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IMPEDANCE VIBRATOR WITH ARBITRARY POINT OF EXCITATION

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Abstract—The approximate analytical solution of the integral equation concerning the current in a thin straight vibrator with complex surface impedance has been obtained. The vibrator is located in unlimited space and is excited in an arbitrary point along its length. The calculations have been made and the plots of electrodynamic characteristics of the vibrator, depending of the value and the type of its surface impedance and the excitation point location, are represented. The comparative analysis between the calculated and the experimental data and also the results, obtained by the method of moments, are represented for perfectly conducting vibrators.

1. INTRODUCTION

At present thin vibrators have wide application as different resonant elements of antenna-waveguide devices. The variety of available vibrator structures and also creation of new constructions on their basis stipulate constant interest of investigators to the problems of their analysis and synthesis. So, for example, location of different active and passive elements in definite points along the length of a vibrator's antenna and also vibrator's excitation in a distinctive from its centre point create additional opportunities to form the set electrodynamic characteristics of vibrator radiators [1–5]. From our point of view, thin vibrators, on the surface of which impedance boundary conditions are performed, can serve these aims [6–21]. In the proposed paper the approximate analytical solution of the current integral equation in a thin impedance vibrator with the arbitrary point of excitation has been obtained. In order to check reliability of the obtained solution the comparison with the experimental values and calculated results, obtained by the method of moments, has been made. The results, showing the efficiency, possibilities and perspectives of the vibrator's use with the distributed surface impedance in practical application, are also represented.

2. PROBLEM FORMULATION AND INITIAL INTEGRAL EQUATIONS

Let us formulate a general problem of electromagnetic waves scattering (radiation) by finite-size conducting bodies. Let the field of the $\vec{E}_0(\vec{r})$ and $\vec{H}_0(\vec{r})$ impressed sources, dependent of the time t as $e^{i\omega t}$, influences the material body of the V volume, which is bounded by the S smooth closed surface and is characterized by the permittivity ε and the permeability $\mu = 1$, and the conductivity σ. This field can be set either as the wave field, falling on the obstacle (a scattering problem), or as the field of the impressed electromotive forces (EMF), applied to the body, different from null only in some part of the volume V (a radiation problem). It is necessary to find the scattering (radiation) field in any point of space, characterized by permittivity ε_1 and the permeability μ_1 (in a general case of a complex type) in the presence of the set interfaces of mediums.

The whole electromagnetic field $\vec{E}(\vec{r})$ and $\vec{H}(\vec{r})$, satisfying the Maxwell's equations and the boundary conditions on the body surface, is defined by integral equations of macroscopical electrodynamics [22]:

$$\begin{aligned} \vec{E}(\vec{r}) &= \vec{E}_0(\vec{r}) + (\operatorname{grad}\operatorname{div} + k^2\varepsilon_1\mu_1)\vec{\Pi}^e(\vec{r}), \\ \vec{H}(\vec{r}) &= \vec{H}_0(\vec{r}) + ik\varepsilon_1 \operatorname{rot}\vec{\Pi}^e(\vec{r}), \end{aligned} \tag{1}$$

in which $\vec{\Pi}^e(\vec{r})$ is the Hertz's electrical vector

$$\vec{\Pi}^e(\vec{r}) = \frac{1}{i\omega\varepsilon_1}\int\limits_V \hat{G}^e(\vec{r},\vec{r}\,')\vec{J}^e(\vec{r}\,')\mathrm{d}\vec{r}\,', \tag{2}$$

$\vec{J}^e(\vec{r})$ is the volume density of the electrical current, and $\hat{G}^e(\vec{r},\vec{r}\,')$ is the electrical tensor Green's function for the vector potential, satisfying the Helmholtz's equation

$$\Delta\hat{G}^e(\vec{r},\vec{r}\,') + k^2\varepsilon_1\mu_1\hat{G}^e(\vec{r},\vec{r}\,') = -4\pi\hat{I}\delta(\vec{r}-\vec{r}\,') \tag{3}$$

and corresponding conditions on the mediums interface, if there is a such one. Here $\hat{I}$ is the unit tensor, $\vec{r}$ is the radius-vector of the observation point, $\vec{r}\,'$ is the radius-vector of the source point, $\omega = 2\pi f$

is the circular frequency (f is the frequency, measured in Hertz), $k = 2\pi/\lambda$ (λ is the wavelength in free space), $\delta(\vec{r} - \vec{r}\,')$ is the Dirac's delta-function.

