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BAND STRUCTURES AND ABNORMAL BEHAVIOR OF ONE DIMENSIONAL PHOTONIC CRYSTAL CONTAINING NEGATIVE INDEX MATERIALS

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Abstract—We have studied the optical properties, band structures and group velocities, of one dimensional photonic crystal (1-D PC) containing negative index materials using translational matrix method (TMM). The 1-D PC containing negative index materials is a periodic arrangement of positive index material (PIM) and negative index material (NIM). The observed group velocity of such structure is larger than the speed of light in certain range of normalized frequency which shown abnormal behaviors. The group velocity of the PC containing negative index materials have found larger than the speed of light. The values of the group velocity become zero, negative and positive for normalized frequencies. The structure containing negative index

materials can be used to trap the photons inside the PC i.e., light localization occurs without introducing the defects.

1. INTRODUCTION

The artificial micro-structured materials with a periodically modulated dielectric functions are known as photonic crystals (PCs). Intensive investigations on PCs have been carried out since the idea of PCs was proposed [1, 2]. Up to now, most calculations of the photonic band structures and the corresponding electric/magnetic field distributions have been performed based on numerical methods, such as the plane wave expansion method and the finite difference time domain method [3, 4]. Some qualitative conclusions have been derived from the related numerical analysis. It is well known that the exact analytical solutions are quite useful for us to understand the physics of PCs. In a sense, the simplest one-dimensional (1D) PC with Kronig-Penney periodic dielectric structure [5, 6] is the only exactly solvable theoretical model. The purpose is to draw new physics out of the 1-D PC by means of the exact analytical solutions.

From Maxwell's equations the refractive index is given by the Maxwell relation, $n^2 = \varepsilon\mu$, where ε is the relative dielectric permittivity and μ is the relative magnetic permeability of the medium. Generally materials have taken positive for the both ε and μ, and refractive index is taken as $\sqrt{\varepsilon\mu}$ without any problems. In 1968 Veselago [7] has considered the case of a medium that had both negative dielectric permittivity and negative magnetic permeability at a given frequency and concluded that the medium should be considered to have a negative refractive index (i.e., the negative square root, $n = -\sqrt{\varepsilon\mu}$ had to be chosen). The several interesting effects such as a modified Snell's law of refraction, a reversed Doppler shift and an obtuse angle for Cerenkov radiation have been pointed out in such media. However theoretical proposals [8–11] for structured photonic media whose ε and μ could become negative in certain frequency ranges were developed experimentally [12, 13]. The striking demonstration of negative index materials can be used to make perfect lenses with resolution capabilities not limited by the conventional diffraction limit has given an enormous boost to the interest in negative index materials.

Ramakrishna [14] has examined the problem of making materials with negative material parameters which are mostly micro-structured composites. The design of negative index materials (meta-materials) show a negative refractive index is an extreme form of electromagnetic engineering. The negative material parameters mostly result from

an over-screened, under-damped response due to the structural resonances. A negative refractive index is obtained by interleaving two structures that individually show a negative ε or negative μ. Zhang et al. [15] has calculated the photon tunneling via evanescent fields in the presence of a layer of negative refraction material in contrast to the conventional right-handed materials. They have shown that photon is tunneled through a much greater distance when a NIM has same magnitude of refraction index and thickness as those of the positive index material. Wu et al. [16] have shown the dispersion relation of one-dimensional periodic structure with alternate positive index material (PIM) and negative index material (NIM). The periodic structure has described unusual phenomena such as spurious modes with complex frequencies, discrete modes and photon tunneling modes. Jiang et al. [17] have studied the transmission properties of a one-dimensional photonic crystal containing two kinds of single negative media. This structure can posses a type of photonic band gap with zero effective phase (ϕ_{eff}). The zero-ϕ_{eff} gap distinguishes itself from a Bragg gap in that it is invariant with a change of scale length and is insensitive to thickness fluctuation. Baria et al. [18] have investigated the band structures of one-dimensional superlattices composed of right handed materials and left handed materials. It is shown that such structures can exhibit new type of electromagnetic modes and dispersion curve that do not exist in usual superlattices composed only by right handed materials. With an appropriate choice of parameters, such band structure is possible to realize an absolute band gap for both TE and TM polarization; this proposed an omni-directional reflection of light for both polarizations. The complete band gap in one-dimensional left handed periodic structures has been purposed. The modulation of refractive index in all three spatial directions is required to find a complete band gap and prevent the propagation of electromagnetic waves in all directions [19–21]. Singh et al. [22] have investigated the structural parameters for the formation of omni-directional band gaps in one dimensional photonic crystal. The omni-directional band gaps are increased with increasing the parameters of refractive index contrast and filling fraction. They have found that the omni-directional reflection ranges of the Si-SiO_2 periodic system is gone to a maximum and a zero when filling fraction is increased. Nicolae et al. [23] have demonstrated that photonic superlattices consisting of periodic distribution of PIM and NIM superlattices have found zero-averaged index band gap. Such band gap is invariant to the geometrical scaling of the superlattices or the direction of wave propagation in the superlattices. Jiang et al. [24] have shown the property of one dimensional photonic crystals with an ε-negative and a μ-negative

defect. With suitable parameters, the pair defect is equivalent to a transparent material with zero effective refractive index. Due to increasing the pair of defect, the gap-edge field can be a highly localized wave instead of the usual standing wave. Recently Ramakrishna et al. [25] have shown that the arrival times for electromagnetic pulses is measured through the rate of absorption in an ideal impedance matched detector are equivalent to the arrival times using the average flow of optical energy. They have also investigated that the transport of optical pulses have not occurred resonant effect through dispersive media with negative dielectric permittivity and negative refractive index. Motivated this, we have studied the band structures and group velocities of the conventional photonic crystal and photonic crystal containing negative index materials with different filling fractions and contrast of refractive indices. The optical properties, band structures and group velocities, are calculated using translational matrix method.

2. THEORY

The dispersion relations, reflection characteristics, group velocities and n_{eff} of the periodic structure contain negative index materials is calculated numerically using transfer matrix method [16, 19, 26]. The geometry of the structure under study is shown in Fig. 1. The propagation of electromagnetic wave is considered along the x-axis normal to the interface in one-dimensional system composed of periodic arrays of two different materials with a refractive index n_1 and n_2 and layer thickness d_1 and d_2. The refractive index profile of the structure is

$$n(x) = \begin{cases} n_1, & 0 < x < d_1 \\ n_2, & d_1 < x < d \end{cases} \tag{1}$$

with $n(x) = n(x + d)$. Here, $d = d_1 + d_2$ is the period of the lattice (or lattice constant).

