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COMPARISON OF n_{fsm}, n_{eff}, AND THE SECOND AND THIRD ORDER DISPERSIONS OF PHOTONIC CRYSTAL FIBERS CALCULATED BY SCALAR EFFECTIVE INDEX METHOD AND EMPIRICAL RELATIONS METHODS

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Abstract—To design less costly and time consuming Photonic Crystal Fibers it is better to use Empirical Relations Method instead of Scalar Effective Index Method. If we compare both empirical relations method and scalar effective index method by accurate and powerful methods like Full-Vector Finite Element Method, we find that empirical relations method has less error than scalar effective index method in calculating PCF parameters such as n_{fsm}, n_{eff}, and the second order dispersion. According to the investigations, we concluded, the inherent error of scalar effective index method approximately increases when pitch decreases. In large pitches the calculation of dispersion by scalar effective index method reveals less error in low wavelengths than high wavelengths and finally we calculated the third order dispersion which is important in some applications.

1. INTRODUCTION

Optical fiber, as an important optical instrument is used for high speed data communications, sensor technology, spectroscopy, and medicine [1, 2]. During recent years lots of studies are done about PCFs or holy fibers [3]. This is due to capabilities of these fibers in handling propagation modes through themselves [4]. This aspect has turned these devices into the most popular and applicable optical instruments such as channel allocation in the wavelength division multiplexing transmission system and Pressure Sensor Applications [5].

PCFs are categorized as mono-material fibers which have a central light guiding area surrounded by rods in a triangular lattice [3]. These rods are filled by air and their diameters and hole pitches are almost the same as the amount of wavelength. This novel structure of PCF makes new properties such as wide single-mode wavelength range, unusual chromatic dispersion and high or low non-linearity [1].

There are several methods to analyze these fibers including: Effective Index Method, (EIM), Localized Basis Function Method, Finite Element Method (FEM), Finite Difference Method (FDM), Plane Wave Expansion Method (PWM) and Multi-Pole Method [1, 4, 5].

Numerical methods are too time consuming and needs huge and iterative computation [1]. Usually these methods are too mighty and their broad capabilities are not required for studying PCFs. Despite of limitations and accuracies, other analytic methods are introduced to replace these methods [1]. In present paper two scalar effective index method (SEIM) and empirical relations method (ERM) are studied.

In SEIM the effective cladding index of a hexagonal unit cell which consists a Fiber rod is calculated with respect to rod diameter and pitch (Λ), then the effective index of PCF is obtained by using the effective cladding index [3].

In ERM, empirical relations for V parameter (Normalized Frequency) and W parameter (Normalized Transverse Attenuation Constant) of PCFs with respect to the basic geometrical parameters (i.e., the air hole diameter and the hole pitch) are formed. Then V and W are computed and used to calculate PCF's basic parameters [1].

The obtained results of these two methods are compared and we show that the accuracy of the methods changes by Λ and wavelength.

One of the main problems in optical fibers is calculation dispersion [9]. We present the calculation result of the second order dispersion of a chromatic dispersion in PCFs with well known properties. Since some properties optical networks strongly depends on the impact of higher order dispersion [10] we also illustrate the calculation result of the third order dispersion. The Sellmeier relation has been used to calculate material dispersion.

2. SCALAR EFFECTIVE INDEX METHOD

One of the analytical methods is scalar effective index method (SEIM) which is valid for the LP01 fiber mode based on weak-guidance approximation [4]. The core refractive index is supposed to be the same as the refractive index of core material which is given by the Sellmeier relation. But the cladding refractive index is determined based on total reflection [6]. Fundamental space-filling mode (FSM) of a PCF is considered to be the mode with the largest modal index of the infinite two-dimensional photonic crystal structure that surrounds the PCF's core [7, 8].

