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DIFFRACTION OF EM PLANE WAVE BY A SLIT IN AN IMPEDANCE PLANE USING MALIUZHINETS FUNCTION

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Abstract—Diffraction of an electromagnetic plane wave from a slit in an impedance plane has been presented. The method is based on Maliuzhinets technique for impedance surfaces. Comparison of the field patterns with different values of the face impedance are also presented.

1. INTRODUCTION

The double perfectly conducting wedge scattering was investigated by Elsherbeni and Hamid [1]. They presented the solution for the diffraction of an incident plane wave by a slit in a perfectly conducting plane considering it as composed of two wedges. In recent years, high frequency description of the scattering from edges in non perfectly conducting surfaces has attracted the interest of many researchers due to its practical applications. Impedance surfaces represent a canonical approach for the coated conducting geometries. Moreover, the application of geometrical theory of diffraction (GTD) to the scattering from a wedge with impedance faces is applicable to the analysis of practical structures involving edges in an impedance surface. It may be noted that scattering from slits, half plane, impedance surfaces and study of high frequency diffraction are topics of current interest [2–13].

For a plane wave incidence in a plane perpendicular to the edge of an impedance wedge, a method of formulation for arbitrary face impedance and wedge angle, was developed by Maliuzhinets [14]. He expressed the total scattered field with a spectrum integral of plane waves, which contains an unknown weight function. The weight function is found by considering boundary, radiation and edge conditions.

The Maliuzhinets function [15, 16] plays a key role in the theory of diffraction by wedges with imperfectly conducting surfaces. Nearly all available exact solutions for nonmetallic wedges are expressed in terms of Maliuzhinets function.

2. FORMULATION OF THE PROBLEM

A slit may be viewed as two coplanar wedges each with zero wedge angle, separated by a certain width. The slit geometry and co-ordinates of the problem are shown in Fig. 1. Width of the slit is $2d$. A dual cylindrical coordinate system is used for the slit geometry, because it has been considered as combination of two wedges. We consider a two dimensional case in which the edge is a straight line and the incident rays all lie in planes normal to the edge. Then the diffracted rays are also normal to the edge, and emanate from it in all directions. Let the incident field is

$$\begin{bmatrix} E_z^i \\ H_z^i \end{bmatrix} = \begin{bmatrix} E_0 \\ H_0 \end{bmatrix} \exp\left[jk(x\cos\phi_0 + y\sin\phi_0)\right] \tag{1}$$

The uniform expression for the field diffracted from a wedge has the form

$$\begin{bmatrix} E_z^d \\ H_z^d \end{bmatrix} = \frac{\exp[-jk\rho)]}{\sqrt{\rho}} D_{\substack{s \\ h}}(\rho,\, \phi,\, \phi_0;\, p) E^i \tag{2}$$

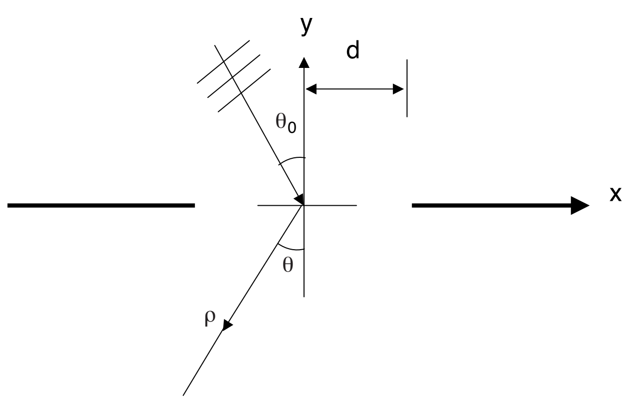

Figure 1. Impedance slit.