To solve the electric field vectors of the Bloch wave, we used translational matrix method [16, 25]. The electromagnetic field distribution within each layer can be expressed as the sum of right- and left-hand side propagating waves. The electric field within the both layers of the nth unit cell can be written as:

$$\begin{aligned} E_1(x) &= \left[\left(a_n e^{-ik_1(x-nd)}\right) + b_n e^{ik_1(x-nd)}\right] e^{i\omega t} \\ E_2(x) &= \left[\left(c_n e^{-ik_2(x-nd)}\right) + d_n e^{ik_2(x-nd)}\right] e^{i\omega t} \end{aligned} \tag{2}$$

where $k_i = \left[\left(\frac{n_i\omega}{c}\right)^2 - \beta^2\right]^{1/2} = \frac{n_i\omega}{c}\cos\theta_i$; θ_i is the ray angle in the ith layer ($i = 1, 2$), β is the propagation constant and $n_i = \sqrt{\varepsilon_i\mu_i}$, where

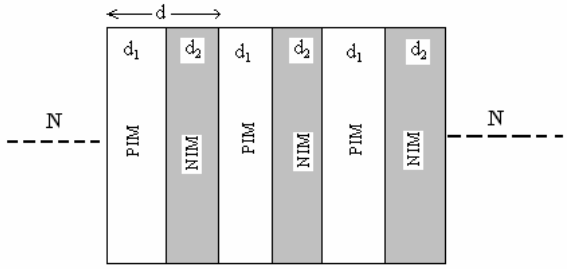

Figure 1. Schematic representation of the proposed multilayer structure.

ε_i and μ_i are the dielectric permittivity and magnetic permeability of the constituent layers. The coefficients a_n, b_n, c_n, and d_n are related through the continuity conditions at the interfaces $x = (n-1)d$ and $x = (n-1)d + d_2$. This continuity condition leads to the matrix equations, which relates the coefficient in the first layer of the nth cell, is given as:

$$\begin{bmatrix} a_{n-1} \\ b_{n-1} \end{bmatrix} = T_n \begin{bmatrix} a_n \\ b_n \end{bmatrix} \tag{3}$$

where T_n is called the transfer matrix given by

$$T_n = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{4}$$

The matrix elements A, B, C and D are given by:

$$\begin{aligned} A &= e^{ik_1d_1}\left[\cos k_2d_2 + \frac{1}{2}i\left(\eta + \frac{1}{\eta}\right)\sin k_2d_2\right]; \\ B &= e^{-ik_1d_1}\left[\frac{1}{2}i\left(\eta - \frac{1}{\eta}\right)\sin k_2d_2\right]; \\ C &= e^{ik_1d_1}\left[-\frac{1}{2}i\left(\eta - \frac{1}{\eta}\right)\sin k_2d_2\right] \& \\ D &= e^{-ik_1d_1}\left[\cos k_2b - \frac{1}{2}i\left(\eta + \frac{1}{\eta}\right)\sin k_2d_2\right] \end{aligned} \tag{5}$$

The parameter η depends on the polarizations. For the TE- and TM-mode of polarizations, η is given by

$$\eta_{\mathrm{TE}} = \frac{k_1}{k_2} \quad \text{and} \quad \eta_{\mathrm{TM}} = \frac{k_1 n_2^2}{k_2 n_1^2} \tag{6}$$

For finite stacks the coefficients of right and left hand sides propagating waves in both sides of the multiplayer structure a_N and b_N, are calculated by multiplying transfer matrix of each cell as [26];

$$\begin{bmatrix} a_0 \\ b_0 \end{bmatrix} = T_1 T_2 \dots T_N \begin{bmatrix} a_N \\ b_N \end{bmatrix}, \tag{7}$$

where N is the total number of the cell. The coefficient of reflection is given by solving above matrix equation with the condition $b_N = 0$ as:

$$r_N = \left(\frac{b_0}{a_0} \right). \tag{8}$$

Thus the reflectivity (or reflectance) of the structure may be calculated as:

$$R_N = |r_N|^2 . \tag{9}$$

Now, according to Bloch theorem, the electric field vector is of the form $E = E_K(x)e^{i(\omega t - Kx)}$, where $E_K(x)$ is periodic with the period 'd'. For to determine of K as a function of eigen value, the equation is written as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} a_n \\ b_n \end{bmatrix} = e^{iKd} \begin{bmatrix} a_n \\ b_n \end{bmatrix} \tag{10}$$

The solution of this matrix equation leads to the dispersion relation for the PC structure containing the alternate stack of positive index materials (PIMs) is given by:

$$K(\omega) = \left(\frac{1}{d} \right) \cos^{-1} \left[\cos(k_1 d_1) \cos(k_2 d_2) - \frac{1}{2} \left(\eta + \frac{1}{\eta} \right) \sin(k_1 d_1) \sin(k_2 d_2) \right] \tag{11}$$

The dispersion relation for the PC structure containing the alternate layer of positive index material (PIM) and negative index material (NIM) is given by [15]:

$$K(\omega) = \left(\frac{1}{d} \right) \cos^{-1} \left[\cos(k_1 d_1) \cosh(k_2 d_2) - \frac{1}{2} \left(\eta + \frac{1}{\eta} \right) \sin(k_1 d_1) \sinh(k_2 d_2) \right] \tag{12}$$

which is different from the normal PC structure, since for LHMs, $k_2 < 0$ (because $n_2 < 0$).

The medium characterized by the refractive index variation $n(\omega)$. The phase velocity of the wave in PC containing NIM is $v_p = \frac{\omega}{k(\omega)}$. For

homogeneous and isotropic periodic dielectric stack, k is the dispersion relation of photonic crystals i.e., $\vec{K}(\omega)$. This dispersion relation shows the radiation modes of the photonic crystal: if wave function can also be calculated. In addition to the eigen frequency and eigen function, there are several parameters that characterized the radiational waves, one of them is the wave velocity. In contrast to the case particles for which the velocity has single meaning; waves have three different kinds of velocities i.e., phase velocity, group velocity and energy velocity. These velocities are equal to each other in uniform materials with dielectric contrasts is real and independent of frequency. So that effective phase index is $n_{eff}(p) = \frac{c}{v_p}$ [27]. The phase velocity is defined as the velocity of the propagation of an equi-phase surface as discussed earlier in this section. This velocity has a definite meaning, for example, for plane waves and spherical waves for which equi-phase surface can be defined without ambiguity. In the photonic crystals, however, the equi-phase surface can not be defined rigorously, since its eigen function is a superposition of plane waves. This means that the phase velocity can not defined appropriately in the photonic crystals [28, 29]. Hence, the calculation of the group velocity is an essential task for the understanding of their optical properties. The group velocity of the radiation modes has a very important role in light propagation and optical response in photonic crystals. The group velocity and effective group index in one-dimensional photonic crystal, which is given by:

$$v_g = \frac{d\omega}{dk(\omega)} \quad \text{and} \quad n_{eff}(g) = \frac{c}{v_g} \tag{13}$$

Table 1.

	n_1	n_2	a	b
(a)	1.0	-3.6	$0.8d$	$0.2d$
(b)	1.0	-1.95	$0.8d$	$0.2d$
(c)	1.0	-1.35	$0.8d$	$0.2d$
(d)	1.0	-1.35	$0.5d$	$0.5d$
(e)	1.0	-1.95	$0.5d$	$0.5d$
(f)	1.0	-3.6	$0.5d$	$0.5d$

Table 2.

	n_1	n_2	a	b
(g)	1.0	1.35	$0.8d$	$0.2d$
(h)	1.0	1.95	$0.8d$	$0.2d$
(i)	1.0	3.6	$0.8d$	$0.2d$
(j)	1.0	1.35	$0.5d$	$0.5d$
(k)	1.0	1.95	$0.5d$	$0.5d$
(l)	1.0	3.6	$0.5d$	$0.5d$

3. RESULTS AND DISCUSSION

In this section, we have studied the dispersion characteristics, group velocities and effective group indices of the structure containing alternate layers of PIM-NIM and the results are compared with the