The induced current is generally concentrated near the body surface at a strong skin-effect, so it is expedient to transform the volume integral in (2) in the following way:

$$\vec{\Pi}^e(\vec{r}\,) = \frac{1}{4\pi ik\varepsilon_1}\int\limits_S \hat{G}^e(\vec{r}, \vec{r}\,')[\vec{n}, \vec{H}(\vec{r}\,')]\mathrm{d}\vec{r}\,'. \tag{4}$$

It turns out to be useful, when the field on the scattering obstacle is situated from some additional considerations. In fact, neglecting the skin-layer thickness, one can use the approximate Leontovich-Shchukin impedance boundary conditions [23]

$$[\vec{n}, \vec{E}(\vec{r}\,)] = \overline{Z}_S(\vec{r}\,)[\vec{n}, [\vec{n}, \vec{H}(\vec{r}\,)]], \tag{5}$$

in which $\vec{n}$ is the outward normal to the S surface, $\overline{Z}_S(\vec{r}\,) = \overline{R}_S(\vec{r}\,) + i\overline{X}_S(\vec{r}\,) = Z_S(\vec{r}\,)/Z_0$ is the normalized surface impedance ($Z_0 = 120\pi$ Ohm is the wave impedance of free space), which can vary on the body surface from one point to another in a general case. Thus, the scattering field in the whole environment is fully defined by setting tangential components of the field on the S boundary of the V volume.

Locating the observation point on the body surface, we obtain the following integral equation (the "e" index is omitted) due to (1)–(5):

$$\begin{aligned} Z_S(\vec{r}\,)\vec{J}(\vec{r}\,) \;=\; & \vec{E}_0(\vec{r}\,) + \frac{1}{i\omega\varepsilon_1}(\mathrm{grad}\,\mathrm{div} + k^2\varepsilon_1\mu_1)\int\limits_S \hat{G}(\vec{r}, \vec{r}\,')\vec{J}(\vec{r}\,')\mathrm{d}\vec{r}\,' \\ & + \frac{Z_S(\vec{r}\,)}{4\pi}\mathrm{rot}\int\limits_S \hat{G}(\vec{r}, \vec{r}\,')[\vec{n}, \vec{J}(\vec{r}\,')]\mathrm{d}\vec{r}\,' \end{aligned} \tag{6}$$

concerning the density of the surface electrical current

$$\vec{J}(\vec{r}\,) = \frac{1}{Z_0}[\vec{n}, \vec{H}(\vec{r}\,)]. \tag{7}$$

When solving the equation (6) directly, some known mathematical difficulties appear; however, it is considerably simplified for the conducting cylinders, the cross-section perimeter of which is small in comparison with their length and the wavelength in the environment (thin vibrators). Besides, one manages to spread the boundary condition of the kind (5) on the cylindrical surfaces with the arbitrary complex impedance, independent of the excitation field structure and the material electrophysical characteristics the vibrator is made of, in this case. The expressions to calculate different construction realizations of the surface impedance are given in [24].

Let us transform the integral equation (6) applicably to the thin vibrator, representing a limited circular cylindrical wire of the radius r and the length $2L$ (in a general case of the curvature axial configuration) for which the following ratios are:

$$\frac{r}{2L} \ll 1, \qquad \left|\frac{r}{\lambda_1}\right| \ll 1, \qquad \frac{r}{\tilde{R}} \ll 1, \tag{8}$$

where λ_1 is the wavelength in the environment, $\tilde{R}$ is the radius of the curvature of the vibrator's axial line.

These inequalities allow us to consider, that the induced current density has only a longitudinal component

$$\vec{J}(\vec{r}) = \vec{e}_s J(s)\psi(\rho, \varphi), \tag{9}$$

and it is distributed along the section as in a quasi-stationary case [1], and what is more,

$$\int\limits_{\perp} \psi(\rho, \varphi)\rho \mathrm{d}\rho \mathrm{d}\varphi = 1. \tag{10}$$

In the expressions (9), (10) $\vec{e}_s$ is the unit vector along tangent to the s axis, coupled with the vibrator, $\psi(\rho, \varphi)$ is the function of the transverse ($\perp$) polar coordinates ρ and φ, $J(s)$ is the unknown current, submitted to the boundary conditions on the vibrator ends:

$$J(-L) = J(L) = 0. \tag{11}$$

Taking all this into consideration and taking into account, that the Green's function of unlimited space $\hat{G}(\vec{r}, \vec{r}\,') = \hat{I}G(\vec{r}, \vec{r}\,')$, where

$$G(\vec{r}, \vec{r}\,') = \frac{e^{-ik\sqrt{\varepsilon_1\mu_1}|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}, \tag{12}$$

we obtain the equation concerning the current in the thin vibrator, located in homogeneous isotropic infinitely extended medium:

$$z_i(s)J(s) = E_{0s}(s) + \frac{1}{i\omega\varepsilon_1}\int\limits_{-L}^{L}\left[\frac{\partial}{\partial s}\frac{\partial J(s')}{\partial s'} + k^2\varepsilon_1\mu_1(\vec{e}_s\vec{e}_{s'})J(s')\right]G_s(s, s')\mathrm{d}s'. \tag{13}$$

