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REFLECTION AND TRANSMISSION OF WAVES AT THE INTERFACE OF PERFECT ELECTROMAGNETIC CONDUCTOR (PEMC)

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Abstract—Perfect electromagnetic conductor (PEMC) has been introduced as a generalization of both the perfect electric conductor (PEC) and the perfect magnetic conductor (PMC). In the present paper, the basic problem of reflection and transmission of an obliquely incident plane wave at the interface of a PEMC half space or slab is considered. It is found that the field outside the PEMC medium can be uniquely determined but the interior field requires some assumptions to be unique. In particular, definition of the PEMC-medium condition as a limit of a Tellegen-medium condition and extraction of certain virtual fields (metafields) are shown to make the field inside the PEMC medium unique.

1. INTRODUCTION

The concept of perfect electromagnetic conductor (PEMC) has been defined by medium conditions of the form [1, 2]

$$\mathbf{D} = M\mathbf{B}, \qquad \mathbf{H} = -M\mathbf{E}, \tag{1}$$

where M is the PEMC admittance parameter, also called the axion parameter by the physicists [3, 4]. The PEMC concept has given rise to numerous studies on the topic [10–14]. In this study, the medium is assumed lossless, whence M is restricted to have real values, positive or negative [5]. In fact, for real M the complex Poynting vector is imaginary,

$$\mathbf{S} = \frac{1}{2}\mathbf{E} \times \mathbf{H}^* = -\frac{M}{2}\mathbf{E} \times \mathbf{E}^* = -\mathbf{S}^*, \tag{2}$$

whence the fields in PEMC do not convey energy. Actually, one can show that the Maxwell energy-stress dyadic [3, 6] vanishes for all possible fields if and only if the medium is PEMC. Thus, there is no energy involved in the fields occupying the PEMC.

The classical perfect electric conductor (PEC) and perfect magnetic conductor (PMC) are two special cases corresponding to the respective parameter values $1/M = 0$ and $M = 0$. For convenience in expressions we can also define the PEMC medium in terms of an angle parameter ϑ defined by

$$M = \frac{1}{\eta_o}\tan\vartheta, \qquad \eta_o = \sqrt{\mu_o/\epsilon_o}. \tag{3}$$

In this case PEC corresponds to $\vartheta = 0$ and PMC to $\vartheta = \pm\pi/2$.

PEMC can also be defined as a Tellegen medium obeying the medium conditions of the form [1]

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix} = q \begin{pmatrix} M & 1 \\ 1 & 1/M \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}, \tag{4}$$

in the limit of the parameter $|q| \to \infty$. In fact, the conditions (4) can be expressed in equivalent form as

$$\mathbf{D} - M\mathbf{B} = 0, \qquad \mathbf{H} + M\mathbf{E} = \mathbf{D}/q, \tag{5}$$

from which we find that $\mathbf{D} = M\mathbf{B}$ is exactly satisfied for any q, while the condition $\mathbf{H} = -M\mathbf{E}$ will be approached in the limiting process. (4) cannot be inverted, because the determinant of the matrix is zero. However, another similar process can be defined as

$$\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = q \begin{pmatrix} 1/M & -1 \\ -1 & M \end{pmatrix} \begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix}, \tag{6}$$

or

$$\mathbf{H} + M\mathbf{E} = 0, \qquad \mathbf{D} - M\mathbf{B} = -\mathbf{H}/q, \tag{7}$$

which, again, approaches (1) in the limit $|q| \to \infty$.

One may ask does it make any difference what definition we use, (1), (4) or (6) in the analysis? It will turn out that, when looking at a PEMC body from the outside, indeed, it does not matter. However, from the inside the situation is different, as will be seen. The conditions (1) appear to be too loose as they fail to define the fields uniquely, which is not satisfactory from the physical point of view. If another definition, (4) or (6), for example, would lead to a unique solution, it could be used as defining true PEMC medium conditions. This problem is approached by analyzing wave reflection from and transmission through a planar interface between an isotropic half space and a half space of PEMC medium. A similar problem was studied by Jancewicz [7] for a normally incident wave. Starting from the condition (1), he found that the fields inside the PEMC medium could not be uniquely determined, while the reflected fields of [1, 2] were reproduced. In the present study we approach the same problem with an obliquely incident time-harmonic wave which can be interpreted as a Fourier component of the most general field. For convenience, expressions related to incident and reflected plane waves, although quite well known in the literature [8], will be first derived to introduce the notation used in this paper.

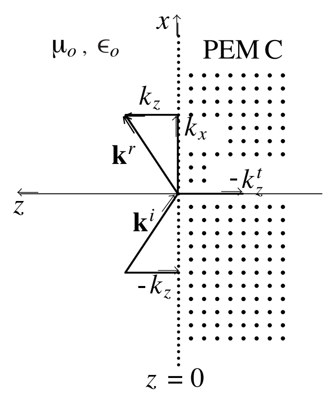

Figure 1. Plane-wave incident upon the interface of PEMC half space creates a transmitted field propagating along the interface with the wave number k_x.