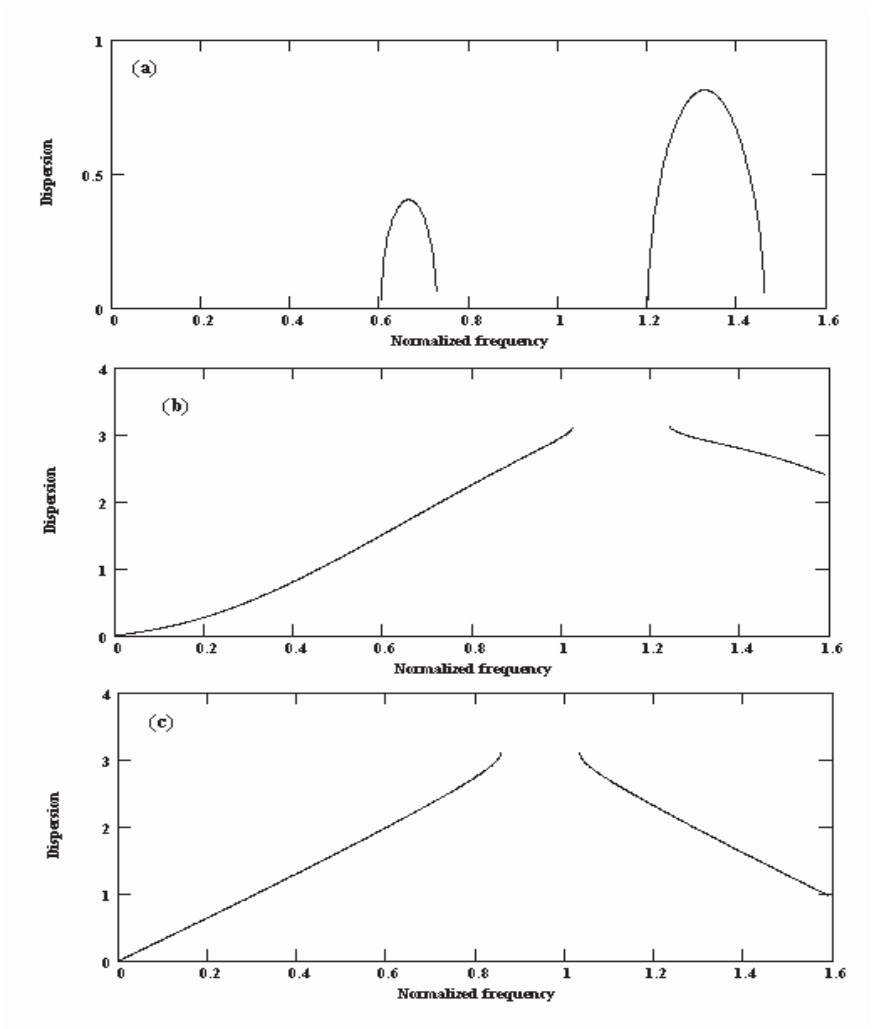

Figure 2. Band structures (xd) versus normalized frequency (in units of c/d) with $0.8d$ and $0.2d$ of thicknesses of PIM-NIM respectively (a) $n_1 = 1.0$, $n_2 = -3.6$; (b) $n_1 = 1.0$, $n_2 = -1.95$; (c) $n_1 = 1.0$, $n_2 = -1.35$.

conventional structure consisting alternate layers of PIM-PIM. For the numerical calculations refractive indices of the alternate layers and their thickness are shown in Tables 1 and 2.

The dispersion curves calculated from Equations (12) for the photonic crystal containing PIM-NIM materials, physical parameters given in Table 1, are shown in the Fig. 2 & Fig. 3. Using Equation (11),

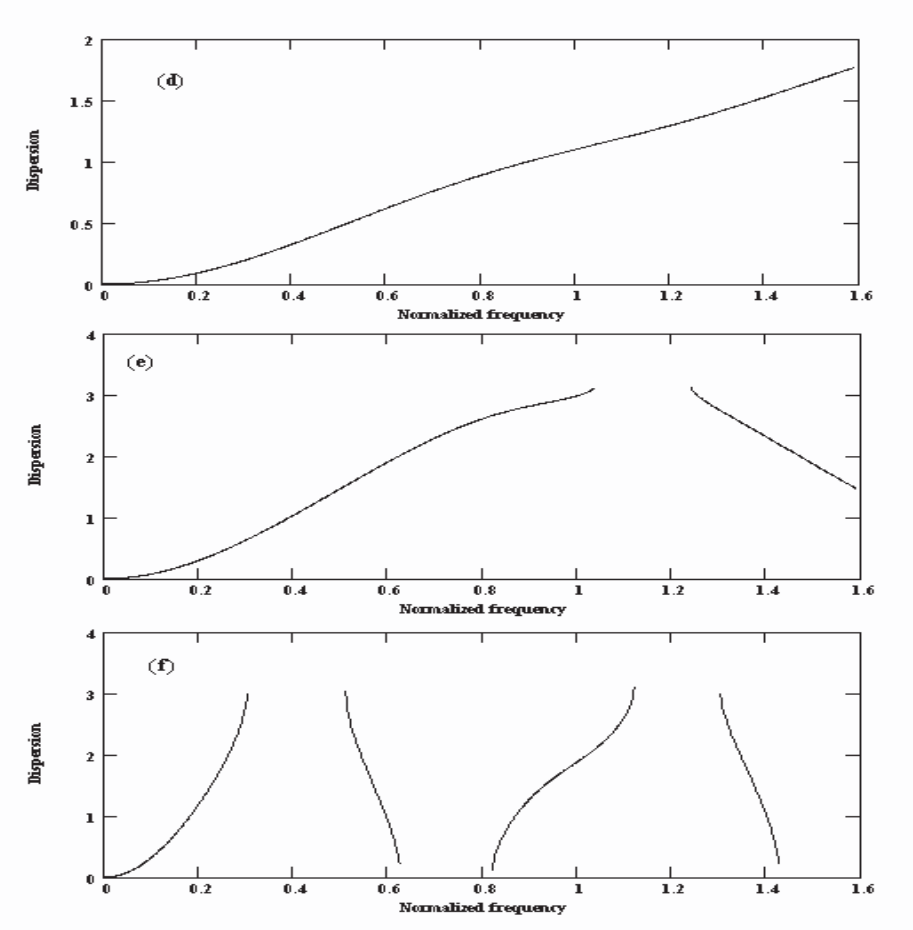

Figure 3. Band structures (xd) versus normalized frequency (in units of c/d) with $0.5d$ and $0.5d$ of thicknesses of PIM-NIM respectively (d) $n_1 = 1.0$, $n_2 = -1.35$; (e) $n_1 = 1.0$, $n_2 = -1.95$; (f) $n_1 = 1.0$, $n_2 = -3.6$.

the dispersion curve for the conventional photonic crystal containing PIM-PIM materials, physical parameters given in Table 2, are shown in the Fig. 4 & Fig. 5. From the study of these figures, it is found that the structure containing the alternate layers of PIM-NIM have larger band gaps than those of PIM-PIM structures in a given normalized frequency range (in units of c/d). Also it is found that for $n_1 = 1.0$, and $n_2 = -3.6$, $a = 0.8d$ and $b = 0.2d$ where d is total thickness of lattices,