Here $E_{0s}(s)$ is the impressed field projection, parallel to the vector $\vec{e}_s$, $z_i(s)$ is the internal impedance per unit length, $\vec{e}_{s'}$ is the ort of the s' axis, coupled with the vibrator surface,

$$G_s(s,s') = \int_{-\pi}^{\pi} \frac{e^{-ik\sqrt{\varepsilon_1\mu_1}\sqrt{(s-s')^2+[2r\sin(\varphi/2)]^2}}}{\sqrt{(s-s')^2+[2r\sin(\varphi/2)]^2}}\psi(r,\varphi)r\mathrm{d}\varphi. \qquad (14)$$

The integral equation solution (13) with the precise kernel in the form of (14) is connected with serious difficulties, so we use the thin-wire approximation [1] further

$$G_s(s,s') = \frac{e^{-ik\sqrt{\varepsilon_1\mu_1}R(s,s')}}{R(s,s')}, \qquad R(s,s') = \sqrt{(s-s')^2+r^2}. \qquad (15)$$

The $G_s(s,s')$ function everywhere is continuous in this case, and the equation for the current is considerably simplified without notable aggravation of preciseness [25]. Applying integration in parts, taking into consideration the condition (11) further in the equation (13), for the straight conductor $((\vec{e}_s\vec{e}_{s'})=1)$ we, finally, have:

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}s^2}+k^2\varepsilon_1\mu_1\right)\int_{-L}^{L} J(s')\frac{e^{-ik\sqrt{\varepsilon_1\mu_1}R(s,s')}}{R(s,s')}\mathrm{d}s' = -i\omega\varepsilon_1 E_{0s}(s) + i\omega\varepsilon_1 z_i(s)J(s). \qquad (16)$$

Thus, the scattering (radiation) problem of electromagnetic waves by thin impedance vibrators in homogeneous isotropic medium is formulated as rigorous boundary problem of macroscopical electrodynamics and is reduced to the current integral equations. Their solution is a main stage, because when the current is obtained, the fields of scattering or radiation are calculated due to (1), that does not meet any principle difficulties.

3. SOLUTION OF THE INTEGRAL EQUATION FOR THE CURRENT

The equation (16) has the following form for the vibrator with the constant along its length impedance ($z_i(s) = z_i$), located in the free space ($\varepsilon_1 = \mu_1 = 1$):

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}s^2}+k^2\right)\int_{-L}^{L} J(s')\frac{e^{-ikR(s,s')}}{R(s,s')}\mathrm{d}s' = -i\omega E_{0s}(s) + i\omega z_i J(s). \qquad (17)$$

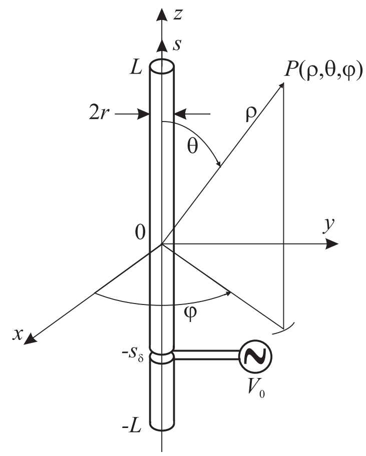


Figure 1. The problem geometry and the symbols used.

Let the vibrator is excited in the $s = -s_\delta$ point by the V_0 voltage generator (Figure 1). Then

$$\begin{aligned} E_{0s}(s) &= V_0\delta(s+s_\delta) = E^s_{0s}(s) + E^a_{0s}(s), \\ E^s_{0s}(s) &= \frac{V_0}{2}[\delta(s+s_\delta)+\delta(s-s_\delta)], \\ E^a_{0s}(s) &= \frac{V_0}{2}[\delta(s+s_\delta)-\delta(s-s_\delta)], \end{aligned} \tag{18}$$

where $E^s_{0s}(s)$ is the symmetrical ($E^s_{0s}(s) = E^s_{0s}(-s)$, the upper index "$s$"), and $E^a_{0s}(s)$ is the antisymmetrical ($E^a_{0s}(s) = -E^a_{0s}(-s)$, the upper index "$a$") relatively to the vibrator's centre components of the impressed field. At this, naturally, the vibrator current will also consist of two parts $J(s) = J^s(s) + J^a(s)$, and the equation (17) will transform into the system of two independent integral equations, concerning the unknown currents $J^s(s)$ and $J^a(s)$:

$$\left\{ \begin{aligned} &\left(\frac{d^2}{ds^2}+k^2\right)\int\limits_{-L}^{L} J^s(s')G_s(s,s')ds' = -i\omega E^s_{0s}(s) + i\omega z_i J^s(s), \\ &\left(\frac{d^2}{ds^2}+k^2\right)\int\limits_{-L}^{L} J^a(s')G_s(s,s')ds' = -i\omega E^a_{0s}(s) + i\omega z_i J^a(s). \end{aligned} \right. \tag{19}$$