2. REFLECTION FROM A BOUNDARY

2.1. Plane-wave Fields

Let us consider a time-harmonic plane wave incident in the half space $z > 0$ to an interface of a second medium at $z = 0$ which causes a reflected wave. The incident and reflected electric and magnetic fields are assumed to have the form

$$\mathbf{E}^i(\mathbf{r}) = \mathbf{E}^i \exp\left(-j\mathbf{k}^i \cdot \mathbf{r}\right), \qquad \mathbf{H}^i(\mathbf{r}) = \mathbf{H}^i \exp\left(-j\mathbf{k}^i \cdot \mathbf{r}\right), \tag{8}$$

$$\mathbf{E}^r(\mathbf{r}) = \mathbf{E}^r \exp\left(-j\mathbf{k}^r \cdot \mathbf{r}\right), \qquad \mathbf{H}^r(\mathbf{r}) = \mathbf{H}^r \exp\left(-j\mathbf{k}^r \cdot \mathbf{r}\right), \tag{9}$$

where the two wave vectors are defined by

$$\mathbf{k}^i = -k_z\mathbf{u}_z + k_x\mathbf{u}_x, \qquad \mathbf{k}^r = k_z\mathbf{u}_z + k_x\mathbf{u}_x, \tag{10}$$

and k_x is assumed to be a positive real number. The half space $z > 0$ is assumed empty, whence the wave-vector components satisfy

$$k_z^2 + k_x^2 = k_o^2 = \omega^2 \mu_o \epsilon_o. \tag{11}$$

The Maxwell equations for a plane wave in the source-free region $z > 0$ have the form

$$\mathbf{k} \times \mathbf{E} = k_o \eta_o \mathbf{H}, \qquad \eta_o \mathbf{k} \times \mathbf{H} = -k_o \mathbf{E}, \tag{12}$$

with $\eta_o = \sqrt{\mu_o/\epsilon_o}$. Because the electric fields satisfy the orthogonality conditions

$$\mathbf{k}^i \cdot \mathbf{E}^i = 0, \qquad \mathbf{k}^r \cdot \mathbf{E}^r = 0, \tag{13}$$

they can be expressed in terms of their components transverse to the z axis as

$$\mathbf{E}^i = \frac{1}{k_z} \left(k_x \mathbf{u}_z \mathbf{u}_x + k_z \overline{\overline{\mathsf{I}}}_t \right) \cdot \mathbf{E}_t^i, \tag{14}$$

$$\mathbf{E}^r = \frac{1}{k_z} \left(-k_x \mathbf{u}_z \mathbf{u}_x + k_z \overline{\overline{\mathsf{I}}}_t \right) \cdot \mathbf{E}_t^r. \tag{15}$$

Vectors transverse to $\mathbf{u}_z$ are defined through the transverse projection dyadic $\overline{\overline{\mathsf{I}}}_t$ and denoted by the subscript $_t$ as

$$\mathbf{a}_t = \overline{\overline{\mathsf{I}}}_t \cdot \mathbf{a}, \qquad \overline{\overline{\mathsf{I}}}_t = \mathbf{u}_x \mathbf{u}_x + \mathbf{u}_y \mathbf{u}_y. \tag{16}$$

Inserting (8) and (9) in (12), we obtain

$$\eta_o \mathbf{H}^i = \frac{1}{k_z k_o} \mathbf{k}^i \times \left(k_x \mathbf{u}_z \mathbf{u}_x + k_z \overline{\overline{\mathsf{I}}}_t \right) \cdot \mathbf{E}_t^i, \tag{17}$$

$$\eta_o \mathbf{H}^r = \frac{1}{k_z k_o} \mathbf{k}^r \times \left(-k_x \mathbf{u}_z \mathbf{u}_x + k_z \overline{\overline{\mathsf{I}}}_t \right) \cdot \mathbf{E}_t^r, \tag{18}$$

whence the relations of the transverse field components can be expressed in the compact form

$$\mathbf{H}_t^i = \overline{\overline{\mathsf{Y}}}_t \cdot \mathbf{E}_t^i, \qquad \mathbf{H}_t^r = -\overline{\overline{\mathsf{Y}}}_t \cdot \mathbf{E}_t^r. \tag{19}$$

$\overline{\overline{\mathsf{Y}}}_t$ is the free-space admittance dyadic. It can be expressed in the form

$$\overline{\overline{\mathsf{Y}}}_t = \frac{1}{\eta_o} \overline{\overline{\mathsf{J}}}_t, \tag{20}$$

where $\overline{\overline{\mathsf{J}}}_t$ is the dimensionless dyadic

$$\overline{\overline{\mathsf{J}}}_t = \frac{1}{k_o k_z} \left(k_z^2 \mathbf{u}_x \mathbf{u}_y - k_o^2 \mathbf{u}_y \mathbf{u}_x \right) \tag{21}$$

satisfying

$$\overline{\overline{\mathsf{J}}}_t^2 = -\overline{\overline{\mathsf{I}}}_t. \tag{22}$$

Because of this property, the normalized admittance dyadic $\overline{\overline{\mathsf{J}}}_t$ resembles the imaginary unit and it creates what is called an almost-complex structure in the space of two-dimensional vectors [3, 9].