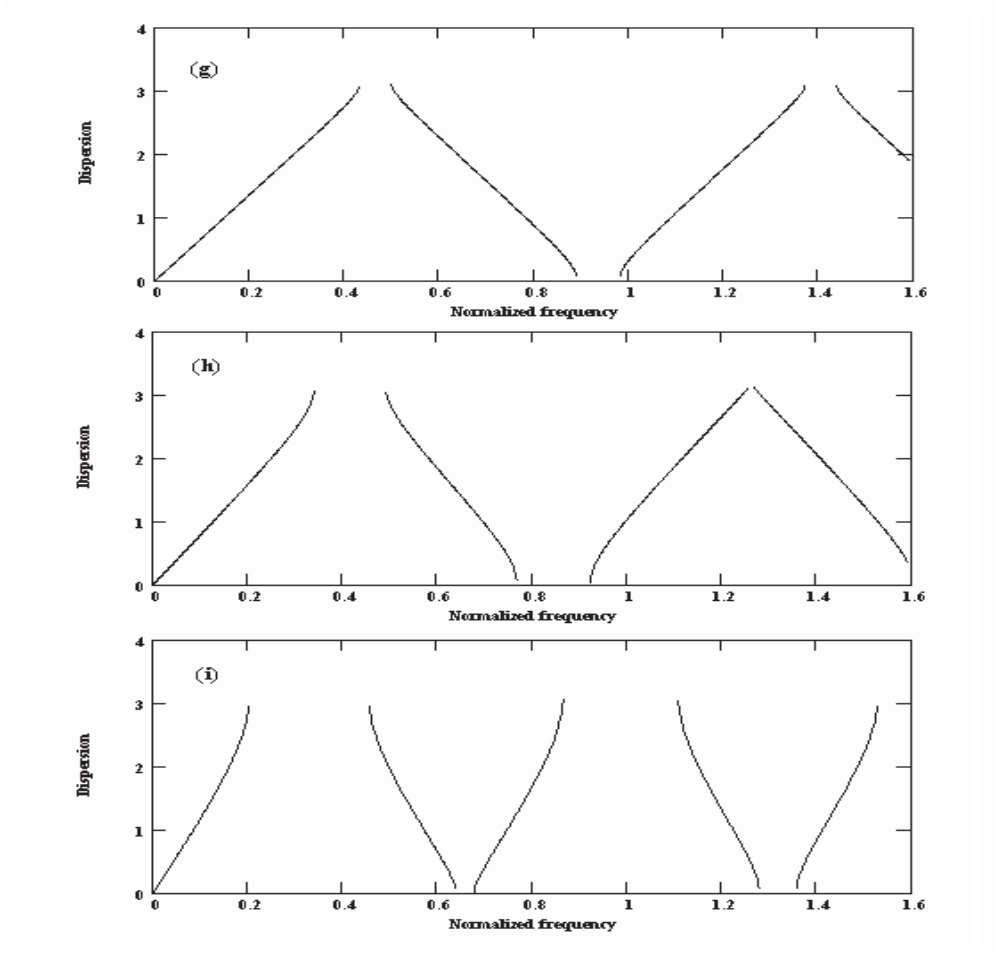

Figure 4. Band structures (xd) versus normalized frequency (in units of c/d) with $0.8d$ and $0.2d$ of thicknesses of PIM-PIM respectively (g) $n_1 = 1.0$, $n_2 = 1.35$; (h) $n_1 = 1.0$, $n_2 = 1.95$; (i) $n_1 = 1.0$, $n_2 = 3.6$.

Fig. 2(a), the nature of curve is different than normal dispersion curves and the width of photonic band gap is maximum among the structures considered here shown in Fig. 4(i). But for the structures in which $n_1 = 1.0$, $n_2 = -1.35$, $a = 0.8d$ & $b = 0.2d$ and $n_1 = 1.0$, $n_2 = -1.95$, $a = 0.8d$ & $b = 0.2d$, Fig. 2(c) & Fig. 2(b), the nature of the dispersion curve is usual as for the normal photonic band gap containing PIM-PIM materials shown in the Figs. 4(g) and 4(h). From these results it may be inferred that when the difference of refractive

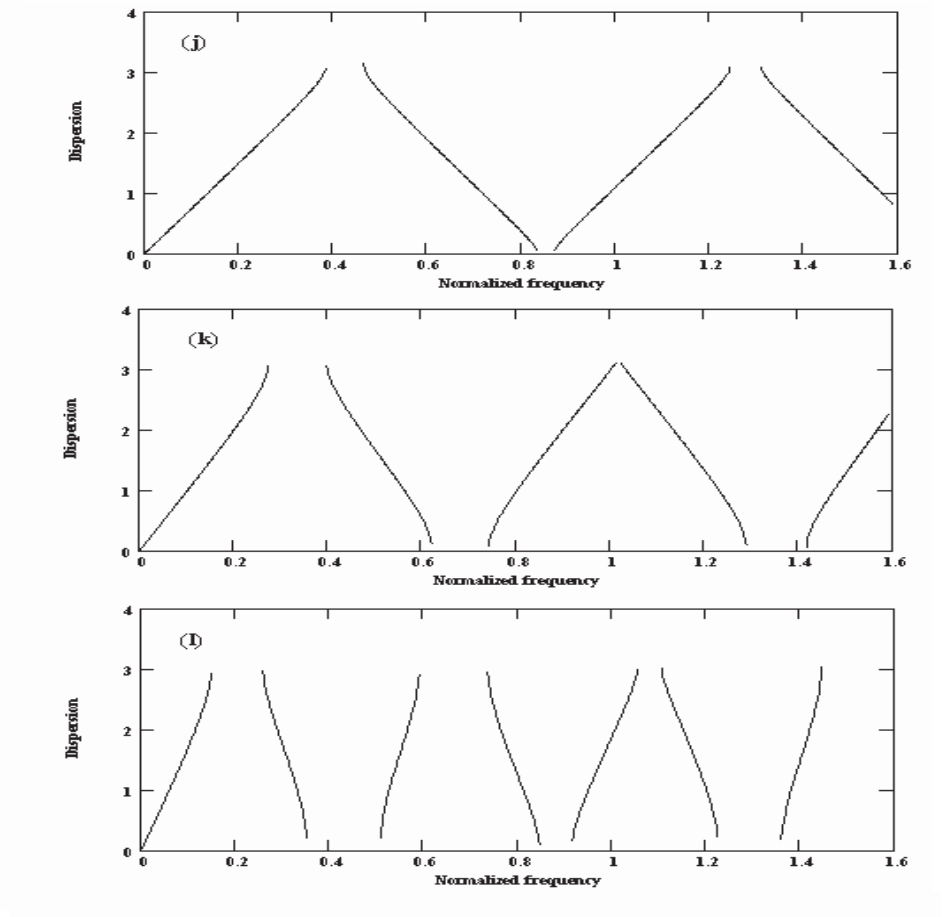

Figure 5. Band structures (xd) versus normalized frequency (in units of c/d) with $0.5d$ and $0.5d$ of thicknesses of PIM-PIM respectively (j) $n_1 = 1.0$, $n_2 = 1.35$; (k) $n_1 = 1.0$, $n_2 = 1.95$; (l) $n_1 = 1.0$, $n_2 = 3.6$.