Let us represent the vibrator currents in the form of the product of the $J^{s,a}_n$ unknown amplitudes on the given distribution functions

$f_n^{s,a}(s)\,(n=0,1)$

$$J^{s,a}(s) = J_0^{s,a} f_0^{s,a}(s) + J_1^{s,a} f_1^{s,a}(s), \quad f_n^{s,a}(\pm L) = 0, \tag{20}$$

and let us use "the induced electromotive forces method" (IEMFM) to solve the equations system (19), which is analogical to "the induced magnetomotive forces method", approbated by us to solve the problems of electromagnetic coupling of the electrodynamic volumes via narrow slots in their common walls [26]. Then the equations (19) are transformed into the system of algebraic equations of the fourth order:

$$\left\{\begin{array}{l} J_0^s\left(Z_{00}^s+\tilde{Z}_{00}^s\right)+J_1^s\left(Z_{01}^s+\tilde{Z}_{01}^s\right)=-(i\omega/2k)E_0^s, \\ J_0^s\left(Z_{10}^s+\tilde{Z}_{10}^s\right)+J_1^s\left(Z_{11}^s+\tilde{Z}_{11}^s\right)=-(i\omega/2k)E_1^s, \\ J_0^a\left(Z_{00}^a+\tilde{Z}_{00}^a\right)+J_1^a\left(Z_{01}^a+\tilde{Z}_{01}^a\right)=-(i\omega/2k)E_0^a, \\ J_0^a\left(Z_{10}^a+\tilde{Z}_{10}^a\right)+J_1^a\left(Z_{11}^a+\tilde{Z}_{11}^a\right)=-(i\omega/2k)E_1^a. \end{array}\right. \tag{21}$$

The following designations are accepted in (21) ($m=0,1;\ n=0,1$):

$$Z_{mn}^{s,a} = \frac{1}{2k}\left\{-\frac{\mathrm{d}f_m^{s,a}(s)}{\mathrm{d}s}A_n^{s,a}(s)\bigg|_{-L}^{L} + \int_{-L}^{L}\left[\frac{\mathrm{d}^2 f_m^{s,a}(s)}{\mathrm{d}s^2}+k^2 f_m^{s,a}(s)\right]A_n^{s,a}(s)\mathrm{d}s\right\},$$

$$\begin{aligned} A_n^{s,a}(s) &= \int_{-L}^{L} f_n^{s,a}(s')G_s(s,s')\mathrm{d}s', \\ \tilde{Z}_{mn}^{s,a} &= \frac{\overline{Z}_S}{ir}\int_{-L}^{L} f_m^{s,a}(s) f_n^{s,a}(s)\mathrm{d}s, \\ E_m^{s,a} &= \int_{-L}^{L} f_m^{s,a}(s)E_{0s}^{s,a}(s)\mathrm{d}s. \end{aligned} \tag{22}$$

When solving the equations system (21) and substituting the obtained values of $J_n^{s,a}$ in (20), we obtain the final expression of the vibrator current:

$$\begin{aligned} J^{s,a}(s) &= -\frac{i\omega}{2k}V_0\left[\frac{\tilde{E}_0^{s,a}Z_{11}^{(s,a)\Sigma}-\tilde{E}_1^{s,a}Z_{01}^{(s,a)\Sigma}}{\tilde{Z}_{00}^{(s,a)\Sigma}Z_{11}^{(s,a)\Sigma}-Z_{10}^{(s,a)\Sigma}Z_{01}^{(s,a)\Sigma}}f_0^{s,a}(s)\right. \\ &\qquad\left.+\frac{\tilde{E}_1^{s,a}Z_{00}^{(s,a)\Sigma}-\tilde{E}_0^{s,a}Z_{10}^{(s,a)\Sigma}}{\tilde{Z}_{00}^{(s,a)\Sigma}Z_{11}^{(s,a)\Sigma}-Z_{10}^{(s,a)\Sigma}Z_{01}^{(s,a)\Sigma}}f_1^{s,a}(s)\right], \\ J(s) &= J^s(s)+J^a(s), \qquad Z_{mn}^{(s,a)\Sigma}=Z_{mn}^{s,a}+\tilde{Z}_{mn}^{s,a}. \end{aligned} \tag{23}$$

