

A NEW LINEAR SPACE-TIME BLOCK CODE FOR WIRELESS CHANNELS WITH CORRELATED FADING COEFFICIENTS

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Abstract—In the recent years, extensive studies have been done to design space-time codes appropriate for communications over fading channels in multiple input-multiple output (MIMO) systems. Most of these designs have been based upon the assumption that the channel fading coefficients are uncorrelated hence independent jointly Gaussian random variables. Naturally the best strategy in such situations that the elements of the channel matrix are independent is to employ diversity techniques to combat the adverse effects of these fading media and thus the most famous space-time codes, i.e., orthogonal and trellis codes have been designed with an eye to realizing the maximum attainable diversity order in a MIMO system. In this paper, we will remove this almost ever-present yet practically difficult to meet condition and shall introduce a new linear space-time block code that due to having some inherent redundancy as well as diversity is well-suited to correlated fading channels. We will discuss the properties of the proposed code, derive its maximum likelihood (ML) decoder and provide simulation results which show its superiority to the highly used orthogonal space-time block codes in a wide range of signal to noise ratios in correlated fading channels.

1. INTRODUCTION

Man's insatiable appetite for achieving higher data rates and increased channel capacities at no bandwidth expense has led to the emergence of MIMO systems, which in turn have given birth to the topic of space-time coding to ensure reliable wireless communication over the fading multipath channel encountered by these systems. Tarokh, Seshadri, and Calderbank [1] were the first researchers to derive performance criteria for designing such codes. They introduced space-time trellis codes, an extension of the conventional convolutional codes which had

an encoding/decoding complexity comparable to that of trellis codes employed in practice over Gaussian channels and were shown to have excellent performance; providing the best tradeoff between data rate, diversity advantage, and coding complexity. Next, Alamouti [2] came up with a bright idea and put forward a linear block code for a MIMO system with two transmit and an optional number of receive antennas which could also achieve the maximum diversity order and due to its simple orthogonal structure, had a simple ML decoding that required only linear processing at the receiver. Despite its inferiority to space-time trellis codes in performance, its linear decoding characteristic and ease of implementation placed it at the focal point of attention. This fundamental work was shortly consummated by Tarokh, Jafarkhani, and Calderbank [3] who elegantly generalized the topic of orthogonal space-time block codes for both real and complex constellations and for any number of transmit antennas.

A keystone condition assumed by the majority of space-time code designers is that the channel fading coefficients are uncorrelated (for instance, see [3–5]), however, in many practical situations; this assumption may not be well-founded. Meeting it calls for large distances between the antennas at both the transmitting and receiving ends. Realization of this very condition is especially difficult in mobile communications where the hand-held user's units are expected to be small and economical.

Therefore, due to the above and a whole host of other constraints such as angle spread and the lack of rich scattering that may come to the fore in practice, special attention must be paid to the design of space-time codes in the case that the channel fading coefficients are correlated. The effect of such correlations on the system performance is studied in [6]. In [6], Bolcksei quantified the system's diversity order as a function of the ranks of the transmit and receive correlation matrices. The recent work on correlated fading includes [7–11]. In [7], Ivrlac et al. study the effects of fading correlations and transmitter channel knowledge on the capacity and cutoff rate for MIMO systems, and in [8], Chiani et al. derive closed form expression for the characteristic functions for MIMO system capacity for the correlated fading case. Smith et al. also study the capacity of MIMO systems, but they focus on semi-correlated flat fading [9]. Hong et al. investigate the design and performance of spatial multiplexing for MIMO correlated fading channels in [10]. In [11, 12], Abouda et al. study the effect of coupling on capacity of MIMO wireless channels in High SNR scenario. Hedayat et al. present a comprehensive analysis of MIMO systems under correlated fading and use their derived expressions for pairwise-error-probability to calculate union bounds on the performance of a

broad spectrum of space-time codes in different fading channels [13].

Needless to say that the higher the correlation between these coefficients, the greater the resemblance between MIMO and single input-single output (SISO) systems. In such circumstances, it is pretty obvious that depending solely on diversity and repetition techniques as orthogonal space-time block codes do is not the best adoptable strategy. Instead we believe that increasing the coding gain through adding some redundancy while maintaining a reasonable amount of diversity can improve the performance of linear space-time block codes in such likely to arise situations. Based on this reasoning, we propose a new linear space-time block code that compared with its orthogonal counterpart attains a smaller diversity order, yet a larger coding gain and as the simulation results verify, outperforms orthogonal space-time block codes in a rather wide range of signal to noise ratios in correlated fading channels.

The outline of the paper is as follows. In Section 2, we describe a mathematical model for the MIMO communication system. In Section 3, a new linear space-time code will be introduced. Section 4 deals with the properties of the proposed code such as its diversity gain, coding gain, etc. In Section 5, the ML decoder for the new code will be found. In Section 6, the performance of the proposed code is compared to that of its orthogonal counterpart via simulations. Finally Section 7 is dedicated to concluding remarks.