index between the two layers is less than -1, the structure shows the normal dispersion curves as obtained for the normal photonic band gap containing PIM-PIM materials. While for the structure where the difference of refractive index between the two layers is greater than -1

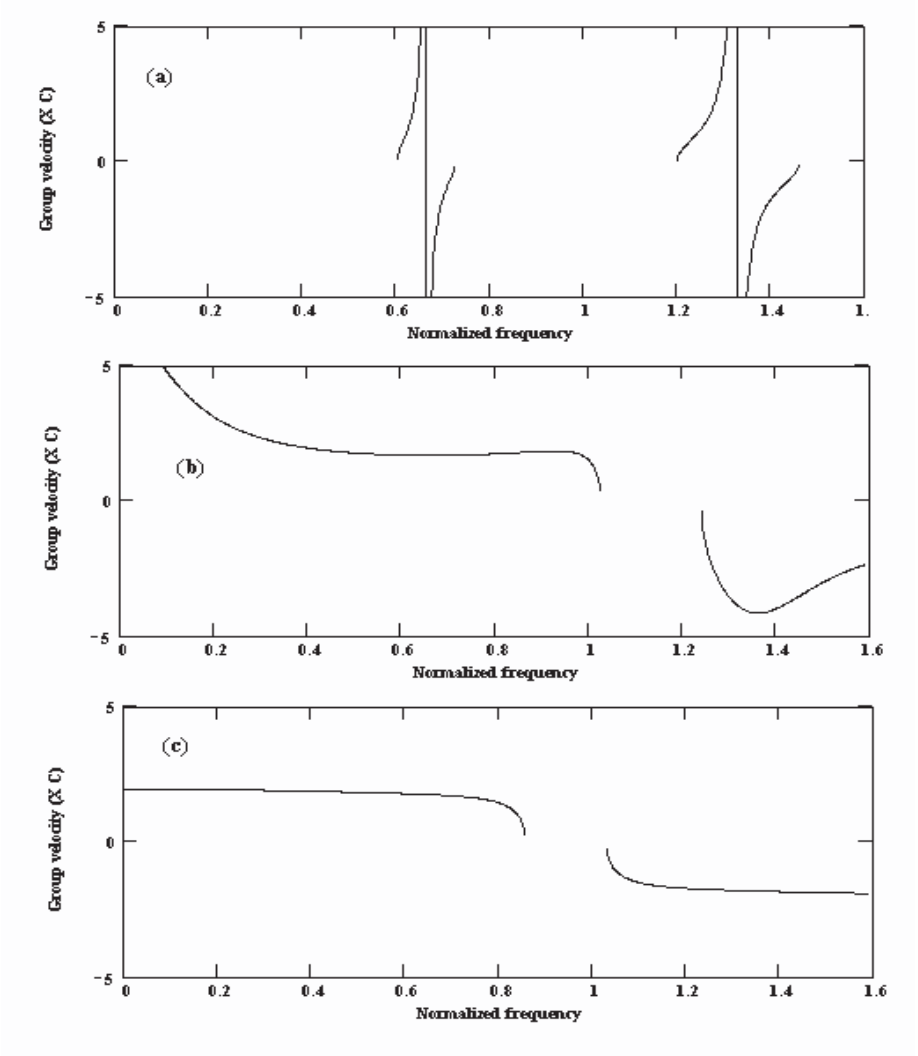

Figure 6. Group velocities (m/s) versus normalized frequency (in units of c/d) with $0.8d$ and $0.2d$ of thicknesses of PIM-NIM respectively (a) $n_1 = 1.0$, $n_2 = -3.6$; (b) $n_1 = 1.0$, $n_2 = -1.95$; (c) $n_1 = 1.0$, $n_2 = -1.35$.

and $a = 0.8d$, $b = 0.2d$, the structure shows the abnormal dispersion curves. It is also interesting to note that through the difference in refractive index of the two layer is greater than -1, the structure shows that the normal dispersion curves for lattice parameters $a = 0.5d$ and $b = 0.5d$ shown in Figs. 3(d), 3(e) & 3(f); Figs. 5(j), 5(k) & 5(l).

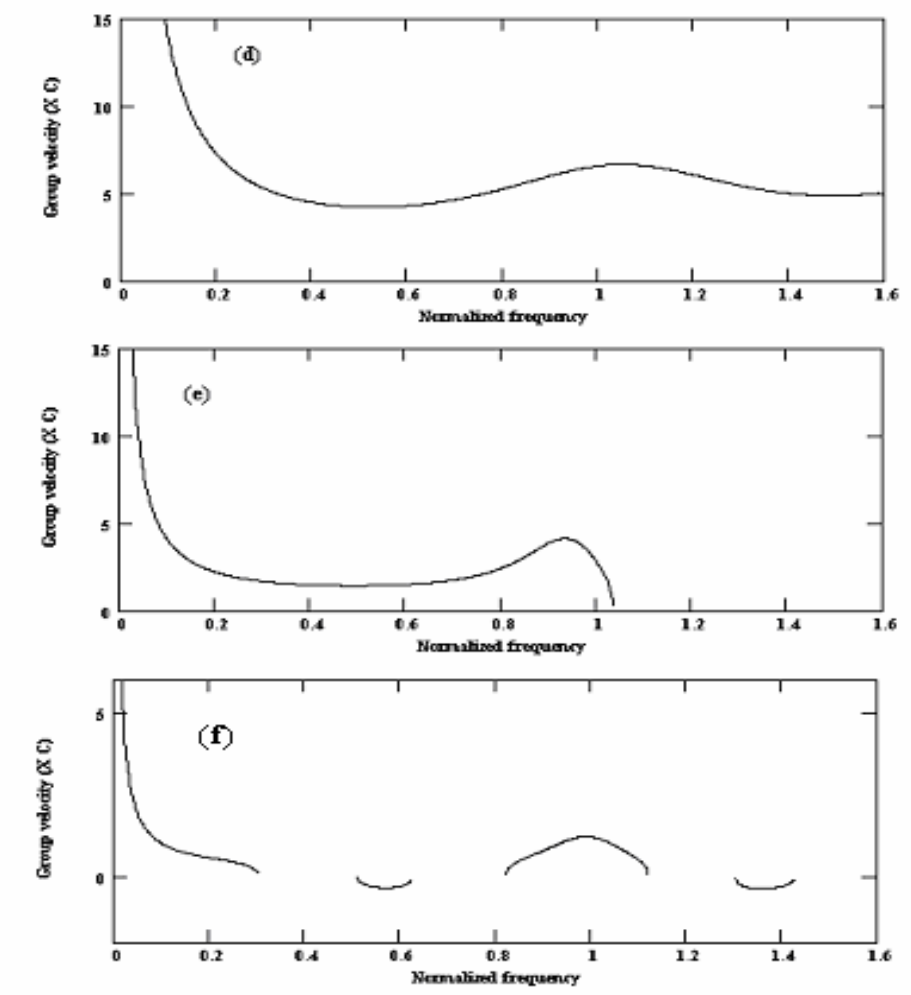

Figure 7. Group velocities (m/s) versus normalized frequency (in units of c/d) with $0.5d$ and $0.5d$ of thicknesses of PIM-NIM respectively (d) $n_1 = 1.0$, $n_2 = -1.35$; (e) $n_1 = 1.0$, $n_2 = -1.95$; (f) $n_1 = 1.0$, $n_2 = -3.6$.

