y^2}, \quad \phi \in (0,\pi) \tag{31b}$$

which map the integration line L onto Λ shown in Fig. 3. Thus we have

$$u^s(x,y,t) = \frac{\sqrt{c}}{16i\pi^3}\int_{\beta=-\infty}^{\infty}\beta^{-1/2}f_1(\beta)e^{-i\beta t}$$



(a) The case when $\beta > 0$



(b) The case when $\beta < 0$

Figure 3. The integration line Λ and the steepest descent line Λ_s in the complex τ-plane.

$$\times \left[\int_{\Lambda} \frac{\sin \tau e^{i \frac{\beta}{c} R \cos(\tau - \phi)}}{(\cos \tau + \tilde{\beta}_1)\sqrt{1 - \cos \tau}} d\tau \right] d\beta$$

$$+ \frac{\sqrt{c}}{16 i \pi^3} \int_{\beta=-\infty}^{\infty} \beta^{-1/2} f_2(\beta) e^{-i\beta t}$$

$$\times \left[\int_{\Lambda} \frac{\sin \tau e^{i \frac{\beta}{c} R \cos(\tau - \phi)}}{(\cos \tau + \tilde{\beta}_2)\sqrt{1 - \cos \tau}} d\tau \right] d\beta \qquad (32a)$$

with

$$\tilde{\beta}_1 = \frac{c}{\beta} \beta_1, \qquad \tilde{\beta}_2 = \frac{c}{\beta} \beta_2. \qquad (32b)$$

Now let us use the Cauchy theorem and translate Λ onto the steepest-descent line Λ_s which passes through the saddle point $\tau_s = \phi$. When $\beta > 0$, one has $\tau_s = \phi < \tau_2 = \arccos(-\tilde{\beta}_2)$ and crosses the pole at $\tau_2 = \arccos(-\tilde{\beta}_2)$ in translating Λ onto Λ_s (see Fig. 3(a)). Conversely, when $\beta < 0$, one has $\tau_s = \phi < \tau_1 = \arccos(-\tilde{\beta}_1)$ and the pole at $\tau_1 = \arccos(-\tilde{\beta}_1)$ is crossed in translating Λ onto Λ_s (see Fig. 3(b)) when $\tau_s = \phi > \tau_1 = \arccos(-\tilde{\beta}_1)$ (see Fig. 3(b)). Thus, by taking also into account the contributions which will come from the poles, (32a) can be rearranged as follows:

$$u^s(x, y, t) = u^p(x, y, t) + u^d(x, y, t) \qquad (33)$$

with

$$u^p(x,y,t)=$$
$$\frac{1}{8\pi^2}\int\limits_{\beta=-\infty}^{\infty}\left[f_1(\beta)\frac{e^{-\lambda_1(\beta)y-i\beta_1x-i\beta t}}{\sqrt{\beta_1+\beta/c}}+f_2(\beta)\frac{e^{-\lambda_2(\beta)y-i\beta_2x-i\beta t}}{\sqrt{\beta_2+\beta/c}}\right]d\beta \quad (34a)$$

and

$$u^d(x,y,t)=$$
$$\frac{\sqrt{c}}{16i\pi^3}\int\limits_{\beta=-\infty}^{\infty}\beta^{-1/2}f_1(\beta)e^{-i\beta t}\left[\int\limits_{\Lambda_s}\frac{\sin\tau e^{i\frac{\beta}{c}R\cos(\tau-\phi)}}{(\cos\tau+\tilde{\beta}_1)\sqrt{1-\cos\tau}}d\tau\right]d\beta$$
$$+\frac{\sqrt{c}}{16i\pi^3}\int\limits_{\beta=-\infty}^{\infty}\beta^{-1/2}f_2(\beta)e^{-i\beta t}\left[\int\limits_{\Lambda_s}\frac{\sin\tau e^{i\frac{\beta}{c}R\cos(\tau-\phi)}}{(\cos\tau+\tilde{\beta}_2)\sqrt{1-\cos\tau}}d\tau\right]d\beta. \quad (34b)$$

$u^p(x,y,t)$ consists of the pole contributions. By using (21a), (21b) in (34a) one can easily show that

$$u^p(x,y,t)=-u^{ref}(x,y,t) \quad (35)$$

in the region defined by $\tau_1<\phi<\tau_2$.