where $p = \frac{1}{n} = \frac{\pi}{\phi_\omega}$ is the parameter associated with the wedge angle ϕ_ω. The diffraction coefficient can be defined as [17]

$$\begin{aligned} D_{\substack{s\\h}}(\rho,\,\phi,\,\phi_0,\,n) = -\sqrt{\rho}\Bigg\{ & \frac{\Psi(\Phi-\pi-\phi)}{\Psi(\Phi-\phi_0)} sign\left(\sin\left(\frac{\pi+\phi-\phi_0}{2n}\right)\right) \\ & \cos\left(\frac{\pi+\phi-\phi_0}{2n}\right) F\left[\sqrt{2k\rho n}\left|\sin\left(\frac{\pi+\phi-\phi_0}{2n}\right)\right|\right] \\ & + \frac{\Psi(\Phi+\pi-\phi)}{\Psi(\Phi-\phi_0)} sign\left(\sin\left(\frac{\pi-(\phi-\phi_0)}{2n}\right)\right) \\ & \cos\left(\frac{\pi-(\phi-\phi_0)}{2n}\right) F\left[\sqrt{2k\rho n}\left|\sin\left(\frac{\pi-(\phi-\phi_0)}{2n}\right)\right|\right] \\ & - \frac{\Psi(\Phi-\pi-\phi)}{\Psi(\Phi-\phi_0)} sign\left(\sin\left(\frac{\pi+\phi+\phi_0}{2n}\right)\right) \\ & \cos\left(\frac{\pi+\phi+\phi_0}{2n}\right) F\left[\sqrt{2k\rho n}\left|\sin\left(\frac{\pi+\phi+\phi_0}{2n}\right)\right|\right] \\ & - \frac{\Psi(\Phi+\pi-\phi)}{\Psi(\Phi-\phi_0)} sign\left(\sin\left(\frac{\pi-(\phi+\phi_0)}{2n}\right)\right) \\ & \cos\left(\frac{\pi-(\phi+\phi_0)}{2n}\right) F\left[\sqrt{2k\rho n}\left|\sin\left(\frac{\pi-(\phi+\phi_0)}{2n}\right)\right|\right]\Bigg\} \end{aligned} \quad (3)$$

D_s and D_h are the diffraction coefficients of E and H polarizations respectively with the same form except the definition of $\sin\theta_\pm$ contained in the functions $\Psi(\alpha)$. Function $F(x)$ is the Fresnel integral defined as

$$F(x) = \frac{1}{\pi}\exp\left(jx^2 + j\frac{\pi}{4}\right)\int_x^\infty \exp(-j\mu^2)d\mu \quad (4)$$

and

$$\Psi(\alpha) = \psi_\Phi\left(\alpha+\theta_+ + \frac{\pi}{2}\right)\psi_\Phi\left(\alpha-\theta_+ + \frac{3\pi}{2}\right)\psi_\Phi\left(\alpha+\theta_- - \frac{3\pi}{2}\right)\psi_\Phi\left(\alpha-\theta_- - \frac{\pi}{2}\right)$$

where $\psi_\phi(.)$ is the Maliuzhinets function.

For the half plane, $n = 2$, $\phi_\omega = 2\pi$ and $\Phi = \pi$, therefore

$$\begin{aligned} D_{\substack{s\\h}}(\rho,\,\phi,\,\phi_0,\,2) = -\sqrt{\rho}\Bigg\{ & \frac{\Psi(-\phi)}{\Psi(\pi-\phi_0)} sign\left(\sin\left(\frac{\pi+\phi-\phi_0}{4}\right)\right) \\ & \cos\left(\frac{\pi+\phi-\phi_0}{4}\right) F\left[\sqrt{8k\rho}\left|\sin\left(\frac{\pi+\phi-\phi_0}{4}\right)\right|\right] \\ & + \frac{\Psi(2\pi-\phi)}{\Psi(\pi-\phi_0)} sign\left(\sin\left(\frac{\pi-(\phi-\phi_0)}{4}\right)\right) \end{aligned}$$