Notation: Bold uppercase letters denote matrices. \mathbf{I}_M denotes the identity matrix of size M . $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^*$ denote transpose, Hermitian transpose, and complex conjugate respectively. For a matrix \mathbf{X} , $\mathbf{X}_{i,j}$ denotes its (i, j) th entry, and $\mathbf{X}^{(L)}$ denotes the matrix formed by selecting the first L columns of \mathbf{X} . The vectorizing operator $\text{vec}(\mathbf{X})$ stacks the columns of the matrix \mathbf{X} in a column vector. For a complex number b , $|b|$ represents its absolute value.

2. THE MIMO CHANNEL MODEL

In this section, we describe a mathematical model for the wireless MIMO communication system subject to quasi-static and flat fading.

Consider a wireless communication system where the base station and the receiver have n and m antennas respectively. At each time slot t , signals c_t^i , $i = 1, 2, \dots, n$ are transmitted simultaneously from the n transmit antennas. The fading coefficient \mathbf{H}_{ij} is the path gain from transmit antenna j to receive antenna i . As mentioned earlier, these gains are modeled as correlated complex Gaussian random variables with unit variance and constitute an $m \times n$ channel matrix \mathbf{H} . The wireless channel is assumed to be quasi-static so that the channel

matrix remains unchanged over a frame of length L , i.e., L consecutive time slots and varies from one frame to another.

At time t the signal received at antenna j is given by

$$r_t^j = \sum_{i=1}^n \mathbf{H}_{ji} c_t^i + \eta_t^j \quad (1)$$

where η_t^j are independent zero-mean complex Gaussian random variables with variance $\frac{1}{2\text{SNR}}$ per dimension. The average energy of the symbols transmitted from each antenna is normalized to be $\frac{1}{n}$.

3. THE NEW LINEAR SPACE-TIME BLOCK CODE

In this section, we shall introduce a new linear space-time block code that borrows ideas from orthogonal designs [3] and Reed-Solomon codes. It is assumed that transmission at the baseband employs a signal constellation A with 2^b elements. At each time slot, Lb bits arrive at the encoder and select constellation signals s_1, s_2, \dots, s_L which are then arranged into a matrix $\mathbf{O}^T(s_1, s_2, \dots, s_L)$ where \mathbf{O} denotes an $L \times L$ orthogonal design introduced in [3], and $L < n$. Next we will add $(n - L)$ rows of zeros to $\mathbf{O}^T(s_1, s_2, \dots, s_L)$ to obtain an $n \times L$ matrix \mathbf{C} . Finally, the matrix \mathbf{C} undergoes a two-dimensional unitary transform to make an $n \times L$ matrix Θ . At each time slot $t = 1, 2, \dots, L$, the entries Θ_{it} , $i = 1, 2, \dots, n$ are transmitted simultaneously from transmit antennas $1, 2, \dots, n$ using a PAM or a QPSK constellation depending on the real or complex nature of the matrix Θ . Since we use nL symbols to transmit Lb bits, the bit/symbol rate of this coding scheme is $\frac{b}{n}$. For further clarification, the encoding procedure is described below for $n = 4$ and $L = 2$.

1 — $2b$ bits arrive at the encoder, selecting constellation signals s_1 and s_2 .

2 — These signals are arranged in an orthogonal matrix as follows:

$$\mathbf{O}^T(s_1, s_2) = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix}$$

3 — Two rows of zeros are added to $\mathbf{O}^T(s_1, s_2)$ to make the 4×2 matrix

$$\mathbf{C} = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

4 — As the final step, a two-dimensional unitary transform will be applied to the matrix \mathbf{C} and the columns of the thus obtained matrix will be transmitted via the MIMO system using a PAM or a QPSK constellation.

Necessary though it is not, we usually let $L \approx n/2$ to maintain a good balance between the amount of repetition, i.e., diversity and redundancy, i.e., coding.

4. PROPERTIES OF THE NEW SPACE-TIME CODE

In [1], two performance bench marks were derived for space-time codes, namely the rank criterion and the determinant criterion which are briefly restated here.

Given perfect channel state information at the receiver, we may approximate the probability that the receiver decides erroneously in favour of a signal $\mathbf{e} = e_1^1 e_1^2 \dots e_1^n e_2^1 e_2^2 \dots e_2^n \dots e_L^1 e_L^2 \dots e_L^n$ assuming that $\mathbf{c} = c_1^1 c_1^2 \dots c_1^n c_2^1 c_2^2 \dots c_2^n \dots c_L^1 c_L^2 \dots c_L^n$ was transmitted as follows [1]:

$$P(\mathbf{c} \rightarrow \mathbf{e} | \mathbf{H}) \leq \exp(-d^2(\mathbf{c}, \mathbf{e}) / (4N_0)) \quad (2)$$

where \mathbf{H} is the channel matrix, $N_0/2$ is the variance of noise per dimension, and

$$d^2(\mathbf{c}, \mathbf{e}) = \sum_{j=1}^m \sum_{t=1}^L \left| \sum_{i=1}^n \mathbf{H}_{ji} (c_t^i - e_t^i) \right|^2. \quad (3)$$

Further calculations in [1] lead to the following design criteria.