It is natural to suppose, that the induced electromotive forces method gives more precise integral equation solution, if the approximating $f_n^{s,a}(s)$ current functions are chosen righter. Hence let us choose the functions, obtained in [15] at solution of the integral equation (17) for the current in the impedance vibrator by the asymptotic method of averaging as $f_0^{s,a}(s)$:

$$f_0^s(s) = \cos\tilde{k}s_\delta \sin\tilde{k}L \cos\tilde{k}s - (1/2)\cos\tilde{k}L(\sin\tilde{k}|s-s_\delta| + \sin|s+s_\delta|),$$
$$f_0^a(s) = \sin\tilde{k}s_\delta \cos\tilde{k}L \sin\tilde{k}s + (1/2)\sin\tilde{k}L(\sin\tilde{k}|s-s_\delta| - \sin|s+s_\delta|), \tag{24}$$

where $\tilde{k} = k - \frac{i\overline{Z}_S}{r\Omega}$, $\Omega = 2\ln(2L/r)$. Let us note, that $f_0^s(s) = \sin\tilde{k}(L-|s|)$, $f_0^a(s) = 0$ at the vibrator excitation in its centre ($s_\delta = 0$). We use the expressions, obtained from [2, 27] when investigating the integral equation (17) characteristics for the function $f_1^{s,a}(s)$ in the case, when $z_i = 0$:

$$\begin{aligned} f_1^s(s) &= \cos ks - \cos kL, \\ f_1^a(s) &= \sin ks - (s/L)\sin kL. \end{aligned} \tag{25}$$

Now substituting the expressions for the functions $f_n^{s,a}(s)$ into the formulas (22), we obtain all coefficients in the equations system (21), defining, in their turn, the current formulas (23). The expressions for $Z_{mn}^{s,a}$ and $\tilde{E}_{mn}^{s,a}$ are given in Appendix A.

Knowledge of real distribution of the $J(s)$ current permits us to calculate all electrodynamic characteristics of the impedance vibrator. So, the input impedance $Z_{in} = R_{in} + iX_{in}$ or the input admittance $Y_{in} = G_{in} + iB_{in}$ in the excitation point are defined by means of the following expressions:

$$Z_{in}[\text{Ohms}] = \frac{V_0}{J(s_\delta)} = \frac{60i}{J_0^s f_0^s(s_\delta) + J_1^s f_1^s(s_\delta) + J_0^a f_0^a(s_\delta) + J_1^a f_1^a(s_\delta)},$$
$$Y_{in}[\text{millimhos}] = \frac{10^3}{Z_{in}}. \tag{26}$$

Then the voltage standing wave ratio (VSWR) into the antenna feeder with the W wave impedance equals:

$$\text{VSWR} = \frac{1+|S_{11}|}{1-|S_{11}|}, \qquad S_{11} = \frac{Z_{in} - W}{Z_{in} + W}, \tag{27}$$

where S_{11} is the reflection coefficient in the feeder. And, finally, the radiation field of the vibrator in the far zone ($\rho \to \infty$, $\rho \gg 2L$) has the following form due to (1):

$$\begin{aligned} E_\theta(\rho,\theta) &= \frac{ik_1^2}{\omega\varepsilon_1}\sin\theta\frac{e^{-ik_1\rho}}{\rho}\int_{-L}^{L} J(s)e^{ik_1 s\cos\theta}\mathrm{d}s, \\ H_\varphi(\rho,\theta) &= \frac{ik_1 k}{\omega}\sin\theta\frac{e^{-ik_1\rho}}{\rho}\int_{-L}^{L} J(s)e^{ik_1 s\cos\theta}\mathrm{d}s, \end{aligned} \tag{28}$$

where $k_1 = k\sqrt{\varepsilon_1\mu_1}$, $(E_\theta/H_\varphi) = \sqrt{\mu_1/\varepsilon_1}$ is the characteristic impedance of the environment.

4. NUMERICAL RESULTS

In order to check reliability of the obtained approximate expression for the current (23) the dependencies of real G_{in} and imaginary B_{in} parts of the input admittance of the perfectly conducting vibrator, excited in the centre, from its electrical length (the solid curves in Figure 2) have been calculated. The experimental data from the monograph [27] (the circles, $f = 663\,\mathrm{MHz}$) and the calculated values (the dashed curves), obtained by the Galerkin's method at the current approximation by means of trigonometric functions of the whole region

$$J(s) = \sum_{n=1}^{N} J_n \sin\frac{n\pi(L+s)}{2L}, \tag{29}$$

are given here for comparison, what is more, to reach necessary accuracy the functions number in formula (29) are chosen to be equal $N = 24$. The comparison of calculation and the experimental curves between each other in Figure 2 allows us to make conclusion of adequacy of the current chosen approximating functions (24) and (25) to real physical process. It is also proved by the plots in Figure 3, where the dependencies of input admittance of perfectly conducting vibrator from $2L/\lambda$ ($\Omega = 2\ln(2L/r) = 10$) at different location of the excitation point are represented.