Moreover, for $a = 0.5d$, $b = 0.5d$ width of the forbidden band is much smaller then that of the structure containing alternate layers of PIM-PIM for the same lattice parameters, also the number of bands is much lesser. The group velocities and effective group indices of the conventional photonic crystal are shown in Figs. 8(g), 8(h), 8(i); Figs. 9(j), 9(k), 9(l) and Figs. 12(g), 12(h), 12(i); Figs. 13(j), 13(k),

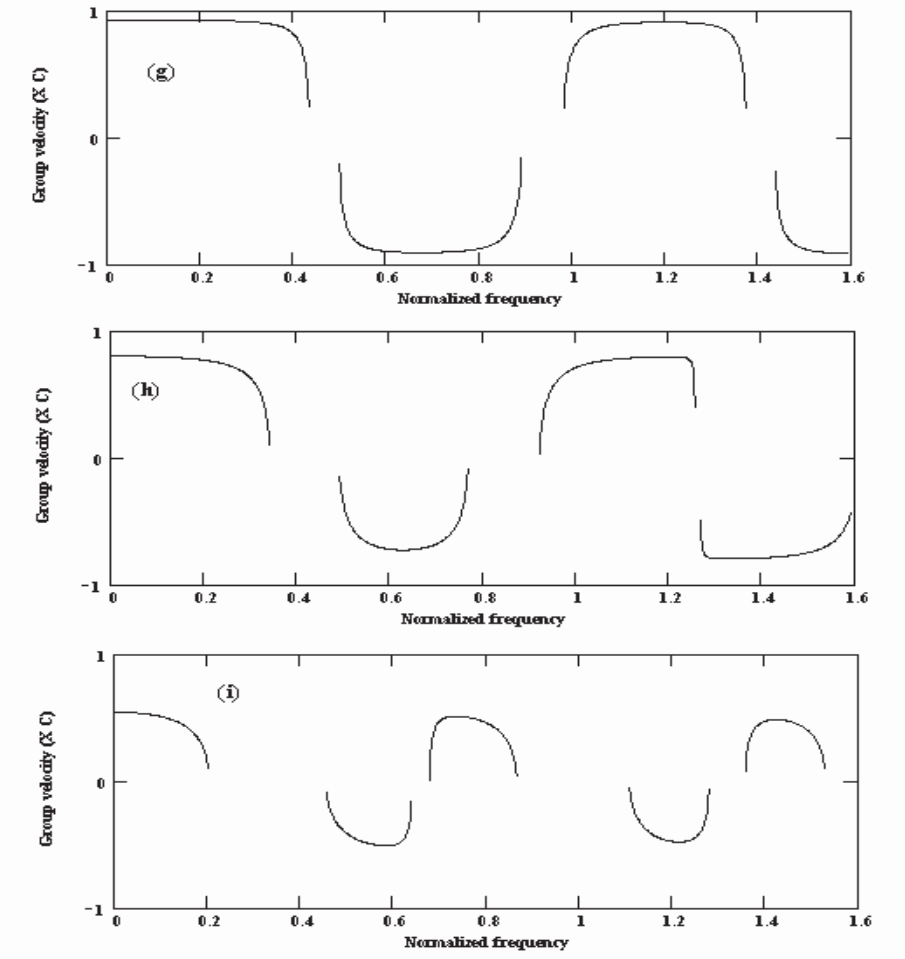

Figure 8. Group velocities (m/s) versus normalized frequency (in units of c/d) with $0.8d$ and $0.2d$ of thicknesses of PIM-PIM respectively (g) $n_1 = 1.0$, $n_2 = 1.35$; (h) $n_1 = 1.0$, $n_2 = 1.95$; (i) $n_1 = 1.0$, $n_2 = 3.6$.

13(l) respectively. The study of group velocities and effective group indices in the conventional photonic crystal shown in Figs. 6(a), 6(b), 6(c); Figs. 7(d), 7(e), 7(f) and Figs. 10(a), 10(b), 10(c); Figs. 11(d), 11(e), 11(f) respectively. The group velocity tends to zero at the edges of the bands and it becomes negative in certain range of normalized

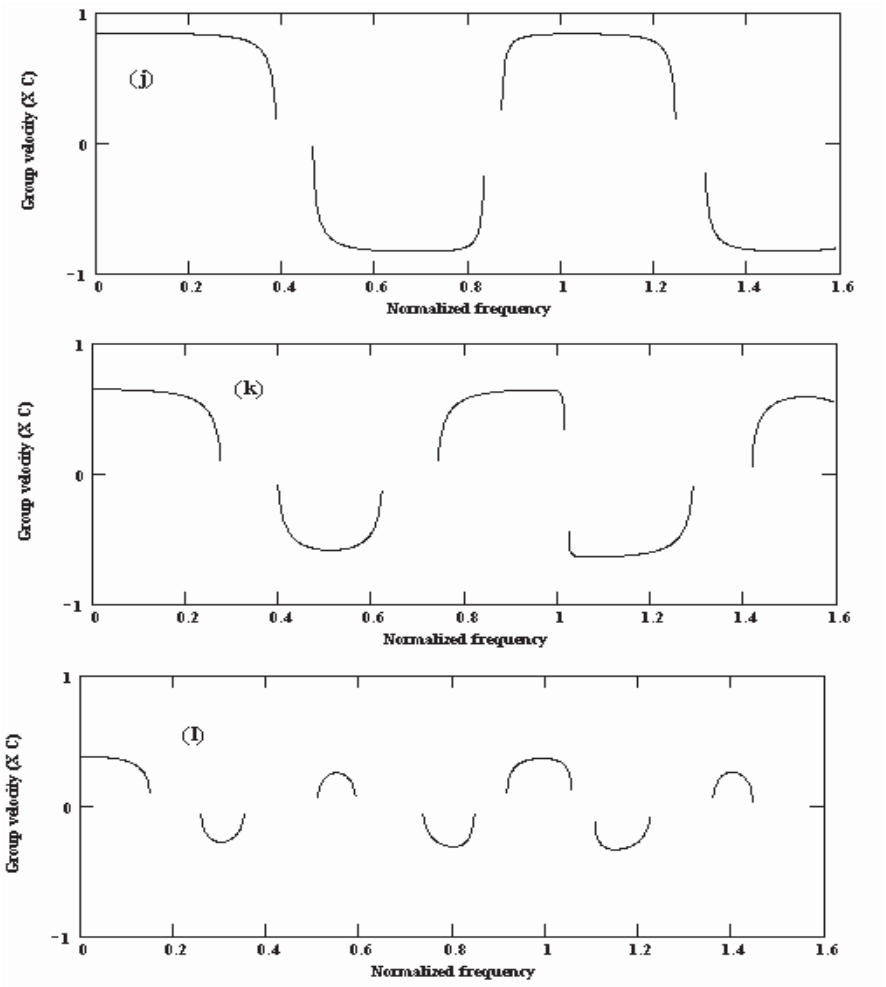

Figure 9. Group velocities (m/s) versus normalized frequency (in units of c/d) with $0.5d$ and $0.5d$ of thicknesses of PIM-PIM respectively (j) $n_1 = 1.0$, $n_2 = 1.35$; (k) $n_1 = 1.0$, $n_2 = 1.95$; (l) $n_1 = 1.0$, $n_2 = 3.6$.