$$\cos\left(\frac{\pi-(\phi-\phi_0)}{4}\right)F\left[\sqrt{8k\rho}\left|\sin\left(\frac{\pi-(\phi-\phi_0)}{4}\right)\right|\right]$$
$$-\frac{\Psi(-\phi)}{\Psi(\pi-\phi_0)}sign\left(\sin\left(\frac{\pi+\phi+\phi_0}{4}\right)\right)$$
$$\cos\left(\frac{\pi+\phi+\phi_0}{4}\right)F\left[\sqrt{8k\rho}\left|\sin\left(\frac{\pi+\phi+\phi_0}{4}\right)\right|\right]$$
$$-\frac{\Psi(2\pi-\phi)}{\Psi(\pi-\phi_0)}sign\left(\sin\left(\frac{\pi-(\phi+\phi_0)}{4}\right)\right)$$
$$\cos\left(\frac{\pi-(\phi+\phi_0)}{4}\right)F\left[\sqrt{8k\rho}\left|\sin\left(\frac{\pi-(\phi+\phi_0)}{4}\right)\right|\right]\Bigg\} \tag{5}$$

and

$$\begin{aligned}\Psi(\alpha) \;=\; &\psi_\pi\left(\alpha+\theta_+ +\frac{\pi}{2}\right)\psi_\pi\left(\alpha-\theta_+ +\frac{3\pi}{2}\right)\\ &\psi_\pi\left(\alpha+\theta_- -\frac{3\pi}{2}\right)\psi_\pi\left(\alpha-\theta_- -\frac{\pi}{2}\right)\end{aligned} \tag{6}$$

The Maliuzhinets function $\psi_\pi(\alpha)$ for the half-plane is given by

$$\Psi_\pi(\alpha)=\exp\left[-\frac{1}{8\pi}\int_0^\alpha\frac{\pi\sin t-2\sqrt{2}\pi\sin\frac{t}{2}+2t}{\pi\cos t}\,dt\right] \tag{7}$$

$\sin\theta_\pm = \frac{1}{\zeta_\pm}$ for E-polarization, $\sin\theta_\pm = \zeta_\pm$ for H-polarization. The angles between the incident and diffracted rays and normal to the half plane are ϕ and ϕ_0 respectively.

For large argument, Fresnel integral can be given as

$$F(x)=\frac{1}{2\sqrt{\pi x}}\exp\left[-j\frac{\pi}{4}\right] \tag{8}$$

3. ELECTROMAGNETIC FIELD DIFFRACTED FROM THE SLIT

We shall apply (2) and (5) to determine the field diffracted by an infinitely long slit of width "$2d$" in a thin impedance plane. Let the edges of the slit are at $x=\pm d$, $y=0$. The singly diffracted field at an observation point, E_1^d is the sum of two terms of the form (5).

3.1. Field from Right Edge

The field diffracted from right edge is

$$E_r^d(\rho_1,\,\phi_1)=D_r(\rho_1,\,\phi_{01},\,\phi_1)\rho_1^{-\frac{1}{2}}\exp(-jk\rho_1)E^i \tag{9}$$

where

$$\begin{aligned} D_r(\rho_1, \phi_{01}, \phi_1) = & -\frac{1}{4\sqrt{2\pi k}}\left[\frac{\Psi(-\phi_1)}{\Psi(\pi-\phi_{01})}\cot\frac{\pi+\phi_1-\phi_{01}}{4}\right. \\ & +\frac{\Psi(2\pi-\phi_1)}{\Psi(\pi-\phi_{01})}\cot\frac{\pi-\phi_1+\phi_{01}}{4} \\ & -\frac{\Psi(-\phi_1)}{\Psi(\pi-\phi_{01})}\cot\frac{\pi+\phi_1+\phi_{01}}{4} \\ & \left.-\frac{\Psi(2\pi-\phi_1)}{\Psi(\pi-\phi_{01})}\cot\frac{\pi-\phi_1-\phi_{01}}{4}\right] \end{aligned} \tag{10}$$