The wave equation in cladding area is defined as $[\nabla_t^2 + ((\frac{w}{c})^2 n^2 - \beta^2)]\psi = 0$, where ∇_t, w, c, n and β are transverse Laplacian operator in cylindrical coordinates, angular frequency, the light velocity in a vacuum, material index of cladding and propagation constant, respectively. ψ can be either the electric or magnetic field [3]. The hexagonal unit cell is replaced by a circular unit cell of radius R (Fig. 1). At any point P on the boundary of unit cell, ψ must satisfy $\frac{\partial\psi}{\partial\vec{n}} = 0$ and $\frac{\partial\psi}{\partial\vec{n}}$ is continuous at the interface of inner and outer side of unit cell, where $\vec{n}$ is the outward unit vector normal to the boundary of unit cell [3].

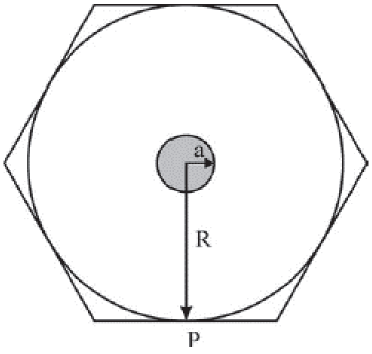


Figure 1. The hexagonal unit cell and its circular equivalent.

We can get the following equation in the inner and outer areas of the air hole under the assumption of weak guidance [9, 3].

$$K(w)\frac{I_1[K(w)a]}{I_0[K(w)a]}\left\{J_0[T(w)a] - Y_0[T(w)a]\frac{J_1[T(w)R]}{Y_1[T(w)R]}\right\} = -T(w)\left\{J_1[T(w)a] - Y_1[T(w)a]\frac{J_1[T(w)R]}{Y_1[T(w)R]}\right\} \tag{1}$$

Where I, J, Y are Bessel functions, $K^2(w) = \beta^2(w) - n_{air}^2(\frac{w}{c})^2$ and $T^2(w) = (\frac{w}{c})^2 n_{silica}^2(w) - \beta^2(w)$.

The optimal radius for SEIM is $R = \frac{\Lambda}{2}$ [4].

By solving equation (1) for $\beta(\omega)$ we can calculate the effective refractive index of the fundamental space-filling mode using $n_{fsm}(w) = \beta(w)\frac{c}{w}$.

Once having obtained $n_{fsm}(w)$, one solves the characteristic equation for the propagation constant $\beta_c(\omega)$ of the LP01 mode of the approximate step-index fiber by $\eta(w)\frac{J_1[\eta(w)r_c]}{J_0[\eta(w)r_c]} = \gamma(w)\frac{K_1[\gamma(w)r_c]}{K_0[\gamma(w)r_c]}$, where K and J are also Bessel functions, and $\eta^2(w) = (\frac{w}{c})^2 n_c^2(w) - \beta_c^2(w)$, $\gamma^2(w) = \beta_c^2(w) - (\frac{w}{c})^2 n_{\mathit{eff}}^2(w)$, with $n_c(\omega)$ being the refractive index of the core material and $r_c = \Lambda - a$ being the core radius [3]. afterward by using $n_{\mathit{eff}}(w) = \beta_c(w)\frac{c}{w}$, the effective index of PCF is obtained.

3. EMPIRICAL RELATIONS METHOD

In this method, the refractive index of silica is constant and supposed to be $n_{core} = 1.45$. The effective core radius is defined $a_{\mathit{eff}} = \Lambda/\sqrt{3}$ [1].

Recently, it has been claimed that the triangular PCFs can be well parameterized in terms of the V parameter [10] that is given by

$$V = \frac{2\pi}{\lambda} a_{\mathit{eff}} \left(n_{core}^2 - n_{\mathit{fsm}}^2\right)^{0.5} = (U^2 + W^2)^{0.5} \tag{2}$$

where $U = \frac{2\pi}{\lambda} a_{\mathit{eff}} (n_{core}^2 - n_{\mathit{fsm}}^2)^{0.5}$ and

$$W = \frac{2\pi}{\lambda} a_{\mathit{eff}} \left(n_{\mathit{eff}}^2 - n_{\mathit{fsm}}^2\right)^{0.5} \tag{3}$$