It is necessary, that the main lobe of the antenna radiation pattern (RP) deviates from the direction $\theta = 90°$ in series of important practical applications. Such a situation takes place, for example, when antennas base stations for cellular mobile communication [28] are exploited. In [4] it is proposed to use the vibrator with the length $2L = 0.8\lambda$ and with a shifted relatively to the vibrator's centre excitation point $s_\delta = -0.2\lambda$ (the direction of the main lobe of the antenna RP equals $\theta_{\max} = 57.7°$). Figure 4 gives the curves

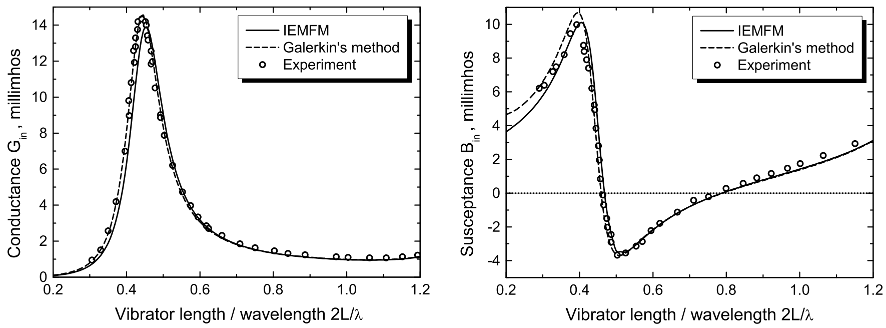


Figure 2. The input admittance of the perfectly conducting vibrator at $r/\lambda = 0.007022$.

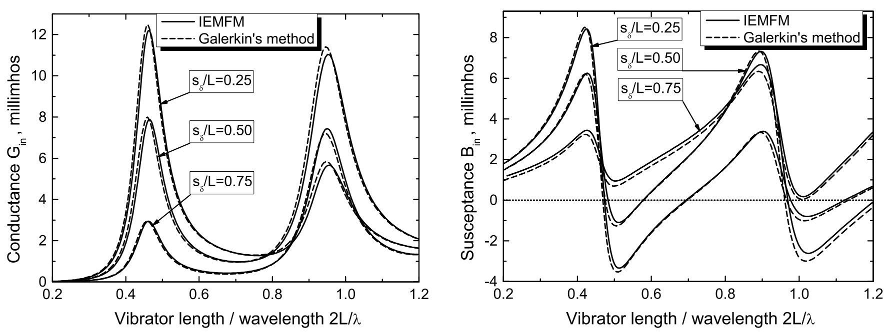


Figure 3. The input admittance of the perfectly conducting vibrator at $\Omega = 10$.

of the current amplitude-phase distribution $J(s) = |J(s)|e^{i \arg J(s)}$ along the perfectly conducting vibrator, calculated by the method of moments [4] (circles) and the formulas (23)–(25). We also show here, how vibrator's availability of the $\overline{R}_S$ active component of the surface impedance influences current distribution, and Figure 5 represents the corresponding field radiation pattern. As it is seen, $\overline{R}_S$ increases — the current amplitude decreases (at this the character of current phase change does not, practically, vary) and the side lobe level sufficiently decreases in the vibrator's RP.

In [4] it is suggested to use concentrated impedance reactive load, included into the vibrator between the points $s = s_\delta$ and $s = 0$ for the antenna's agreement with the feeding line. At this the minimal values of VSWR at the frequency of $f = 900\,\text{MHz}$ are: VSWR = 2.2 for $W = 50\,\text{Ohm}$, VSWR = 1.46 for $W = 75\,\text{Ohm}$. We propose to use non-concentrated but distributed along the vibrator surface impedance

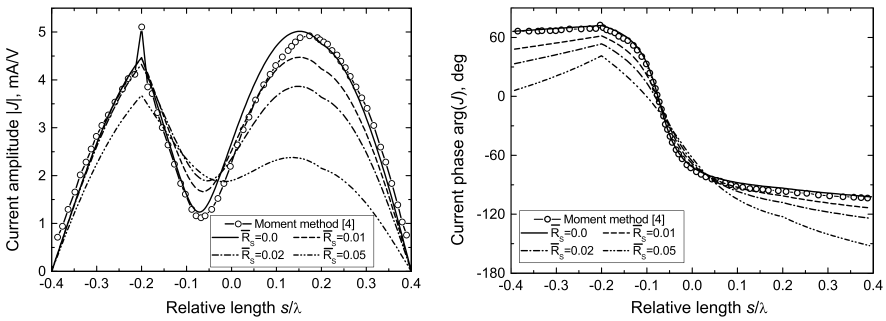


Figure 4. The amplitude-phase current distribution in the vibrator at $f = 900\,\text{MHz}$, $2L = 0.8\lambda$, $r = 0.005\lambda$, $s_\delta = -0.2\lambda$.