2.2. Interface Conditions

Let us assume that the boundary at $z = 0$ is an interface of another medium occupying the half space $z < 0$ and the plane wave transmitted through the interface has the form

$$\mathbf{E}^t(\mathbf{r}) = \mathbf{E}^t \exp\left(-j\mathbf{k}^t \cdot \mathbf{r}\right), \quad \mathbf{H}^t(\mathbf{r}) = \mathbf{H}^t \exp\left(-j\mathbf{k}^t \cdot \mathbf{r}\right). \tag{23}$$

Continuity of the fields along the interface requires that the wave vector be of the form

$$\mathbf{k}^t = \mathbf{u}_x k_x + \mathbf{u}_z k_z^t, \tag{24}$$

where k_z^t depends on the medium behind the interface. Continuity of the transverse fields through the interface requires the conditions

$$\mathbf{E}_t^i + \mathbf{E}_t^r = \mathbf{E}_t^t, \tag{25}$$

$$\overline{\overline{\mathsf{J}}}_t \cdot (\mathbf{E}_t^i - \mathbf{E}_t^r) = \eta_o \mathbf{H}_t^t \tag{26}$$

to be valid.

A relation between the incident and transmitted fields can be found by eliminating $\mathbf{E}_t^r$ from (25) and (26):

$$2\mathbf{E}_t^i = \mathbf{E}_t^t - \overline{\overline{\mathsf{J}}}_t \cdot \eta_o \mathbf{H}_t^t. \tag{27}$$

If a linear relation between the transmitted transverse field components is known in the form

$$\mathbf{H}_t^t = \overline{\overline{\mathsf{Y}}}_t^t \cdot \mathbf{E}_t^t, \tag{28}$$

from (27) the transmitted electric field can be expressed as

$$\mathbf{E}_t^t = 2\left(\overline{\overline{\mathsf{J}}}_t + \eta_o \overline{\overline{\mathsf{Y}}}_t^t\right)^{-1} \cdot \overline{\overline{\mathsf{J}}}_t \cdot \mathbf{E}_t^i. \tag{29}$$

The reflected field can then be found from (25) in the form

$$\mathbf{E}_t^r = \overline{\overline{\mathsf{R}}}_t \cdot \mathbf{E}_t^i, \tag{30}$$

where the reflection dyadic is defined by

$$\overline{\overline{\mathsf{R}}}_t = \left(\overline{\overline{\mathsf{J}}}_t + \eta_o \overline{\overline{\mathsf{Y}}}_t^t\right)^{-1} \cdot \left(\overline{\overline{\mathsf{J}}}_t - \eta_o \overline{\overline{\mathsf{Y}}}_t^t\right). \tag{31}$$

2.3. Metafields

To consider uniqueness for the fields in the medium behind the interface at $z = 0$ one may ask whether there could exist fields $\mathbf{E}_o^t$, $\mathbf{H}_o^t$, $\mathbf{B}_o^t$, $\mathbf{D}_o^t$ in the PEMC half space without any sources or fields causing them in the region $z > 0$. This means that we are looking for fields in the region $z < 0$ satisfying the conditions

$$\mathbf{u}_z \times \mathbf{E}_o^t = 0, \quad \mathbf{u}_z \times \mathbf{H}_o^t = 0, \tag{32}$$

$$\mathbf{u}_z \cdot \mathbf{B}_o^t = 0, \quad \mathbf{u}_z \cdot \mathbf{D}_o^t = 0, \tag{33}$$

at the interface $z = 0$. Existence of such virtual fields or metafields, as they may be called, depends on the medium in the half space $z < 0$. For example, in an isotropic medium with permittivity ϵ and permeability μ, one can easily show that the conditions (32), (33) would lead to vanishing of the field. However, for the PEMC conditions (1) there actually may exist nonzero metafields. For example, assuming the previous exponential dependence on x, the following metafields, derived from an arbitrary scalar function $E_o(z)$, satisfies the interface conditions (32)

$$\mathbf{E}_o^t(\mathbf{r}) = \mathbf{u}_z E_o(z) e^{-jk_x x}, \quad \mathbf{H}_o^t(\mathbf{r}) = -M\mathbf{E}_o^t(\mathbf{r}). \tag{34}$$

From the Maxwell equations we obtain the other field components,

$$\mathbf{B}_o^t(\mathbf{r}) = -\mathbf{u}_y \frac{k_x}{\omega} E_o(z) e^{-jk_x x}, \quad \mathbf{D}_o^t(\mathbf{r}) = \mathbf{B}_o^t(\mathbf{r})/M, \tag{35}$$

which satisfy (33).