First by using Table 1 from [1], we calculate V by using $V(\frac{\lambda}{\Lambda}, \frac{d}{\Lambda}) = A_1 + \frac{A_2}{1+A_3 \exp(A_4\lambda/\Lambda)}$ [1], where

$$A_i = a_{i0} + a_{i1}\left(\frac{d}{\Lambda}\right)^{b_{i1}} + a_{i2}\left(\frac{d}{\Lambda}\right)^{b_{i2}} + a_{i3}\left(\frac{d}{\Lambda}\right)^{b_{i3}}.$$

Afterward, the effective cladding index n_{fsm} is obtained by (2). Then by using Table 2 from [1] and $W(\frac{\lambda}{\Lambda}, \frac{d}{\Lambda}) = B_1 + \frac{B_2}{1+B_3 \exp(B_4\lambda/\Lambda)}$ [1], where

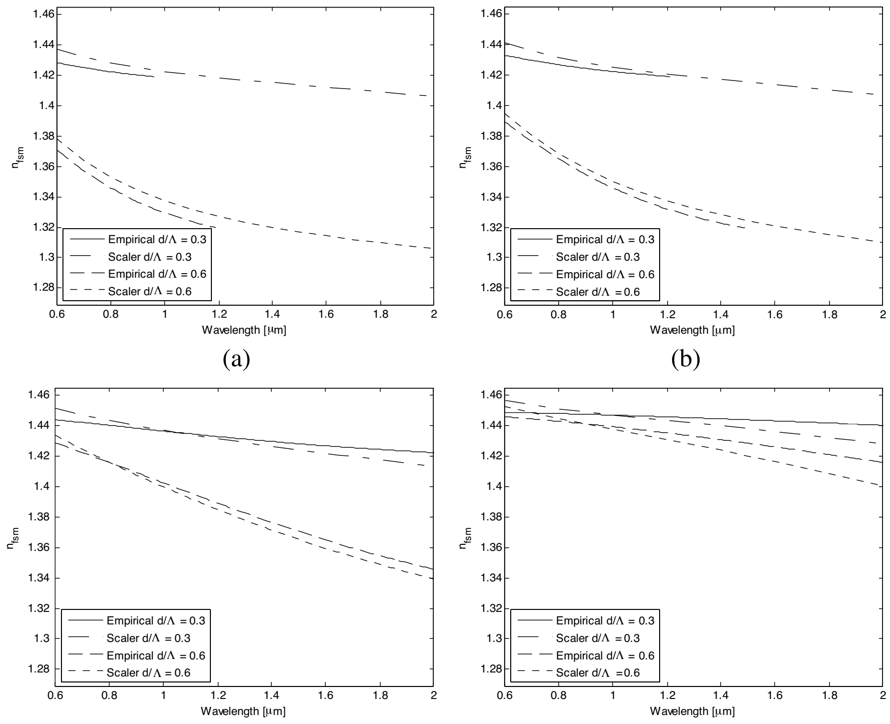
$$B_i = c_{i0} + c_{i1}\left(\frac{d}{\Lambda}\right)^{d_{i1}} + c_{i2}\left(\frac{d}{\Lambda}\right)^{d_{i2}} + c_{i3}\left(\frac{d}{\Lambda}\right)^{d_{i3}},$$

we calculate W.

Finally by using (3) for given W and $n_{\mathit{fsm}}, n_{\mathit{eff}}$ can be obtained.