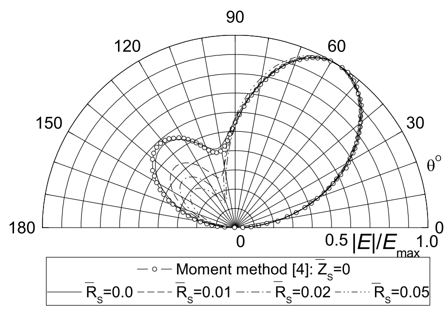


Figure 5. The vibrator's radiation pattern at $f = 900\,\text{MHz}$, $2L = 0.8\lambda$, $r = 0.005\lambda$, $s_\delta = -0.2\lambda$.

for the purposes of agreement. As it is seen from the plots in Figures 6, 7, a definite value of the surface impedance ($\overline{R}_S = 0.0$, $\overline{X}_S = 0.0475$) exists, at which $|S_{11}|$ and VSWR are minimal for different W and s_δ at the antenna's set parameters. The bandwidth on the level VSWR ≤ 2.0 for $W = 75\,\text{Ohm}$ and $s_\delta = -0.2\lambda$ is 27 MHz when using concentrated load $X_L = 203.8\,\text{Ohm}$, located in the point $s_L = -0.175\lambda$ [4]. Availability of vibrator's distributed impedance sufficiently widens the bandwidth (to 50 MHz for level VSWR ≤ 2.0 and 28 MHz for level VSWR ≤ 1.5), what is more, we can attain an acceptable agreement for different values of the W wave impedance of the feeding line by changing the excitation point position.

So, availability of the vibrator's distributed impedance of a definite kind (inductive) and the value $\overline{X}_S = 0.0475$ practically leads to the increase of its "efficient" electrical length (to $2L_{eff} \approx \lambda$) and, as a result, to tuning in the resonance ($B_{in} \approx 0$, Figure 3), the

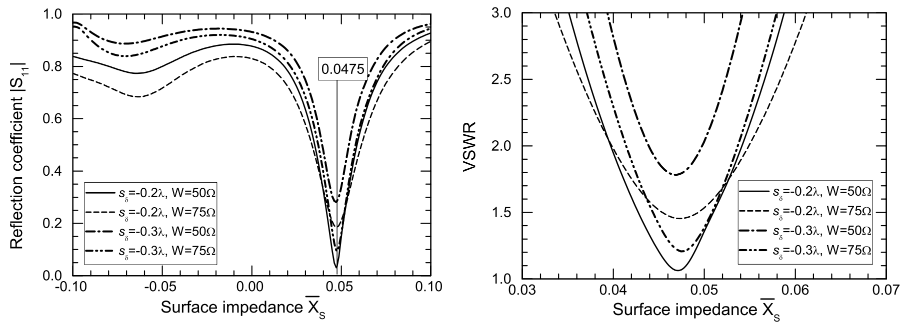


Figure 6. The magnitude dependences of the reflection coefficient $|S_{11}|$ and VSWR on the vibrator's surface impedance at $f = 900\,\text{MHz}$, $2L = 0.8\lambda$, $r = 0.005\lambda$, $\overline{R}_S = 0.0$.

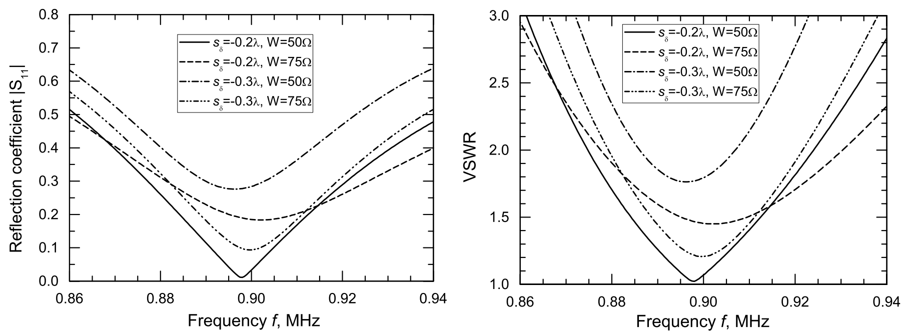
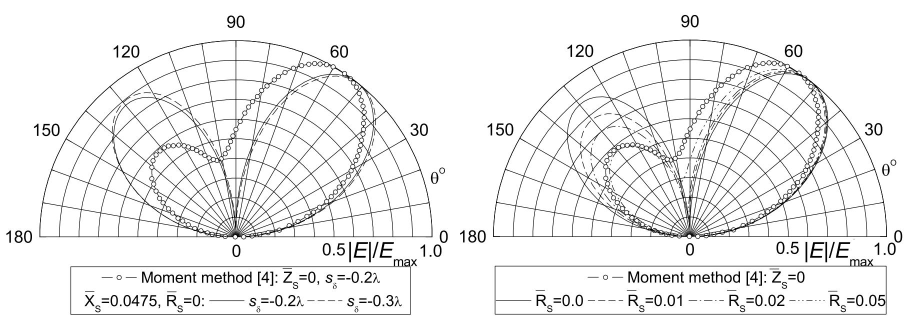