In the far field of the slit ($\rho \gg d$)

$$\rho_1 = \rho - d\sin\theta, \quad \phi_1 = \frac{3\pi}{2}+\theta, \quad \phi_{01} = \frac{\pi}{2}+\theta_0$$

This far-field substitution for ρ_1 is used in the exponential term; in the amplitude term $\rho_1 = \rho$

$$\begin{aligned} D_r\left(\rho, \frac{\pi}{2}+\theta_0, \frac{3\pi}{2}+\theta\right) = & -\frac{1}{4\sqrt{2\pi k}}\left[\frac{\Psi\left(-\frac{3\pi}{2}-\theta\right)}{\Psi\left(\frac{\pi}{2}-\phi_0\right)}\cot\frac{2\pi+\theta-\phi_0}{4}\right. \\ & +\frac{\Psi\left(\frac{\pi}{2}-\theta\right)}{\Psi\left(\frac{\pi}{2}-\phi_0\right)}\cot\frac{\phi_0-\theta}{4} \\ & -\frac{\Psi\left(-\frac{3\pi}{2}-\theta\right)}{\Psi\left(\frac{\pi}{2}-\phi_0\right)}\cot\frac{3\pi+\theta+\phi_0}{4} \\ & \left.-\frac{\Psi\left(\frac{\pi}{2}-\theta\right)}{\Psi\left(\frac{\pi}{2}-\phi_0\right)}\cot\frac{-\pi-\theta-\phi_0}{4}\right] \end{aligned} \tag{11}$$

3.2. Field from Left Edge

The field diffracted from left edge is

$$E_l^d(\rho_2, \phi_2) = D_l(\rho_2, \phi_{02}, \phi_2)\rho_2^{-\frac{1}{2}}\exp(-jk\rho_2)E^i \tag{12}$$

where

$$\begin{aligned} D_l(\rho_2, \phi_{02}, \phi_2) = & -\frac{1}{4\sqrt{2\pi k}}\left[\frac{\Psi(-\phi_2)}{\Psi(\pi-\phi_{02})}\cot\frac{\pi+\phi_2-\phi_{02}}{4}\right. \\ & +\frac{\Psi(2\pi-\phi_2)}{\Psi(\pi-\phi_{02})}\cot\frac{\pi-\phi_1+\phi_{02}}{4} \\ & -\frac{\Psi(-\phi_2)}{\Psi(\pi-\phi_{02})}\cot\frac{\pi+\phi_2+\phi_{02}}{4} \end{aligned}$$

$$-\frac{\Psi(2\pi-\phi_2)}{\Psi(\pi-\phi_{02})}\cot\frac{\pi-\phi_2-\phi_{02}}{4}\Bigg] \tag{13}$$

In the far field of the slit ($\rho \gg d$)

$$\rho_2 = \rho + d\sin\theta, \quad \phi_2 = \frac{3\pi}{2} - \theta, \quad \phi_{02} = \frac{\pi}{2} - \theta_0$$

This far-field substitution for ρ_2 is used in the exponential term; in the amplitude term $\rho_2 = \rho$

$$\begin{aligned}
D_l\left(\rho,\, \frac{\pi}{2}-\theta_0,\, \frac{3\pi}{2}-\theta\right) \;=\; & -\frac{1}{4\sqrt{2\pi k}}\left[\frac{\Psi\left(-\frac{3\pi}{2}+\theta\right)}{\Psi\left(\frac{\pi}{2}+\phi_0\right)}\cot\frac{2\pi-\theta+\phi_0}{4}\right.\\
& +\frac{\Psi\left(\frac{\pi}{2}+\theta\right)}{\Psi\left(\frac{\pi}{2}+\phi_0\right)}\cot\frac{-\phi_0+\theta}{4}\\
& -\frac{\Psi\left(-\frac{3\pi}{2}+\theta\right)}{\Psi\left(\frac{\pi}{2}+\phi_0\right)}\cot\frac{3\pi-\theta-\phi_0}{4}\\
& \left.-\frac{\Psi\left(\frac{\pi}{2}-\theta\right)}{\Psi\left(\frac{\pi}{2}-\phi_0\right)}\cot\frac{-\pi+\theta+\phi_0}{4}\right]
\end{aligned} \tag{14}$$