Thus, for any scalar function $E_o(\mathbf{r})$ such a metafield may exist in the PEMC half space without anyone noticing its existence from the outside. Because such a field appears nonphysical, we could extract it from any solution.

2.4. PEMC Interface

The relation (28) depends on the properties of the medium in the half space $z < 0$. For the PEMC half space the medium conditions (1) are also valid for the transverse field components, whence the interface is defined by the admittance dyadic

$$\overline{\overline{\mathsf{Y}}}_t^t = -M\overline{\overline{\mathsf{I}}}_t. \tag{36}$$

Inserting in (31) we can expand

$$\overline{\overline{\mathsf{R}}}_t = -\frac{\left(\overline{\overline{\mathsf{J}}}_t + \eta_o M\overline{\overline{\mathsf{I}}}_t\right)^2}{1 + (\eta_o M)^2} = \frac{1 - (M\eta_o)^2}{1 + (M\eta_o)^2}\overline{\overline{\mathsf{I}}}_t - \frac{2M\eta_o}{1 + (M\eta_o)^2}\overline{\overline{\mathsf{J}}}_t. \tag{37}$$

For example, for PEC corresponding to $\vartheta = 0$ and $M = \infty$ we obtain $\overline{\overline{\mathsf{R}}}_t = -\overline{\overline{\mathsf{I}}}_t$ and $\mathbf{E}_t^r = -\mathbf{E}_t^i$.

Expressing

$$\eta_o M = \tan\vartheta, \tag{38}$$

we can represent the reflection dyadic compactly as

$$\overline{\overline{\mathsf{R}}}_t = \cos 2\vartheta\ \overline{\overline{\mathsf{I}}}_t - \sin 2\vartheta\ \overline{\overline{\mathsf{J}}}_t = \exp\left(-2\vartheta\overline{\overline{\mathsf{J}}}_t\right), \tag{39}$$

whose validity is based on the Taylor series expansion of the exponential. In the special case of normal incidence, with $k_z = k_o$ and $\overline{\overline{\mathsf{J}}}_t = -\mathbf{u}_z \times \overline{\overline{\mathsf{I}}}$, the reflection dyadic (37), (39) coincides with that derived in [1, 2], when the opposite orientation of $\mathbf{u}_z$ is taken into account.

For general incidence the reflected transverse field becomes

$$\mathbf{E}_t^r = \overline{\overline{\mathsf{R}}}_t \cdot \mathbf{E}_t^i = \cos 2\vartheta \mathbf{E}_t^i - \sin 2\vartheta \overline{\overline{\mathsf{J}}}_t \cdot \mathbf{E}_t^i. \tag{40}$$

The total transverse fields at the interface define the transverse component of the transmitted field as

$$\mathbf{E}_t^t = \mathbf{E}_t^i + \mathbf{E}_t^r = 2\cos\vartheta \exp\left(-\vartheta\overline{\overline{\mathsf{J}}}_t\right) \cdot \mathbf{E}_t^i, \tag{41}$$

or

$$\mathbf{E}_t^t = \frac{2}{1 + (M\eta_o)^2}\left(\mathbf{E}_t^i - M\eta_o\overline{\overline{\mathsf{J}}}_t \cdot \mathbf{E}_t^i\right). \tag{42}$$

The magnetic field can be obtained similarly as

$$\eta_o \mathbf{H}_t^t = \overline{\overline{\mathsf{J}}}_t \cdot \left(\mathbf{E}_t^i - \mathbf{E}_t^r\right) = -2\sin\vartheta \exp\left(-\overline{\overline{\mathsf{J}}}_t\vartheta\right) \cdot \mathbf{E}_t^i, \tag{43}$$

which equals $-\eta_o M \mathbf{E}_t^t$, as expected.

3. FIELDS IN THE PEMC MEDIUM

3.1. PEMC Conditions

Starting from the PEMC conditions (1) does not yield unique transmitted fields, since the two Maxwell equations

$$\mathbf{k}^t \times \mathbf{H}^t = -\omega \mathbf{D}^t, \tag{44}$$
$$\mathbf{k}^t \times \mathbf{E}^t = \omega \mathbf{B}^t \tag{45}$$

actually become one and the same equation. In fact, from the interface conditions we know the transverse components of the fields, $\mathbf{E}_t^t$, $\mathbf{H}_t^t$, and of the wave vector, $\mathbf{k}_t^t = \mathbf{u}_x k_x$, which correspond to five scalar components out of 14. From (44) and (45) D_z^t, B_z^t can be solved which increases the number of known components to 7. The remaining 7 unknown components, $E_z^t, H_z^t, \mathbf{B}_t^t, \mathbf{D}_t^t$ and k_z^t cannot be solved from (44) and (45). Actually, we only need to know two of the unknowns, k_z^t and, e.g., E_z^t, to be able to solve the remaining five from (44) and (45).