- *The Rank Criterion:* In order to achieve the maximum diversity mn , the following error matrix has to be full rank for every pair of distinct codewords \mathbf{c} and \mathbf{e} .

$$\mathbf{B}(\mathbf{c}, \mathbf{e}) = \begin{bmatrix} e_1^1 - c_1^1 & e_2^1 - c_2^1 & \dots & e_L^1 - c_L^1 \\ e_1^2 - c_1^2 & e_2^2 - c_2^2 & \dots & e_L^2 - c_L^2 \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ e_1^n - c_1^n & e_2^n - c_2^n & \dots & e_L^n - c_L^n \end{bmatrix} \quad (4)$$

If $\mathbf{B}(\mathbf{c}, \mathbf{e})$ has minimum rank r over the set of pairs of distinct codewords, then a diversity gain of rm is obtained. This criterion is valid for both Rayleigh and Rician channels.

• *The Determinant Criterion:* Suppose that a diversity order of rm is to be reached. The minimum of r th roots of the sum of determinants of all $r \times r$ principal cofactors of $\mathbf{A}(\mathbf{c}, \mathbf{e}) = \mathbf{B}(\mathbf{c}, \mathbf{e}) \mathbf{B}^H(\mathbf{c}, \mathbf{e})$ taken over all pairs of distinct codewords \mathbf{c} and \mathbf{e} determines the coding gain where r is the rank of $\mathbf{B}(\mathbf{c}, \mathbf{e})$. Indicating the r nonzero eigenvalues of $\mathbf{A}(\mathbf{c}, \mathbf{e})$ by $\{\lambda_i\}_{i=1}^r$, the coding gain can be equivalently written as $(\lambda_1 \lambda_2 \dots \lambda_r)^{1/r}$.

In the remaining of this section, we discuss the properties of the proposed space-time code.

• *The columns of the matrix \mathbf{C} introduced in Section 3 form an orthogonal set of vectors.*

Proof: This property is the immediate result of the orthogonality of $\mathbf{O}^T(s_1, s_2, \dots, s_L)$ which remains unaffected by the zero padding that makes the matrix \mathbf{C} out of it.

• *The rows of the matrix \mathbf{C} form an orthogonal set of vectors.*

Proof: Similar to that of the first property.

$$\bullet \mathbf{C}^H \mathbf{C} = \left(\sum_{i=1}^L |s_i|^2 \right) \mathbf{I}_L.$$

Proof: Let $\mathbf{D} = \mathbf{C}^H \mathbf{C}$. We can write:

$$\mathbf{D}_{ij} = \sum_{k=1}^n \mathbf{C}_{ki}^* \mathbf{C}_{kj}. \quad (5)$$

Due to the first property of the new code, the right side of the recent equality is zero when $i \neq j$. In other words, only the diagonal elements of \mathbf{D} are nonzero and we have:

$$\mathbf{D}_{ii} = \sum_{k=1}^n |\mathbf{C}_{ki}|^2 = \sum_{k=1}^L |\mathbf{C}_{ki}|^2 = \sum_{k=1}^L |s_k|^2 \quad (6)$$

It can be easily verified that $\mathbf{C} \mathbf{C}^H$ is an $n \times n$ matrix, the elements of which are all zero except the first L diagonal ones that are given by

$$\sum_{k=1}^L |s_k|^2.$$

Before proceeding further, we restate a definition from linear algebra. Let $\mathbf{x} = (x_1, x_2, \dots, x_k)$ and $\mathbf{y} = (y_1, y_2, \dots, y_k)$ be two complex vectors. The inner product of \mathbf{x} and \mathbf{y} is given by [14]

$$\mathbf{x} \cdot \mathbf{y} = \sum_{i=1}^k x_i y_i^*$$

- The columns of the matrix Θ form an orthogonal set of vectors.

Proof: We know that the matrix Θ is obtained by applying a two-dimensional unitary transform to \mathbf{C} . Therefore we can write:

$$\Theta = \mathbf{FCG} \quad (7)$$

where $\mathbf{F}_{n \times n}$ and $\mathbf{G}_{L \times L}$ are unitary matrices. Note that $\Theta_i = \mathbf{FCG}_i$ and $\Theta_j = \mathbf{FCG}_j$ where Θ_i represents the i th column of Θ and so on. We have:

$$\begin{aligned} \Theta_j \cdot \Theta_i &= \Theta_i^H \Theta_j = \mathbf{G}_i^H \mathbf{C}^H \mathbf{F}^H \mathbf{FCG}_j = \mathbf{G}_i^H \mathbf{C}^H \mathbf{I}_n \mathbf{CG}_j \\ &= \mathbf{G}_i^H \mathbf{C}^H \mathbf{CG}_j = \left(\sum_{k=1}^L |s_k|^2 \right) \mathbf{G}_i^H \mathbf{G}_j = 0 \end{aligned} \quad (8)$$

Corollary: The diversity gain of the new coding scheme is Lm .