The field diffracted from the slit can be calculated by taking the linear combination of the two terms (9) and (12).

$$E(\rho,\,\phi) = E_r(\rho_1,\,\phi_1) + E_l(\rho_2,\,\phi_2) \tag{15}$$

4. DISCUSSION

In this paper we have investigated the diffracted pattern of a slit in an impedance plane using the Maliuzhinets function. We have derived a simple and convenient expression for the field diffracted by an infinite slit in an impedance plane when the wavelength is greater than or equal to the slit width. The principal result is that this field can be accurately calculated everywhere by considering each half plane composing the slit, to be excited by the incident plane wave. It may be noted that the results are valid for all incident angles. Comparison of E and H-polarized fields are also made, as shown in Fig. 2. Furthermore, the plots shown in Fig. 3. give a comparison of the field pattern with different values of the face impedance.

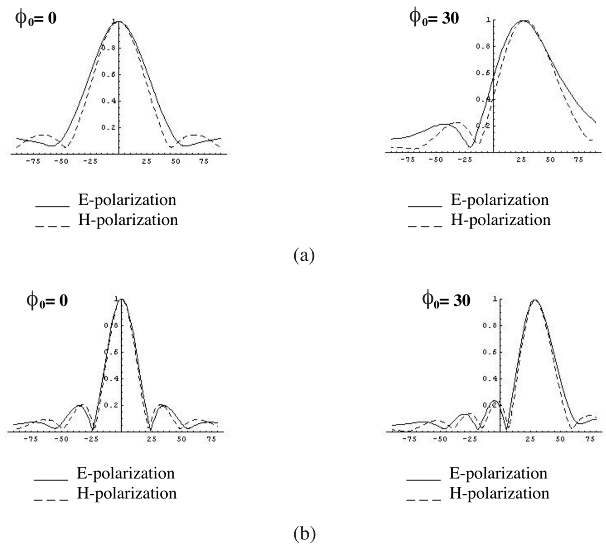

Figure 2. (a) E and H-polarizations for impedance 0.2–0.5j with slit width $kd = 4$, (b) E and H-polarizations for impedance 0.2–0.5j with slit width $kd = 8$.

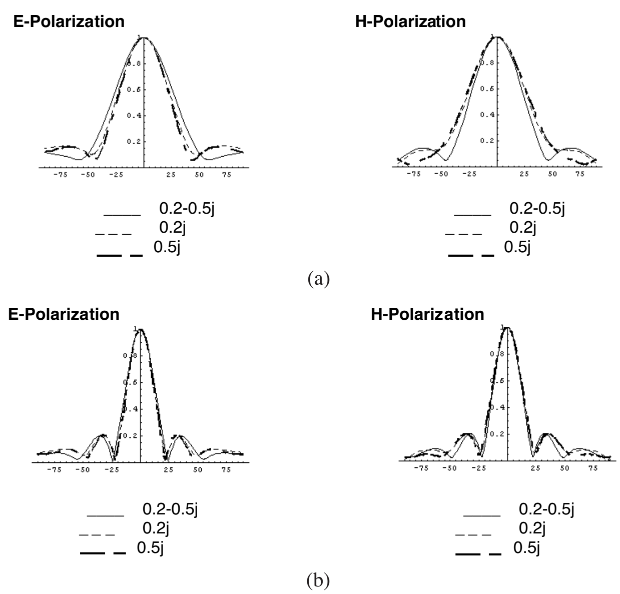

Figure 3. (a) Different values of impedance for slit width $kd = 4$, (b) Different values of impedance for slit width $kd = 8$.

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