It is not much of an improvement to start from the PEMC conditions of the form (6). In fact, eliminating $\mathbf{E}^t$ and $\mathbf{B}^t$, (45) can be written as

$$\mathbf{k}^t \times \mathbf{H}^t = -\omega \left(\mathbf{D}^t + \mathbf{H}^t/q\right), \tag{46}$$

whence together with (44) this would require $\mathbf{H}^t/q = 0$ and, for finite q, $\mathbf{H}^t = 0$ which eventually would lead to vanishing of all fields. To avoid that, we must have $|q| = \infty$ from the start, which means that (6) coincides with (1).

We can try to approach uniqueness by extracting a metafield component from the fields on the physical grounds that a field without a source cannot exist. Since any field of the form (34), (35) is a metafield, we can actually require that the transmitted fields satisfy

$$\mathbf{u}_z \cdot \mathbf{E}^t = 0, \quad \mathbf{u}_z \cdot \mathbf{H}^t = 0. \tag{47}$$

After the metafield extraction there still remains the question about the z dependence of the fields because PEMC allows all possible wave vectors. The most obvious choice would be $k_z^t = 0$. However, if we consider a slab of PEMC instead of the half space, this will cause a problem at the other interface, as will be seen. Thus, k_z^t remains an open parameter and no uniqueness is achieved when starting from the conditions (1) or (5).

3.2. Tellegen Conditions

Let us now start from the Tellegen medium conditions (4) and assume a finite value for the parameter q,

$$\mathbf{D}^t = q\left(\mathbf{H}^t + M\mathbf{E}^t\right), \tag{48}$$

$$M\mathbf{B}^t = q\left(\mathbf{H}^t + M\mathbf{E}^t\right). \tag{49}$$

To simplify the analysis, we temporarily define two auxiliary vectors $\mathbf{F}$, $\mathbf{G}$ by

$$\mathbf{F} = \frac{1}{2M}(M\mathbf{E} + \mathbf{H}), \quad \mathbf{G} = \frac{1}{2M}(M\mathbf{E} - \mathbf{H}), \tag{50}$$

whence the fields can be expressed as

$$\mathbf{E} = \mathbf{F} + \mathbf{G}, \tag{51}$$
$$\mathbf{H} = M(\mathbf{F} - \mathbf{G}). \tag{52}$$

Inserting these in (44) and (45), the resulting Maxwell equations

$$M\mathbf{k}^t \times (\mathbf{F} - \mathbf{G}) = -2\omega q M\mathbf{F}, \tag{53}$$
$$M\mathbf{k}^t \times (\mathbf{F} + \mathbf{G}) = 2\omega q M\mathbf{F}, \tag{54}$$

can be reduced to

$$\mathbf{k}^t \times \mathbf{F} = 0, \tag{55}$$
$$\mathbf{k}^t \times \mathbf{G} = 2\omega q\mathbf{F}. \tag{56}$$

Because of the assumption $k_x > 0$ we have $\mathbf{k}^t \neq 0$ and there must exist a scalar α such that

$$\mathbf{F} = \alpha\mathbf{k}^t. \tag{57}$$

The second Equation (56) can now be written as

$$\mathbf{k}^t \times \mathbf{G} = 2\omega q\alpha\mathbf{k}^t = 2\omega\gamma\mathbf{k}^t. \tag{58}$$

To avoid fields growing infinite when $q \to \infty$, we must assume that simultaneously $\alpha \to 0$ so that

$$\gamma = q\alpha \tag{59}$$

remains finite. (58) implies

$$\mathbf{k}^t \cdot \mathbf{k}^t = 0, \quad \Rightarrow \quad \mathbf{k}^t = k_x(\mathbf{u}_x + j\mathbf{u}_z), \tag{60}$$

with $\mathbf{k}^t$ satisfying

$$\mathbf{u}_y \times \mathbf{k}^t = j\mathbf{k}^t. \tag{61}$$

The other possibility $\mathbf{k}^t = k_x(\mathbf{u}_x - j\mathbf{u}_z)$ is ruled out by requiring that the field must not grow exponentially as $z \to -\infty$. Expanding (58) as

$$\mathbf{u}_y \times \left(\mathbf{k}^t \times \mathbf{G}\right) = \mathbf{k}^t\,(\mathbf{u}_y \cdot \mathbf{G}) = 2j\omega\gamma\mathbf{k}^t, \tag{62}$$

we see that $\mathbf{G}$ must be of the form

$$\mathbf{G} = 2j\omega\gamma\mathbf{u}_y + \beta\,(\mathbf{u}_x + j\mathbf{u}_z)\,, \tag{63}$$

where β may be any scalar.