Proof: We want to show that if $\mathbf{B} = \Theta - \Theta'$, then $\text{rank}(\mathbf{B}) = L$ for every distinct pair of codewords Θ and Θ' . Note that $\mathbf{B} = \mathbf{F}(\mathbf{C} - \mathbf{C}')\mathbf{G}$ where it is assumed that Θ and Θ' are obtained by applying the unitary transform given in (7) to \mathbf{C} and \mathbf{C}' . We observe that:

$$\begin{aligned} \mathbf{B}_j \cdot \mathbf{B}_i &= \mathbf{B}_i^H \mathbf{B}_j = \mathbf{G}_i^H (\mathbf{C} - \mathbf{C}')^H \mathbf{F}^H \mathbf{F} (\mathbf{C} - \mathbf{C}') \mathbf{G}_j \\ &= \mathbf{G}_i^H (\mathbf{C} - \mathbf{C}')^H (\mathbf{C} - \mathbf{C}') \mathbf{G}_j = \left(\sum_{k=1}^L |s_k - s'_k|^2 \right) \mathbf{G}_i^H \mathbf{G}_j = 0 \end{aligned} \quad (9)$$

In other words, the L columns of \mathbf{B} constitute an orthogonal hence linearly independent set of vectors. Therefore, $\text{rank}(\mathbf{B}) = L$ for every pair of distinct codewords, resulting in a diversity gain of Lm .

- The rows of the matrix Θ are not orthogonal.

Proof: We may write $\Theta_i = \mathbf{F}_i \mathbf{CG}$ and $\Theta_j = \mathbf{F}_j \mathbf{CG}$ where Θ_i represents the i th row of Θ and so on. We have:

$$\Theta_i \cdot \Theta_j = \Theta_i \Theta_j^H = \mathbf{F}_i \mathbf{CGG}^H \mathbf{C}^H \mathbf{F}_j^H = \mathbf{F}_i \mathbf{CC}^H \mathbf{F}_j^H \quad (10)$$

Using the third property of the new code reduces (10) to

$$\left(\sum_{k=1}^L |s_k|^2 \right) \left(\sum_{p=1}^L \mathbf{F}_{ip} \mathbf{F}_{jp}^* \right). \quad (11)$$

$\sum_{k=1}^L |s_k|^2$ is clearly nonzero and so is $\sum_p \mathbf{F}_{ip} \mathbf{F}_{jp}^*$ when the index of summation varies from 1 to L and $L < n$, thus ending the proof of the recent property.

Corollary: This property rules out the possibility of ML detection of the transmitted block of signals using only linear processing at the receiver. In fact as will be explained in Section 5, ML decoding of a received block entails an exhaustive search over the set of all b^L possible Θ matrices.

- *The proposed code is maximum distance separable (MDS).*

Proof: By looking at the structure of a typical \mathbf{C} matrix, we may write:

$$\mathbf{C} = \begin{bmatrix} \mathbf{U}_{L \times L} \\ \mathbf{0}_{(n-L) \times L} \end{bmatrix}$$

where $\mathbf{0}_{(n-L) \times L}$ denotes an $(n-L) \times L$ zero matrix and in order to avoid any confusion, we have represented the $L \times L$ orthogonally arranged block of information with \mathbf{U} instead of $\mathbf{O}^T(s_1, s_2, \dots, s_L)$. Note that:

$$\Theta = \mathbf{FCG} = \mathbf{F} \begin{bmatrix} \mathbf{U}_{L \times L} \\ \mathbf{0}_{(n-L) \times L} \end{bmatrix} \mathbf{G} = \mathbf{F} \begin{bmatrix} (\mathbf{UG})_{L \times L} \\ \mathbf{0}_{(n-L) \times L} \end{bmatrix} = \mathbf{FX} \quad (12)$$

where $\mathbf{X} = \mathbf{CG}$. Let $\mathbf{U}' = \mathbf{UG}$ and let Θ_i , \mathbf{U}'_i , and \mathbf{X}_i denote the i th columns of Θ , \mathbf{U}' , and \mathbf{X} respectively. We can write:

$$\begin{aligned} \Theta &= [\Theta_1 | \Theta_2 | \dots | \Theta_L] = \mathbf{F} [\mathbf{X}_1 | \mathbf{X}_2 | \dots | \mathbf{X}_L] = [\mathbf{FX}_1 | \mathbf{FX}_2 | \dots | \mathbf{FX}_L] \\ &= \mathbf{F} \begin{bmatrix} (\mathbf{U}'_1)_{L \times 1} & | & (\mathbf{U}'_2)_{L \times 1} & | & \dots & | & (\mathbf{U}'_L)_{L \times 1} \\ \mathbf{0}_{(n-L) \times 1} & | & \mathbf{0}_{(n-L) \times 1} & | & \dots & | & \mathbf{0}_{(n-L) \times 1} \end{bmatrix} = \mathbf{F}^{(L)} [\mathbf{U}_1 | \mathbf{U}_2 | \dots | \mathbf{U}_L] \end{aligned} \quad (13)$$

where the last equality is a direct result of the fact that:

$$\mathbf{FX}_i = \mathbf{F}^{(L)} \mathbf{U}'_i \quad (14)$$

A closer examination of (14) reveals an interesting fact. One can regard \mathbf{U}'_i as an information vector of length L and $\mathbf{F}^{(L)}$ as the full-rank generator matrix of an (n, L) linear block code. We shall represent the minimum hamming distance of this code with d_{\min} . Also interesting to note is that the remaining $n-L$ orthogonal columns of \mathbf{F} constitute the full-rank parity check matrix of the aforementioned linear block code. It is a well known fact that the minimum hamming distance of a linear block code equals the rank of its parity check matrix plus one (See [15] for a detailed proof of the recent statement). Since the parity check matrix is full-rank, $d_{\min} = n - L + 1$ and maximum distance separability follows suit.

According to the above interpretation, the i th column of the codeword Θ , i.e., Θ_i can be regarded as a codevector produced by the

information vector \mathbf{U}'_i and the generator matrix $\mathbf{F}^{(L)}$. If we represent the minimum hamming distance between two codewords Θ and Θ' or equivalently the minimum hamming distance of the proposed space-time block code by d , we can obviously write:

$$d \geq Ld_{\min} = L(n - L + 1) . \quad (15)$$

- *We believe that due to its inherent redundancy as well as diversity, the new code can provide more coding gain than orthogonal space-time block codes.*

For instance, using a BPSK constellation of $\{\pm 1\}$ at baseband, the new code with the two-dimensional inverse discrete cosine transform (IDCT) as its unitary transform yields a coding gain of 8 when $n = 4$ and $L = 2$ in a Rayleigh channel which is twice that of its orthogonal counterpart, i.e., a 4×4 orthogonal code. This result has been obtained by finding the matrix \mathbf{A} introduced in Section 4 together with its rank r for every pair of distinct codewords and determining the minimum of r th roots of the product of its r nonzero eigenvalues for both coding schemes.

We believe that this most prominent feature of the new space-time code gives it the upper hand in performance at lower signal to noise ratios compared with the conventional orthogonal codes in practical communication scenarios that the elements of the channel matrix are correlated. As will be shown by the simulation results, the higher the correlation between the path gains, the more the similarity between MIMO and SISO systems, the greater the gap in performance between the new code and orthogonal repetition-based codes and the wider the range of signal to noise ratios over which the proposed code performs better.

5. ML DECODING OF THE PROPOSED LINEAR SPACE-TIME BLOCK CODE

Assuming perfect channel information, the ML decoder of a MIMO system suffering from fading and corrupted by additive white Gaussian noise described by (1) computes the decision metric

$$\sum_{t=1}^L \sum_{j=1}^m \left| r_t^j - \sum_{i=1}^n \mathbf{H}_{ji} \Theta_{it} \right|^2 \quad (16)$$

over all distinct codewords Θ and decides in favour of the one that minimizes this sum.

In orthogonal codes, the columns of each transmitted array are all permutations of the first column with possibly different signs. Therefore, having the first column determines the codeword uniquely. Owing to the orthogonality of the rows of the transmitted array, ML decoding can be done by simple linear processing as explained in [3].

In contrast, as proved in Section 4, the rows of the transmitted arrays are not orthogonal in the proposed coding scheme. Thus for these codes, ML decoding cannot be realized via linear processing, instead it entails minimizing (12) over the set of all possible Θ matrices, the number of which is 2^{bL} as we will prove in the following.

Proof: Due to their one to one correspondence, the number of Θ matrices equals the number of \mathbf{C} matrices introduced in Section 3. The structure of a \mathbf{C} matrix is such that having the first L elements of its first column determines it uniquely. Each of these elements can be chosen from a set of 2^b constellation signals, resulting in a total of $(2^b)^L = 2^{bL}$ possibilities for the \mathbf{C} hence Θ matrices.

6. ERROR PERFORMANCE SIMULATIONS

In the simulations, a BPSK baseband constellation of $\{\pm 1\}$ is employed. The additive noise components of the m receive antennas are assumed to be independent identically distributed zero-mean complex Gaussian random variables, each having a variance of $\frac{1}{2\text{SNR}}$ per dimension. The two-dimensional IDCT is used as the unitary transform for the proposed code. The plots in this section show the bit error rate (BER) vs. signal to noise ratio (SNR) for both orthogonal coding scheme and the proposed code. For the comparisons to be fair, the following conditions must be met.