4. RESULTS

Several interesting results are illustrated in Figs. 2, 3 and 4 that compare $n_{\mathit{fsm}}, n_{\mathit{eff}}$ and the second order dispersion for $\Lambda = 0.8, 1.0, 2.0$



(a) (b)

(c) (d)

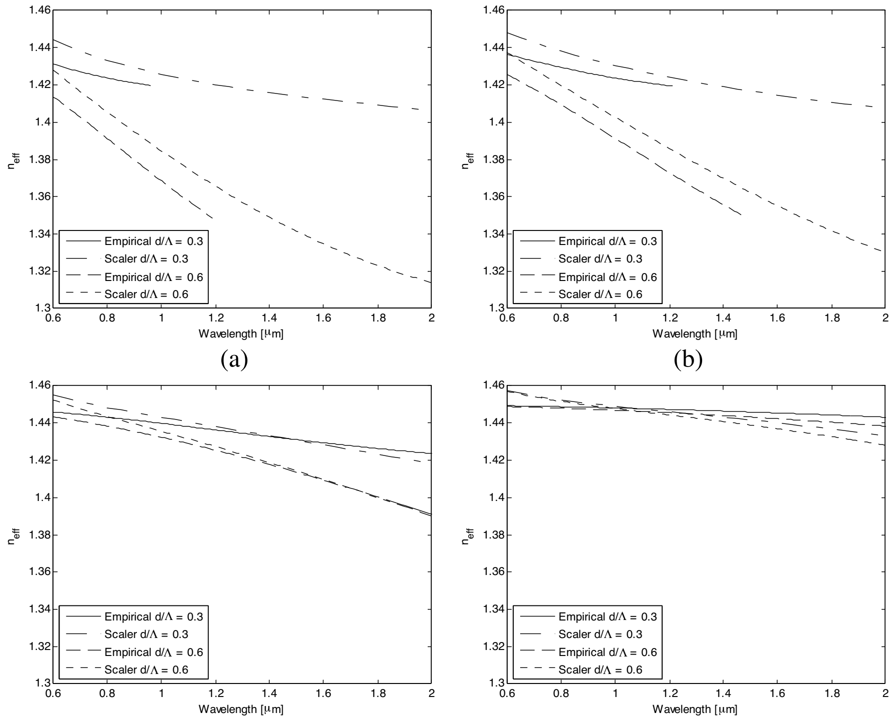
Figure 2. Comparison of the n_{fsm} obtained by ERM and SEIM. The solid line is obtained by ERM for $d/\Lambda = 0.3$, the dashed-dotted line by SEIM for $d/\Lambda = 0.3$, the dashed line by ERM for $d/\Lambda = 0.6$ and the dotted line by SEIM for $d/\Lambda = 0.6$: (a) $\Lambda = 0.8\,\mu$m, (b) $\Lambda = 1.0\,\mu$m, (c) $\Lambda = 2.0\,\mu$m, and (d) $\Lambda = 5.0\,\mu$m.

and $5.0\,\mu$m by using ERM and SEIM. The results are for wavelength in range 0.6 to $2.0\,\mu$m. It is realized that the results obtained by ERM are more accurate than those obtained by SEIM, because they are closer to an accurate method like FVFEM [11].

As one can see, for $\Lambda = 0.8, 1.0$, ERM does not answer for all wavelength in range of 0.6 to $2.0\,\mu$m. It is because of restrictions which are mentioned in [11]. So we conclude that ERM performs better that SEIM at calculating n_{fsm}, n_{eff} and the second order dispersion.

We compared the third order dispersion by ERM with the third order dispersion by SEIM (Fig. 5).

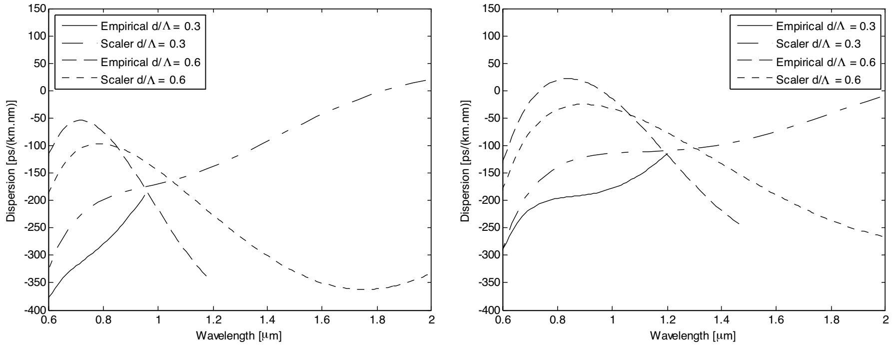
To observe the difference between the methods, we calculate the relative error of n_{fsm} and n_{eff} (Figs. 6, 7). These figures prove that