Figure 7. The magnitude dependences of the reflection coefficient $|S_{11}|$ and VSWR on the frequency at $2L = 0.8\lambda$, $r = 0.005\lambda$ and $\overline{X}_S = 0.0475$ on frequency $f = 900\,\text{MHz}$.



(a) (b)

Figure 8. The vibrator's radiation patterns at $f = 900\,\text{MHz}$, $2L = 0.8\lambda$, $r = 0.005\lambda$ and $\overline{X}_S = 0.0475$: (a) $\overline{R}_S = 0.0$; (b) $s_\delta = -0.2\lambda$.

field radiation pattern is also changed in comparison with the case $2L = 0.8\lambda$ (Figure 8a). However, the possibility of lowering of the pattern side lobe level by including the $\overline{R}_S$ active impedance (Figure 8b) exists, as in the case, represented in Figure 5, though at this the agreement with the feeding line for some W values is worsened.

5. CONCLUSION

The proposed problem solution of the electromagnetic waves radiation by a thin impedance vibrator with the arbitrary excitation point, based on the induced electromotive forces method with choice of the adequate approximating functions to represent the current distribution, permits to calculate all electrodynamic characteristics of the vibrator with the help of simple formulas rather precise. The approximating functions for the symmetrical and antisymmetrical components of the current are chosen both from the approximate analytical solution of the integral equation for the current in the impedance vibrator and from physical considerations when investigating the properties of this equation. The given for a concrete example numerical results allow to conclude the efficiency and wide possibilities of vibrators application with the distributed surface impedance in antenna's devices of this type. The suggested approach to the problem solution of the single impedance vibrator can be used to analyze multi-elements vibrator structures, for example, Yagi-Uda antennas or vibrator phased antenna arrays.

APPENDIX A. THE COEFFICIENTS OF THE ALGEBRAIC EQUATIONS SYSTEM

$$Z_{0n}^{s} = \frac{\tilde{k}}{k}\left[\cos\tilde{k}s_{\delta}A_{n}^{s}(L)-\cos\tilde{k}LA_{n}^{s}(s_{\delta})\right]+\frac{\left(k^{2}-\tilde{k}^{2}\right)}{2k}\int\limits_{-L}^{L}f_{0}^{s}(s)A_{n}^{s}(s)\mathrm{d}s, \tag{A1}$$

$$Z_{1n}^{s} = \sin kLA_{n}^{s}(L)-\frac{k}{2}\cos kL\int\limits_{-L}^{L}A_{n}^{s}(s)\mathrm{d}s, \tag{A2}$$

$$Z_{0n}^{a} = -\frac{\tilde{k}}{k}\left[\sin\tilde{k}s_{\delta}A_{n}^{a}(L)-\sin\tilde{k}LA_{n}^{a}(s_{\delta})\right]+\frac{\left(k^{2}-\tilde{k}^{2}\right)}{2k}\int\limits_{-L}^{L}f_{0}^{a}(s)A_{n}^{a}(s)\mathrm{d}s, \tag{A3}$$

$$Z_{1n}^a = \left(\frac{\sin kL}{kL} - \cos kL\right) A_n^a(L) - \frac{k}{2L}\sin kL \int\limits_{-L}^{L} A_n^a(s) s \mathrm{d}s, \tag{A4}$$

$$\tilde{E}_0^s = \cos\tilde{k}s_\delta \sin\tilde{k}(L-|s_\delta|), \qquad \tilde{E}_1^s = \cos ks_\delta - \cos kL, \tag{A5}$$

$$\tilde{E}_0^a = -\sin\tilde{k}|s_\delta|\sin\tilde{k}(L-|s_\delta|), \quad \tilde{E}_1^a = \sin ks_\delta - (s_\delta/L)\sin kL. \tag{A6}$$

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