- *Both codes must use the same number of transmit and receive antennas.*
- *Both codes must have similar bit/symbol rates. Therefore, we always compare an $n \times n$ orthogonal code with an $n \times L$ proposed code.*
- *The total power radiated per bit by the transmit antennas must be the same for both codes. Since the \mathbf{C} matrices introduced in Section 3 contain zeros in their structure, they must be multiplied by a power equalizing factor of $\sqrt{n/L}$.*

In our simulations, we set $n = 4$, $L = 2$, and $m = 2$. The graphs of this section compare the performance of the codes for different amounts of correlation between the fading coefficients, including the two extreme cases of zero and full correlation.

Let $\mathbf{H}'_{nm \times 1} = \text{vec}(\mathbf{H})$ and name it the channel vector. The

covariance matrices of the five channel vectors employed in the simulations are given below:

$$\mathbf{COV}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{COV}_2 =$$

$$\begin{bmatrix} 1 & 0.36897 & 0.35641 & 0.41552 & 0.98424 & 0.27016 & 0.39232 & 0.37191 \\ 0.36897 & 1 & 0.47387 & 0.57817 & 0.51773 & 0.98768 & 0.46888 & 0.45216 \\ 0.35641 & 0.47387 & 1 & 0.53669 & 0.45548 & 0.36136 & 0.99435 & 0.41374 \\ 0.41552 & 0.57817 & 0.53669 & 1 & 0.522 & 0.47622 & 0.46702 & 0.98415 \\ 0.98424 & 0.51773 & 0.45548 & 0.522 & 1 & 0.41765 & 0.48376 & 0.45893 \\ 0.27016 & 0.98768 & 0.36136 & 0.47622 & 0.41765 & 1 & 0.35976 & 0.35258 \\ 0.39232 & 0.46888 & 0.99435 & 0.46702 & 0.48376 & 0.35976 & 1 & 0.33594 \\ 0.37191 & 0.45216 & 0.41374 & 0.98415 & 0.45893 & 0.35258 & 0.33594 & 1 \end{bmatrix}$$

$$\mathbf{COV}_3 =$$

$$\begin{bmatrix} 1 & 0.77448 & 0.81911 & 0.78613 & 0.99256 & 0.73918 & 0.83248 & 0.69186 \\ 0.77448 & 1 & 0.73125 & 0.80142 & 0.79665 & 0.99704 & 0.7667 & 0.7135 \\ 0.81911 & 0.73125 & 1 & 0.83435 & 0.76733 & 0.73869 & 0.99166 & 0.79874 \\ 0.78613 & 0.80142 & 0.83435 & 1 & 0.79836 & 0.80567 & 0.79168 & 0.98696 \\ 0.99256 & 0.79665 & 0.76733 & 0.79836 & 1 & 0.75887 & 0.77965 & 0.7022 \\ 0.73918 & 0.99704 & 0.73869 & 0.80567 & 0.75887 & 1 & 0.77125 & 0.72592 \\ 0.83248 & 0.7667 & 0.99166 & 0.79168 & 0.77965 & 0.77125 & 1 & 0.73884 \\ 0.69186 & 0.7135 & 0.79874 & 0.98696 & 0.7022 & 0.72592 & 0.73884 & 1 \end{bmatrix}$$

$$\mathbf{COV}_4 =$$

$$\begin{bmatrix} 1 & 0.92306 & 0.92364 & 0.93597 & 0.9962 & 0.95694 & 0.91799 & 0.897 \\ 0.92306 & 1 & 0.89855 & 0.91012 & 0.91898 & 0.99417 & 0.91233 & 0.8636 \\ 0.92364 & 0.89855 & 1 & 0.90411 & 0.88885 & 0.91371 & 0.99788 & 0.93418 \\ 0.93597 & 0.91012 & 0.90411 & 1 & 0.9207 & 0.93984 & 0.91763 & 0.9805 \\ 0.9962 & 0.91898 & 0.88885 & 0.9207 & 1 & 0.95339 & 0.8831 & 0.86528 \\ 0.95694 & 0.99417 & 0.91371 & 0.93984 & 0.95339 & 1 & 0.92398 & 0.89295 \\ 0.91799 & 0.91233 & 0.99788 & 0.91763 & 0.8831 & 0.92398 & 1 & 0.946 \\ 0.897 & 0.8636 & 0.93418 & 0.9805 & 0.86528 & 0.89295 & 0.946 & 1 \end{bmatrix}$$

$$\mathbf{COV}_5 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Figures 1–5 compare the performance of the proposed code and its orthogonal counterpart in Rayleigh fading channels. As Fig. 1 shows, when there is no correlation between the fading coefficients, orthogonal codes outperform the proposed code. However as the amount of correlation increases, this superiority fades away. In fact it is observed that when there is some correlation between the elements of the channel matrix, the new code performs better than its orthogonal counterpart up to some SNR, and then the situation is reversed. It is also seen that as this correlation rises, the new code achieves better performance over a wider range of SNRs, pushing the intersection point of the two BER curves further to the right (See Figs. 2, 3, and 4). Finally, as shown in Fig. 5, in the extreme case of experiencing full correlation between the fading coefficients, i.e., when they are all equal and the MIMO system is not any different from the SISO system, the proposed code attains much better performance in comparison to its