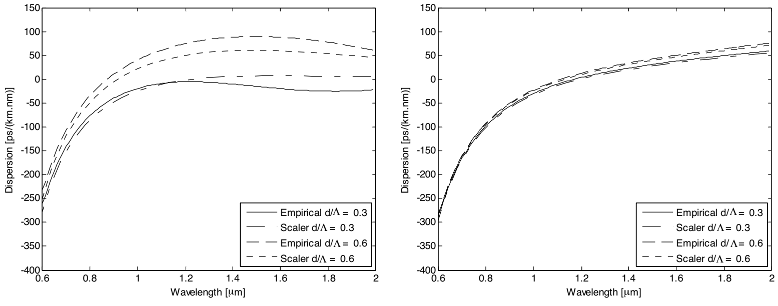
(a) (b)

(c) (d)

Figure 3. Comparison of the n_{eff} obtained by ERM and SEIM. The solid line is obtained by ERM for $d/\Lambda = 0.3$, the dashed-dotted line by SEIM for $d/\Lambda = 0.3$, the dashed line by ERM for $d/\Lambda = 0.6$ and the dotted line by SEIM for $d/\Lambda = 0.6$: (a) $\Lambda = 0.8\,\mu$m, (b) $\Lambda = 1.0\,\mu$m, (c) $\Lambda = 2.0\,\mu$m, and (d) $\Lambda = 5.0\,\mu$m.

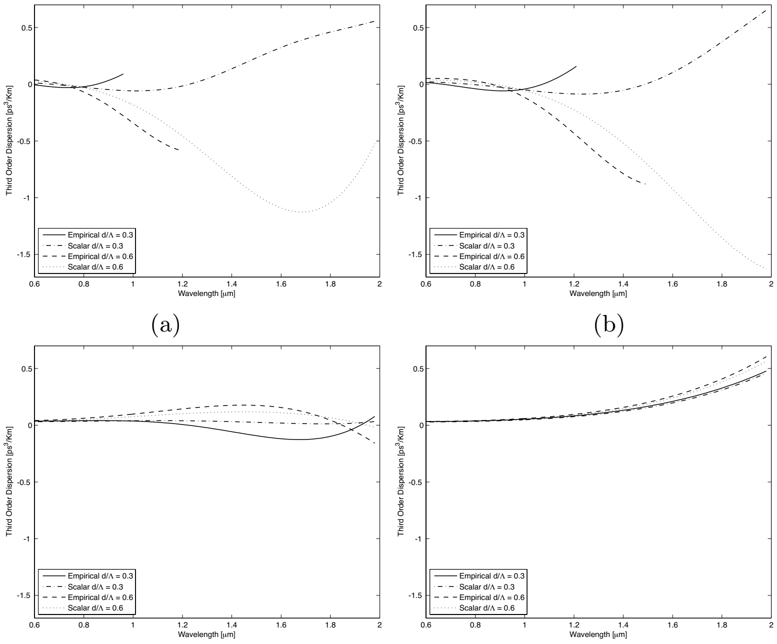


(a) (b)



(c) (d)