frequency for each conventional photonic crystal. But for the photonic crystal with PIM-NIM for which $n_1 = 1.0$, and $n_2 = -3.6$, $a = 0.8d$ and $b = 0.2d$, group velocity as well as the effective group index becomes

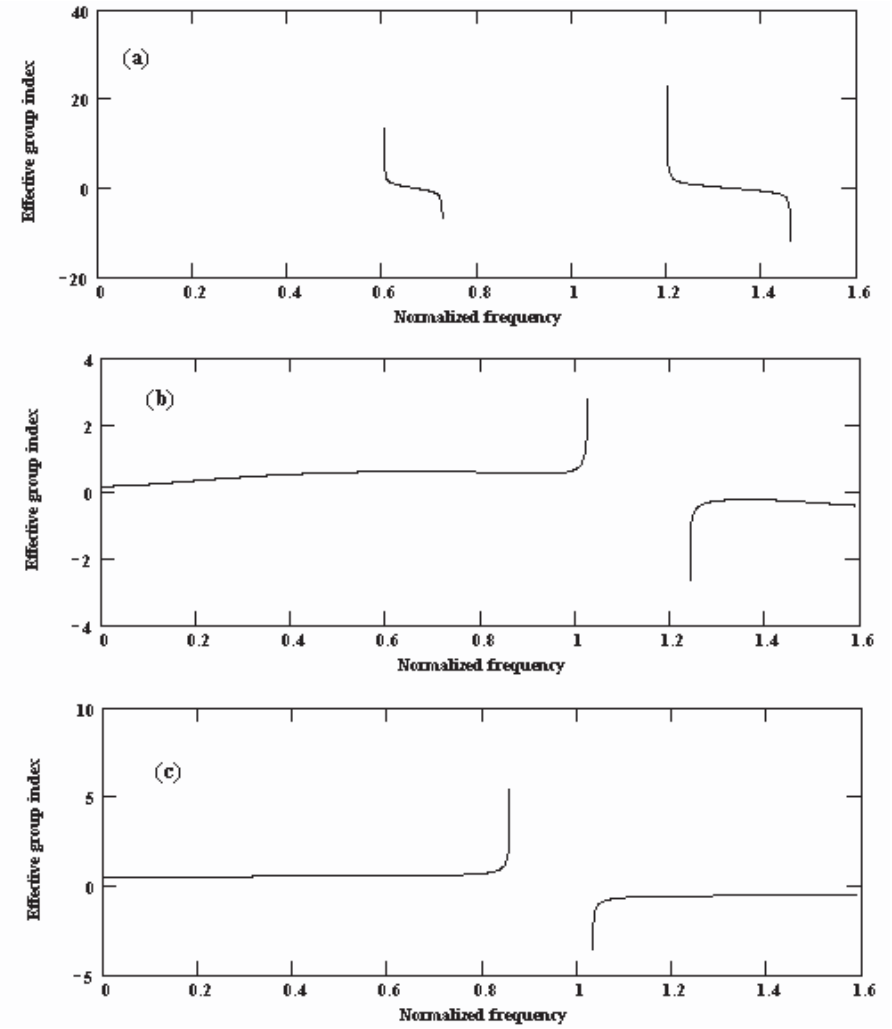

Figure 10. Effective group indices versus normalized frequency (in units of c/d) with $0.8d$ and $0.2d$ of thicknesses of PIM-NIM respectively (a) $n_1 = 1.0$, $n_2 = -3.6$; (b) $n_1 = 1.0$, $n_2 = -1.95$; (c) $n_1 = 1.0$, $n_2 = -1.35$.

exactly equal to zero at certain value of normalized frequency is shown in Figs. 6(a) and 10(a) respectively. In addition to this, the high positive and negative values of group velocity and effective group index is found in $n_1 = 1.0$, and $n_2 = -3.6$, $a = 0.8d$ and $b = 0.2d$. Moreover, the photonic crystal structure containing PIM-NIM, the value of group

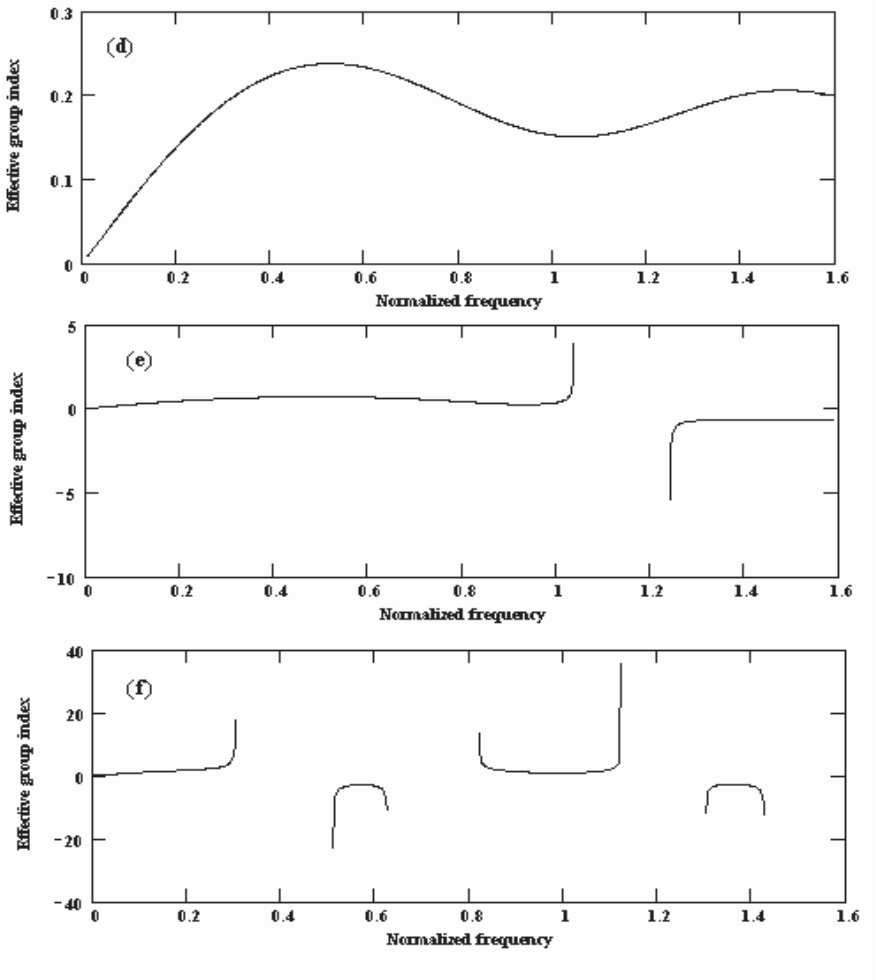

Figure 11. Effective group indices versus normalized frequency (in units of c/d) with $0.5d$ and $0.5d$ of thicknesses of PIM-NIM respectively (d) $n_1 = 1.0$, $n_2 = -1.35$; (e) $n_1 = 1.0$, $n_2 = -1.95$; (f) $n_1 = 1.0$, $n_2 = -3.6$.