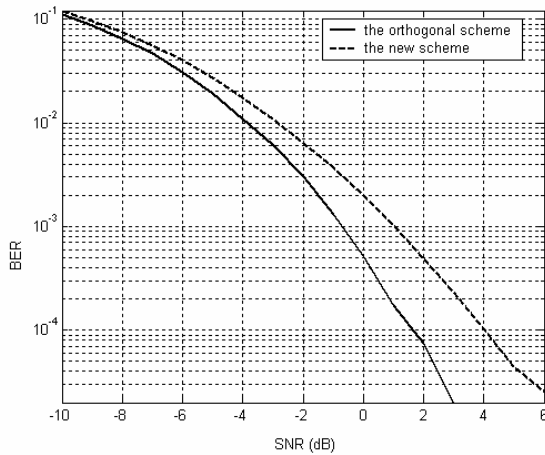


Figure 1. The BER performance comparison of the new code with its orthogonal counterpart in an uncorrelated Rayleigh fading channel (corresponding to $\mathbf{COV1}$) for $n=4$, $L=2$, and $m=2$.

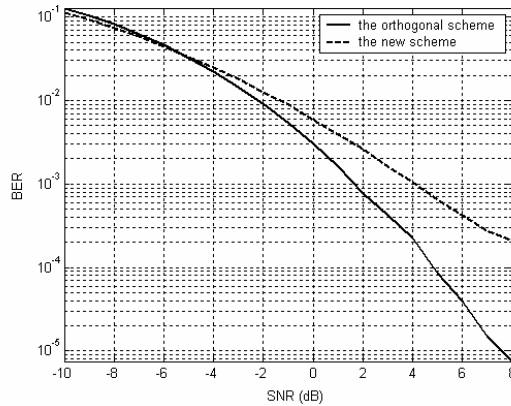


Figure 2. The BER performance comparison of the new code with its orthogonal counterpart in a correlated Rayleigh fading channel (corresponding to \mathbf{COV}_2) for $n = 4$, $L = 2$, and $m = 2$.

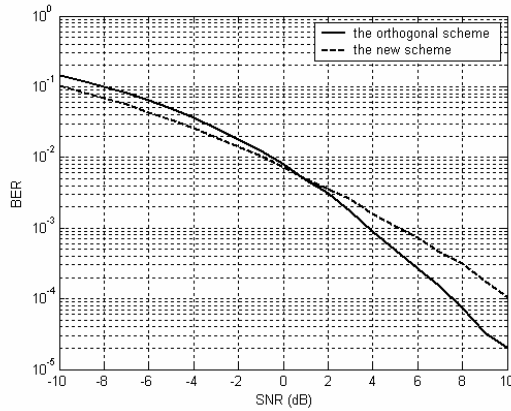


Figure 3. The BER performance comparison of the new code with its orthogonal counterpart in a correlated Rayleigh fading channel (corresponding to \mathbf{COV}_3) for $n = 4$, $L = 2$, and $m = 2$.

orthogonal counterpart for all practical SNRs. Figs. 6–7 compare the performance of the two coding schemes in Ricean fading channels with the line-of-sight component of the signal having as much power as all the other components. In other words, if we denote the ratio of the power of the line-of-sight component of the signal and the power of all the other components by K_f , the simulations have been done with $K_f = 1$. Once again it is observed that a boost in channel correlation

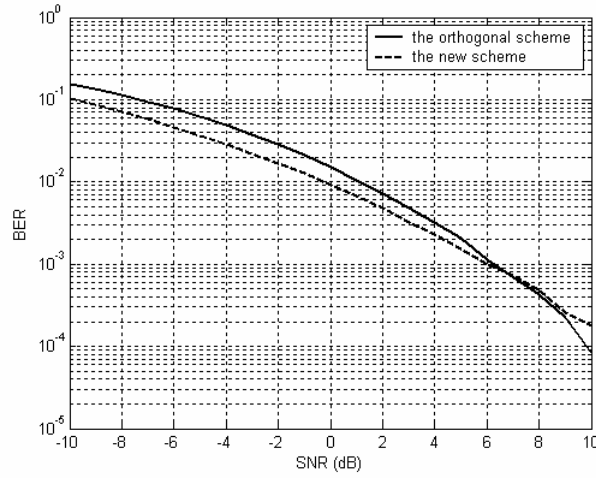


Figure 4. The BER performance comparison of the new code with its orthogonal counterpart in a correlated Rayleigh fading channel (corresponding to \mathbf{COV}_4) for $n = 4$, $L = 2$, and $m = 2$.