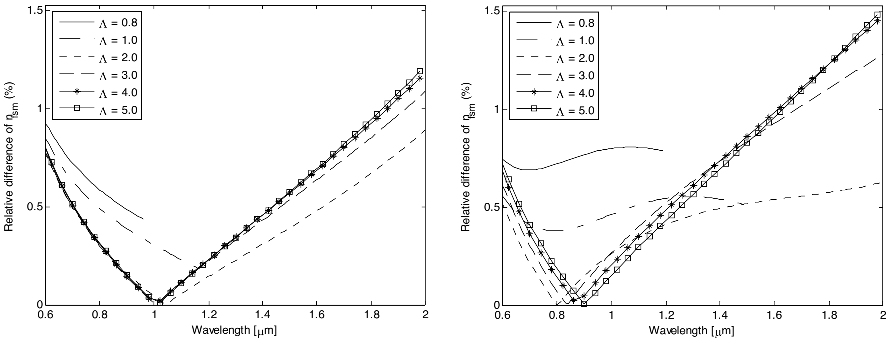
Figure 4. Comparison of the second order dispersion obtained by ERM and SEIM. The solid line is obtained by ERM for $d/\Lambda = 0.3$, the dashed-dotted line by SEIM for $d/\Lambda = 0.3$, the dashed line by ERM for $d/\Lambda = 0.6$ and the dotted line by SEIM for $d/\Lambda = 0.6$: (a) $\Lambda = 0.8\,\mu$m, (b) $\Lambda = 1.0\,\mu$m, (c) $\Lambda = 2.0\,\mu$m, and (d) $\Lambda = 5.0\,\mu$m.



(a) (b)

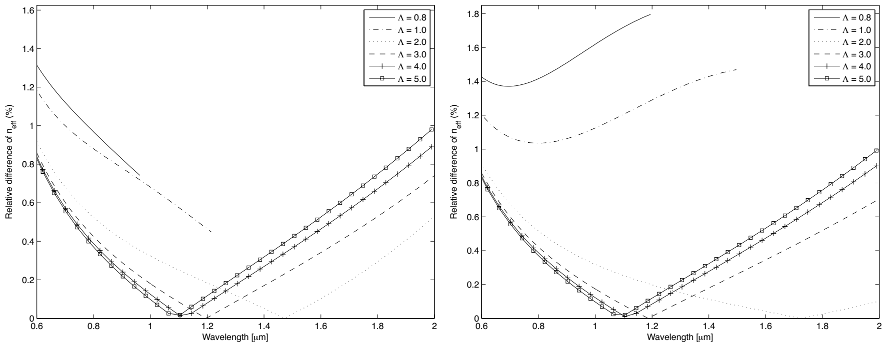
(c) (d)

Figure 5. Comparison of the third order dispersion obtained by ERM and SEIM. The solid line is obtained by ERM for $d/\Lambda = 0.3$, the dashed-dotted line by SEIM for $d/\Lambda = 0.3$, the dashed line by ERM for $d/\Lambda = 0.6$ and the dotted line by SEIM for $d/\Lambda = 0.6$: (a) $\Lambda = 0.8\,\mu$m, (b) $\Lambda = 1.0\,\mu$m, (c) $\Lambda = 2.0\,\mu$m, and (d) $\Lambda = 5.0\,\mu$m.



(a) (b)

Figure 6. Relative difference between n_{fsm} obtained by ERM and SEIM: (a) $\frac{d}{\lambda} = 0.3$, and (b) $\frac{d}{\lambda} = 0.6$.



(a) (b)

Figure 7. Relative difference between n_{eff} obtained by ERM and SEIM: (a) $\frac{d}{\lambda} = 0.3$, and (b) $\frac{d}{\lambda} = 0.6$.

the error of SEIM for small Λs is huge.

The current approximation of mutual effects of each rod to other rods and the core, can be a reason. Because as Λ increases (the distance between rods increases), this mutual effects lowers and the error obeys a fixed pattern.

5. CONCLUSIONS

Studying the results of two methods proves that scalar effective index method is less accurate and slower than empirical relations method. Empirical relations method is found to give more accurate results than scalar effective index method for large pitches. The disadvantages of empirical relations method appears for small air filling fractions and small pitches, when it does not answer for all range of wavelengths. After all we can say that calculating the parameters of photonic crystal fibers by empirical relations method and scalar effective index method for small pitches is not recommended.

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