velocity is found larger than the speed of light at the certain range of normalized frequency, which is given superluminal behavior. But for structure containing PIM-PIM, the group velocity is always found less than the speed of light through it becomes negative at the certain range of frequency. Thus, we have concluded that the PCs containing PIM-NIM have found the group velocity greater than the speed of light in vacuum. It is noticed that the values of the group velocity is become zero, negative and positive at certain range of frequencies. However the conventional photonic crystal, the group velocity is found less than

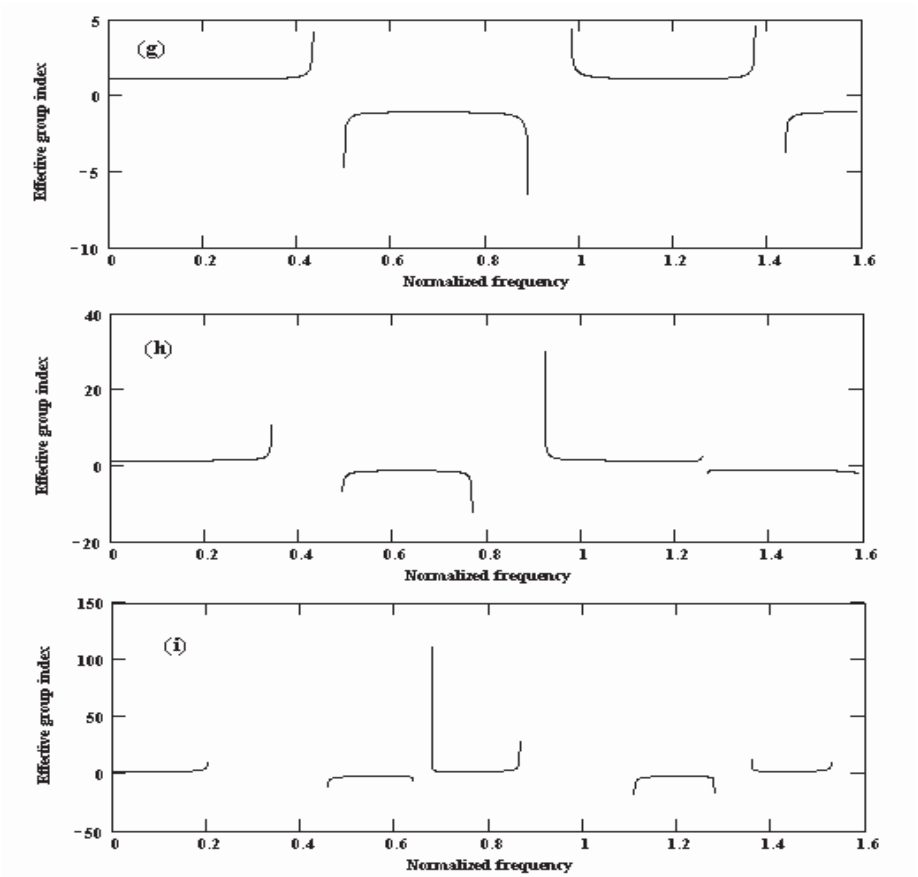

Figure 12. Effective group indices versus normalized frequency (in units of c/d) with $0.8d$ and $0.2d$ of thicknesses of PIM-PIM respectively (g) $n_1 = 1.0$, $n_2 = 1.35$; (h) $n_1 = 1.0$, $n_2 = 1.95$; (i) $n_1 = 1.0$, $n_2 = 3.6$.

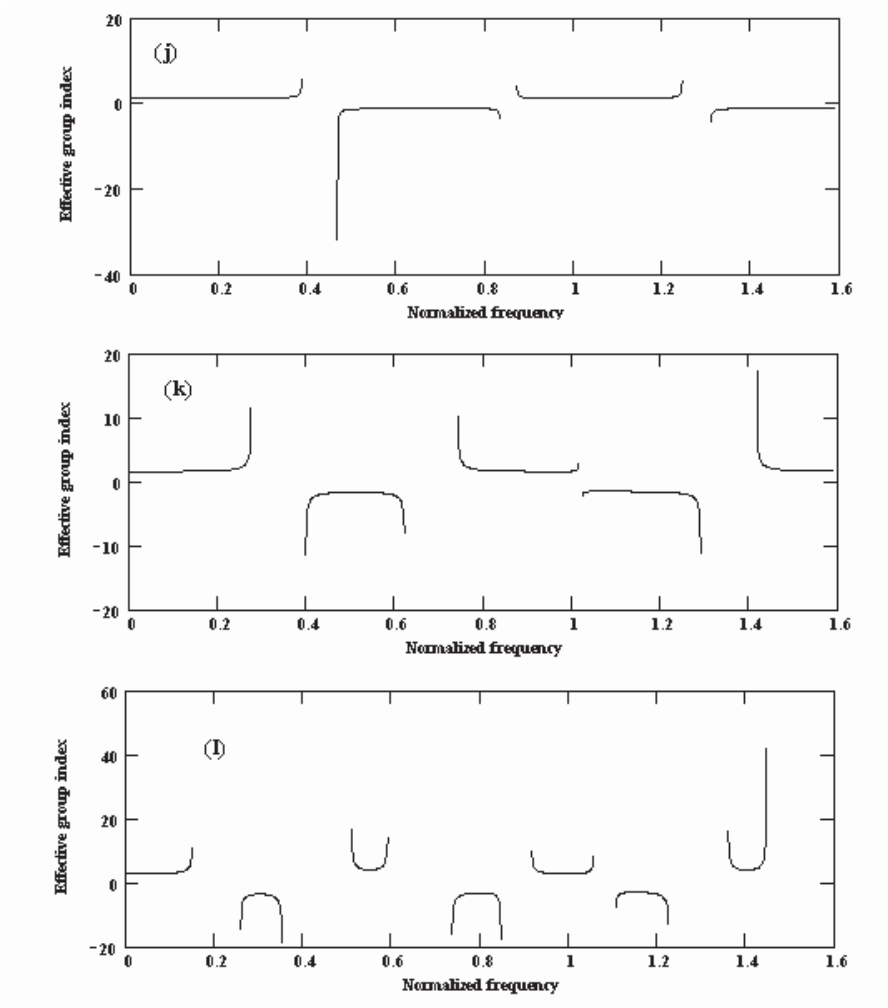

Figure 13. Effective group indices versus normalized frequency (in units of c/d) with $0.5d$ and $0.5d$ of thicknesses of PIM-PIM respectively (j) $n_1 = 1.0$, $n_2 = 1.35$; (k) $n_1 = 1.0$, $n_2 = 1.95$; (l) $n_1 = 1.0$, $n_2 = 3.6$.

speed of light with positive and negative values. Thus, the photonic crystals containing PIM-NIM can be used to trap the photon due to abnormal behavior of the group velocity i.e., light localization occurs without introducing the defects.

4. CONCLUSIONS

The value of group velocity of the one dimensional photonic crystal containing PIM-NIM is grater than the velocity of light in certain range of normalized frequency, which gives superluminal behavior. But the periodic structure containing PIM-PIM, the group velocity is always less than the velocity of light though it becomes negative in a certain range of frequency. Thus, we concluded that the group velocity of the period structure containing PIM-NIM have greater than the velocity of light, in addition to zero, negative and positive value; where as for the photonic crystal having PIM-PIM periodic materials, the group velocity is less than velocity of light though it is positive and negative. Thus, the photonic crystal containing PIM-NIM can be used to trap the photons i.e., light localization occurs without introducing the defects.

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