The electric and magnetic fields can now be expressed as

$$\mathbf{E}^t = \mathbf{F} + \mathbf{G} = (\alpha k_x + \beta)(\mathbf{u}_x + j\mathbf{u}_z) + 2j\omega\gamma\mathbf{u}_y, \quad (64)$$

$$\mathbf{H}^t = M(\mathbf{F} - \mathbf{G}) = M(\alpha k_x - \beta)(\mathbf{u}_x + j\mathbf{u}_z) - 2j\omega M\gamma\mathbf{u}_y. \quad (65)$$

whence the fields satisfy

$$\mathbf{H}^t + M\mathbf{E}^t = 2M\alpha\mathbf{k}^t = \frac{2M\gamma}{q}\mathbf{k}^t. \quad (66)$$

Comparing this with (48) and (49), we can identify

$$\mathbf{D}^t = 2M\gamma\mathbf{k}^t, \qquad \mathbf{B}^t = 2\gamma\mathbf{k}^t. \quad (67)$$

Now we can safely let $q \to \infty$, whence the electric and magnetic fields become

$$\mathbf{E}^t \to \beta(\mathbf{u}_x + j\mathbf{u}_z) + 2j\omega\gamma\mathbf{u}_y, \quad (68)$$

$$\mathbf{H}^t \to -M\beta(\mathbf{u}_x + j\mathbf{u}_z) - 2j\omega M\gamma\mathbf{u}_y. \quad (69)$$

The unknown parameters β, γ can be found by comparing the transverse components of $\mathbf{E}^t$ to those obtained from the interface condition (42), rewritten as

$$\mathbf{E}_t^t = \frac{2}{k_o k_z(1 + (M\eta_o)^2)} \left(\mathbf{u}_x\left(k_o k_z \mathbf{u}_x - M\eta_o k_z^2 \mathbf{u}_y\right) + \mathbf{u}_y\left(k_o k_z \mathbf{u}_y + M\eta_o k_o^2 \mathbf{u}_x\right)\right) \cdot \mathbf{E}_t^i. \quad (70)$$

$$\beta = \frac{2\left(k_o E_x^i - M\eta_o k_z E_y^i\right)}{k_o(1 + (M\eta_o)^2)}, \quad (71)$$

$$\gamma = \frac{k_z E_y^i + M\eta_o k_o E_x^i}{j\omega k_z(1 + (M\eta_o)^2)}. \quad (72)$$

When applying to the normally incident wave with $k_z \to k_o$, $k_x \to 0$, we obtain

$$\beta \to \frac{2\left(E_x^i - M\eta_o E_y^i\right)}{1 + (M\eta_o)^2}, \quad (73)$$

$$\gamma \to \frac{E_y^i + M\eta_o E_x^i}{j\omega(1 + (M\eta_o)^2)}, \quad (74)$$

and

$$\mathbf{E}^t \to \frac{2}{1 + (M\eta_o)^2}\left(\left(\bar{\bar{\mathsf{I}}}_t + M\eta_o\mathbf{u}_z \times \bar{\bar{\mathsf{I}}}\right) \cdot \mathbf{E}^i + j\mathbf{u}_z\left(E_x^i - M\eta_o E_y^i\right)\right). \quad (75)$$

This result contains a discontinuity because the E_z^t component depends on the choice of the x axis: it obtains different values when approaching normal incidence from different directions.

To remove this defect from the solution we again extract the z components of the $\mathbf{E}^t$ and $\mathbf{H}^t$ fields, because they can be interpreted as metafields (34).

Thus, we obtain the final field expressions

$$\mathbf{E}^t(\mathbf{r}) = \frac{2e^{-jk_x x}e^{k_x z}}{k_o k_z(1+(M\eta_o)^2)}\left(k_o k_z \bar{\bar{\mathsf{I}}}_t + M\eta_o\left(k_o^2 \mathbf{u}_y\mathbf{u}_x - k_z^2 \mathbf{u}_x\mathbf{u}_y\right)\right)\cdot\mathbf{E}^i, \quad (76)$$

$$\mathbf{H}^t = -\frac{2Me^{-jk_x x}e^{k_x z}}{k_o k_z(1+(M\eta_o)^2)}\left(k_o k_z \bar{\bar{\mathsf{I}}}_t + M\eta_o\left(k_o^2 \mathbf{u}_y\mathbf{u}_x - k_z^2 \mathbf{u}_x\mathbf{u}_y\right)\right)\cdot\mathbf{E}^i, \quad (77)$$

$$\begin{aligned}\mathbf{B}^t(\mathbf{r}) &= \frac{k_x}{\omega}(\mathbf{u}_x + j\mathbf{u}_z)\times\mathbf{E}^t(\mathbf{r})\\ &= \frac{2k_x e^{-jk_x x}e^{k_x z}}{\omega k_o k_z(1+(M\eta_o)^2)}\\ &\quad \left(k_o k_z(\mathbf{u}_x + j\mathbf{u}_z)\times\bar{\bar{\mathsf{I}}}_t - jM\eta_o\left(k_o^2(\mathbf{u}_x + j\mathbf{u}_z)\mathbf{u}_x + k_z^2\mathbf{u}_y\mathbf{u}_y\right)\right)\cdot\mathbf{E}^i, \quad (78)\\ \mathbf{D}^t(\mathbf{r}) &= M\mathbf{B}^t(\mathbf{r}). \quad (79)\end{aligned}$$