As to the last term $u^d(x,y,t)$ in (33), it gives the edge-excited diffracted wave. The integrals in (34b), written on Λ_s and $\beta\in(-\infty,\infty)$, can be evaluated asymptotically by saddle-point technique [30] for points far away from the edge. The saddle points connected with the parameters τ and β are

$$\tau_s=\phi \quad (36)$$

and

$$\beta_s=\omega\left[1-\frac{v}{c}\frac{\chi(x,y,t)}{\sqrt{b^2/\gamma^2+\chi^2(x,y,t)}}\right], \quad (37)$$

respectively. Here $\chi(x,y,t)$ is given by

$$\chi(x,y,t)=v(t-R/c)+a/\gamma. \quad (38)$$

Thus we get finally

$$u^d(x,y,t)\ \sim\ \frac{\mu Ib\sqrt{c^2-vc}}{4\gamma\sqrt{\pi}}\frac{\sin\phi}{\sqrt{1-\cos\phi}}\frac{1}{\sqrt{R}}\frac{[b^2/\gamma^2+\chi^2]^{-1/2}}{\left[\sqrt{b^2/\gamma^2+\chi^2}-\chi\right]^{1/2}}$$

$$\times \frac{\left[\sqrt{b^2/\gamma^2+\chi^2}-(v/c)\chi\right]}{\left[(v/c-\cos\phi)\sqrt{b^2/\gamma^2+\chi^2}-(1-\cos\phi v/c)\chi\right]}$$
$$\times\Re\left\{F\left[\frac{k\beta_2}{2\omega}R\left(\phi-\arccos(c/v-c\omega/\beta_2\gamma^2)\right)^2\right]\right.$$
$$\left.\times\, e^{i\left(3\pi/4+\varphi+\omega R/c+\omega\sqrt{b^2/\gamma^2+\chi^2}/c-\omega t\right)}\right\}, \tag{39a}$$

where $F(\eta)$ is the following function which can be expressed through the classical error function [32]:

$$F(\eta)=\sqrt{\eta}e^{-i\eta}erfc\left(e^{-i\pi/4}\sqrt{\eta}\right). \tag{39b}$$

4.2. Waves in the Region $0<y<b$

By replacing first $C(\alpha)$ and $B(\alpha)$ in (12) by their expressions given by (16a) and (17a) and then inserting the result into (8b) one gets the expression of the field in the region $0<y<b$. A scrutinization shows that the terms are identical to the terms obtained in the previous Section 4.1. Therefore the wave in the region $y\in(0,b)$ consists of the analytical continuation of that in $y>b$.

4.3. Waves in the Region $y<0$

By considering (13d) and (26a) in (12), the total field in the region can be written from (8b) as follow:

$$u(x,y,t)=\frac{i}{16\pi^3}\int\limits_{\beta=-\infty}^{\infty}\int\limits_{L}\left[\frac{f_1(\beta)}{\alpha-\beta_1+i0}+\frac{f_2(\beta)}{\alpha-\beta_2+i0}\right]\frac{e^{\lambda y-i\alpha x-\beta t}}{\sqrt{\alpha+\beta/c}}d\alpha d\beta. \tag{40}$$

By making the same substitutions given by (31a), (31b) for $\phi\in(-\pi,0)$ and following the same procedure, as explained in the Section 4.1., on the stage of translation the integration line Λ onto the steepest-descent line Λ_s which passes through the saddle point $\tau_s=-\phi$, the field given by (40) can be rearranged as

$$u^s(x,y,t)=u^p(x,y,t)+u^d(x,y,t) \tag{41}$$

where $u^p(x,y,t)$ is the expression of the field which is obtained from the residue contribution of the poles τ_1 and τ_2. The computation of these pole contributions shows that $u^p(x,y,t)$ is identical with the incident field $u^i(x,y,t)$ in the lit region defined by $-\tau_2<\phi<-\tau_1$. In (41) the term $u^d(x,y,t)$ is the diffracted field by the edge whose explicit expression is given by (37a) through the saddle-point technique.

5. AN ILLUSTRATIVE EXAMPLE

In order to give an idea about the variation of the edge-diffracted wave with respect to time normalized by $(\tilde{\lambda}/c)$, one computes the expression of $u^d(x, y, t)$ given by (37a) at the point $Q(-\tilde{\lambda}, 10\tilde{\lambda}, 0)$ for $v = 2c/3$, $I = 10\,\mathrm{mA}$ and $a = b = 100\tilde{\lambda}$. Here $\tilde{\lambda}$ and c stands for the wave-length of the monochromatic wave. Fig. 4. given below shows this variation in the time interval $t \in (0, 20\tilde{\lambda}/c)$.

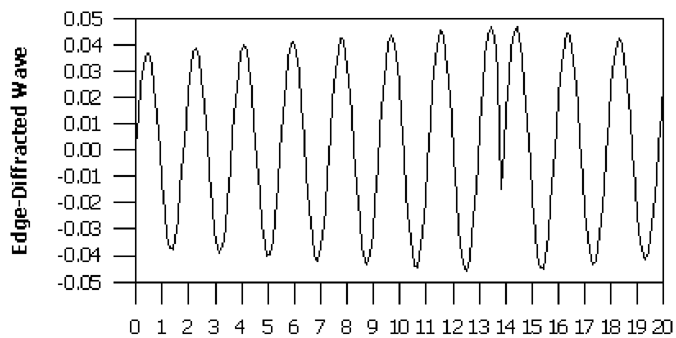


Figure 4. Variation of the edge diffracted wave with normalized time.

From this figure it is obvious to remark that the edge-excited wave has always the same oscillation whenever the source is approaching the observation point or going far away from it. The non-periodic variation of the wave is also observed.

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