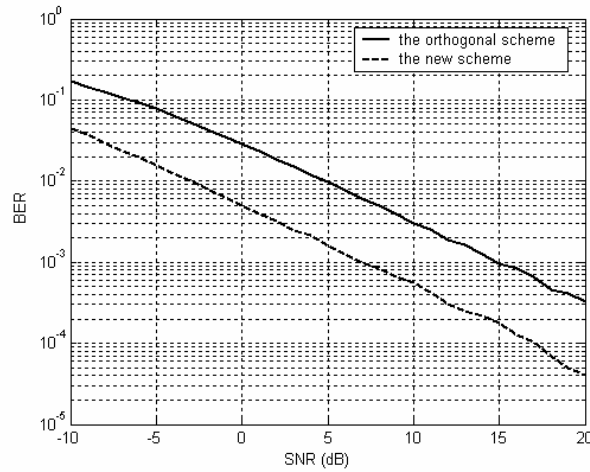


Figure 5. The BER performance comparison of the new code with its orthogonal counterpart in a fully-correlated Rayleigh fading channel (corresponding to \mathbf{COV}_5) for $n = 4$, $L = 2$, and $m = 2$.

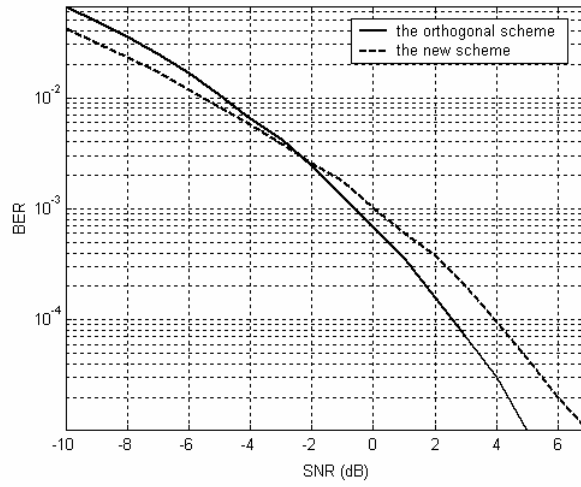


Figure 6. The BER performance comparison of the new code with its orthogonal counterpart in a correlated Ricean fading channel (corresponding to \mathbf{COV}_2) for $n = 4$, $L = 2$, $m = 2$, and $K_f = 1$.

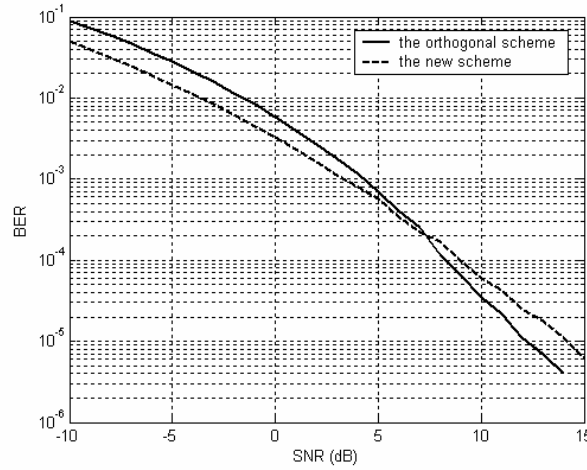


Figure 7. The BER performance comparison of the new code with its orthogonal counterpart in a correlated Ricean fading channel (corresponding to \mathbf{COV}_4) for $n = 4$, $L = 2$, $m = 2$, and $K_f = 1$.

results in the superiority of the new scheme in a wider range of SNRs to its orthogonal counterpart.

7. CONCLUSIONS

In this paper, we presented a new linear MDS space-time block code, discussed its properties and derived its ML decoder. We believe that due to its greater coding gain, it can perform better than the conventional linear orthogonal space-time block codes in a realistic situation when there is some correlation between the fading coefficients. In other words, we think that relying mainly on repetition and attempting to increase the diversity will not yield the best possible results. It seems that increasing the coding gain through adding some redundancy to the sent information while considering a reasonable amount of repetition in the design of the channel code is more fruitful in such situations and can enhance the performance. Bearing witness to the veracity of this claim are the error performance simulation results presented in Section 6. Of course we must add that this improvement comes at the cost of increased decoding complexity in comparison to the conventional orthogonal space-time block codes.

We believe that the studies we initiated here, only scratch the tip of the iceberg, yet they unveil a coding strategy better suited to correlated fading channels than the well known orthogonal space-time block codes in lower signal to noise ratios. It was also shown via simulations that when the correlation between the fading coefficients rose [16–24], the range of signal to noise ratios over which the proposed code outperformed its orthogonal counterpart and the gap between their performance curves widened.

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