In this way we have arrived at a unique representation of the fields inside the PEMC half space. For the normal incidence case $k_x \to 0$ we see that the $\mathbf{B}^t$ and $\mathbf{D}^t$ fields vanish while the other fields become constant in the half space,

$$\mathbf{E}^t(\mathbf{r}) \to \frac{2}{1+(M\eta_o)^2}\left(\bar{\bar{\mathsf{I}}}_t + M\eta_o\mathbf{u}_z\times\bar{\bar{\mathsf{I}}}_t\right)\cdot\mathbf{E}^i, \quad (80)$$

$$\mathbf{H}^t(\mathbf{r}) \to -\frac{2M}{1+(M\eta_o)^2}\left(\bar{\bar{\mathsf{I}}}_t + M\eta_o\mathbf{u}_z\times\bar{\bar{\mathsf{I}}}_t\right)\cdot\mathbf{E}^i, \quad (81)$$

as obtained from (75).

3.3. PEC and PMC

It is not often that fields inside ideal PEC or PMC media are considered and it may be erroneously assumed that all fields vanish in both cases. However, the PEC conditions do not require that the **H** and **D** fields vanish and the PMC conditions do not require that the **E** and **B** fields vanish. Let us consider the fields excited by the incident plane wave in both of these special cases of the PEMC medium.

For the PMC half space corresponding to $M = 0$ (76)–(79) become

$$\mathbf{E}^t(\mathbf{r}) = 2\mathbf{E}^i_t e^{-jk_x x} e^{k_x z}, \qquad \mathbf{H}^t = 0, \tag{82}$$

$$\mathbf{B}^t(\mathbf{r}) = \frac{2k_x}{\omega}(\mathbf{u}_x + j\mathbf{u}_z) \times \mathbf{E}^i_t e^{-jk_x x} e^{k_x z}, \qquad \mathbf{D}^t(\mathbf{r}) = 0. \tag{83}$$

Similarly, for the PEC we obtain

$$\mathbf{E}^t(\mathbf{r}) = 0, \quad \mathbf{H}^t(\mathbf{r}) = -\frac{2e^{-jk_x x} e^{k_x z}}{k_o k_z \eta_o} \left(k_o^2 \mathbf{u}_y \mathbf{u}_x - k_z^2 \mathbf{u}_x \mathbf{u}_y\right) \cdot \mathbf{E}^i, \tag{84}$$

$$\mathbf{B}^t(\mathbf{r}) = 0, \quad \mathbf{D}^t(\mathbf{r}) = \frac{2jk_x e^{-jk_x x} e^{k_x z}}{\omega k_o k_z \eta_o} \left(k_o^2 (\mathbf{u}_x + j\mathbf{u}_z)\mathbf{u}_x + k_z^2 \mathbf{u}_y \mathbf{u}_y\right) \cdot \mathbf{E}^i. \tag{85}$$

As a check we can consider plane-wave transmission into an isotropic medium with parameters $\epsilon = \epsilon_r \epsilon_o$ and $\mu = \mu_r \mu_o$, whence the surface dyadic becomes

$$\eta_o \overline{\overline{\mathsf{Y}}}{}^t_t = \frac{k_z^t}{\mu_r k_o} \mathbf{u}_x \mathbf{u}_y - \frac{\epsilon_r k_o}{k_z^t} \mathbf{u}_y \mathbf{u}_x. \tag{86}$$

For example, the PEC medium is obtained as the limit $\mu_r \to 0$, $\epsilon_r \to \infty$, whence after some algebra, the field vectors in (84) are obtained. In the process $\epsilon_r \to \infty$ and $\mu_r \to 0$ the quantity $\mu_r \epsilon_r$ was left unspecified and its choice determines the wave vector component $k_z^t = \sqrt{\mu_r \epsilon_r k_o^2 - k_x^2}$. Assuming $\mu_r \epsilon_r \to 0$, the above solution is obtained, after which the results (85) are also recovered.

4. PEMC SLAB

The previous analysis can be quite easily extended to take another interface at $z = -d$ into account. In this case the PEMC medium forms a slab in the region $0 > z > -d$. Assuming the same isotropic medium in the two half spaces, the field transmitted into the region $z < -d$ can be assumed to have the form of a plane wave,

$$\mathbf{E}^T(\mathbf{r}) = \mathbf{E}^T \exp\left(-j\mathbf{k}^T \cdot (\mathbf{r} + \mathbf{u}_z d)\right), \tag{87}$$

$$\mathbf{H}^T(\mathbf{r}) = \mathbf{H}^T \exp\left(-j\mathbf{k}^T \cdot (\mathbf{r} + \mathbf{u}_z d)\right). \tag{88}$$

Because of the $\exp(-jk_x x)$ dependence, the wave vector equals that of the incident wave,

$$\mathbf{k}^T = \mathbf{k}^i = \mathbf{u}_x k_x - \mathbf{u}_z k_z, \tag{89}$$

and the magnetic field satisfies

$$\eta_o \mathbf{H}_t^T = \eta_o \overline{\overline{\mathsf{Y}}}_t \cdot \mathbf{E}^T = \overline{\overline{\mathsf{J}}}_t \cdot \mathbf{E}^T. \tag{90}$$

These fields at the interface $z = -d$ must satisfy the PEMC condition, i.e.,

$$\eta_o \mathbf{H}_t^T - M\eta_o \mathbf{E}_t^T = 0, \tag{91}$$

which becomes

$$\left(\overline{\overline{\mathsf{J}}}_t - M\eta_o \overline{\overline{\mathsf{I}}}_t\right) \cdot \mathbf{E}_t^T = 0. \tag{92}$$

Since the dyadic in brackets has the inverse $-\left(\overline{\overline{\mathsf{J}}}_t + M\eta_o \overline{\overline{\mathsf{I}}}_t\right)/(1 + (M\eta_o)^2)$, it follows that $\mathbf{E}_t^T = 0$ and $\mathbf{H}_t^T = 0$, whence all field components vanish in the half space $z < -d$. This is, of course, due to the fact that energy is not conveyed through the PEMC slab [7].

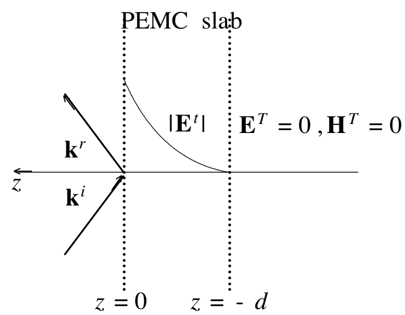

Figure 2. Plane-wave incident on the interface of PEMC slab creates a field transmitted into the slab whose magnitude obeys the function $\sinh(kx(z + d))$. In the region $z < -d$ the fields vanish.

To satisfy the interface condition at $z = 0$ we assume that there exist two plane waves in the PEMC slab denoted by $\mathbf{E}^{t+}$ and $\mathbf{E}^{t-}$. These fields are transverse to the z axis and have the dependence on x and z

$$\mathbf{E}^t(\mathbf{r}) = \exp(-jk_x x)\left(\mathbf{E}^{t+}\exp(k_x z) + \mathbf{E}^{t-}\exp(-k_x z)\right), \tag{93}$$

$$\mathbf{H}^t(\mathbf{r}) = -M\exp(-jk_x x)\left(\mathbf{E}^{t+}\exp(k_x z) + \mathbf{E}^{t-}\exp(-k_x z)\right). \tag{94}$$

Since they vanish at $z = -d$, we can write

$$\mathbf{E}^t(\mathbf{r}) = 2\mathbf{E}^t\exp(-jk_x x)\sinh(k_x(z + d)), \tag{95}$$

$$\mathbf{H}^t(\mathbf{r}) = -2M\mathbf{E}^t\exp(-jk_x x)\sinh(k_x(z + d)), \tag{96}$$

with

$$\mathbf{E}^{t\pm} = \pm\mathbf{E}^t \exp(\pm k_x d). \tag{97}$$

Now the interface conditions at $z = 0$ imply the same reflection as from the PEMC half space. Thus, the condition (42) takes the modified form

$$\mathbf{E}^t = \frac{\mathbf{E}_t^i - M\eta_o \overline{\overline{\mathsf{J}}}_t \cdot \mathbf{E}_t^i}{(1 + (M\eta_o)^2)\sinh(k_x d)}, \tag{98}$$

which inserted in (93) with (97) taken into account yields the total field in the PEMC slab as

$$\mathbf{E}^t(\mathbf{r}) = \frac{2\exp(-jk_x x)\sinh(k_x(z+d))}{(1 + (M\eta_o)^2)\sinh(k_x d)} \left(\mathbf{E}_t^i - M\eta_o \overline{\overline{\mathsf{J}}}_t \cdot \mathbf{E}_t^i\right). \tag{99}$$

As two special cases, for $d \to \infty$ the expression (76) for the PEMC half space is reproduced while for the normal incidence case $k_x \to 0$ of [7], (99) corresponds to the field

$$\mathbf{E}^t(\mathbf{r}) = \frac{2(z+d)}{(1 + (M\eta_o)^2)d} \left(\mathbf{E}_t^i + M\eta_o \mathbf{u}_z \times \mathbf{E}_t^i\right), \tag{100}$$

which represents linear dependence on the z coordinate.

5. CONCLUSION

In this paper the problem of finding fields reflected from and transmitted through the interface of a PEMC half space or slab has been analyzed. In contrast to an earlier study, obliquely incident time-harmonic plane wave was assumed. Assuming the PEMC conditions (1) it was shown that the field inside the PEMC medium is not unique even if we extract an unphysical metafield component which does not depend on the incident field. However, if the PEMC is defined as a limiting case of a Tellegen medium, with infinitely large parameters, uniqueness can be attained.

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