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FRACTIONAL SURFACE WAVEGUIDE

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Abstract—Fractional curl operator has been utilized to study the fractional order surface waveguides. Fractional order surface waveguides may be regarded as intermediate step of two surface waveguides which are related through the principle of duality. Fractional eigenvalue equations are examined at the interface between dielectric medium and free space, for various values of fractional order parameter result in different fractional surface wave modes.

1. INTRODUCTION

Fractional curl operator which is represented as $curl^{\alpha}$, where α denotes the order of the operator, obtains from fractionalization of the usual curl operator. Fractional curl operator constructs new set of solutions to Maxwell equations, which may be described as an intermediate steps between original solution set and the dual to the original solution set. These solutions are named as "fractional field" see for instance [1]. The applications of fractional curl operator was further extended by Naqvi et al. [2], they discussed the behavior of fractional dual solution in an unbounded chiral medium. Veliev and Engheta [3] and Ivakhnychenko et al. [4] utilized the fractional curl operator to a fixed solution and obtained the fractional fields that represent the solution of reflection problem from anisotropic surface impedance. Hussain and Naqvi [5], introduction the idea of fractional nonsymmetric transmission line.

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Hussain et al. [6, 7], extended this idea to study fractional waveguides. Afterward Faryad and Naqvi [8] extended the work of fractional dual parallel plates waveguides and constructed the solutions corresponding to the fractional dual rectangular wave guides.

Propagation through waveguide structures has been studied by various authors [9–17]. In present paper we have extended the idea of fractional dual parallel plates waveguides [7] to surface waveguide or open waveguide. Surface waveguides are particularly used in millimetric-wave circuits. The fractional field expressions are carried out in both dielectric medium and free space. The fractional eigenvalue equations are examined numerically at the interface between dielectric medium and free space, taking $t = \lambda_0/4$ and $t = \lambda_0/2$, for various values of fractional parameter α and results represent various intermediate fractional surface wave modes.

2. FRACTIONAL SURFACE WAVEGUIDE

Consider a surface waveguide composed of a dielectric of thickness "t" coated on plane perfect electric conductor. It is assumed to be of infinite extent in the y and z directions. We assume propagation in the $+z$ direction with $e^{i\beta z}$ propagation factor and no variation in the y direction, that is $(\partial/\partial y = 0)$. The geometry is divided into two regions. The first region $(0 < x < t)$ consists of dielectric material of permittivity $\epsilon = \kappa\epsilon_0$ and the other region $x > t$ represents the free space. The total TM fields for region $0 < x < t$ may be written as

$$\mathbf{E}_z = \hat{z}A\sin(k_d x)\exp(i\beta z) \tag{1a}$$

$$\mathbf{E}_x = \hat{x}\frac{i\beta}{k_d}A\cos(k_d x)\exp(i\beta z) \tag{1b}$$

$$Z\mathbf{H}_y = -\hat{y}\frac{ik}{k_d}A\cos(k_d x)\exp(i\beta z) \tag{1c}$$

where $\beta^2 = k^2 - k_d^2$, $k = \sqrt{\kappa}k_0$, $k_0 = \omega\sqrt{\mu_0\epsilon_0}$, $Z = \sqrt{\frac{\mu_0}{\kappa\epsilon_0}}$, and $\frac{m-1}{2}\pi \leq k_d t \leq \frac{m}{2}\pi$, m is positive integer. Z is the impedance of the dielectric medium. For region $t < x < \infty$

$$\mathbf{E}_z = \hat{z}A\sin(k_d t)\exp(-h(x-t) + i\beta z) \tag{2a}$$

$$\mathbf{E}_x = \hat{x}\frac{i\beta}{h}A\sin(k_d t)\exp(-h(x-t) + i\beta z) \tag{2b}$$

$$Z_0\mathbf{H}_y = -\hat{y}\frac{ik_0}{h}A\sin(k_d t)\exp(-h(x-t) + i\beta z) \tag{2c}$$

where $\beta^2 = k_0^2 - h^2$, $Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ implies that $k_d = ih$ in free space, and $0 < h < 2n$, n is positive integer.

We have first examined the fractional dual solutions of E-waves in the region $0 < x < t$. Total field in this region may be considered as combination of two TEM waves bouncing back and forth between the two boundaries, that is $x = 0$ and $x = t$. The $x = 0$ boundary represent PEC interface. For region $0 < x < t$, fields given in (1) may be written as

$$\mathbf{E}_z = -i\hat{z}\frac{A}{2}[\exp(ik_d x + i\beta z) - \exp(-ik_d x + i\beta z)] \tag{3a}$$

$$\mathbf{E}_x = \hat{x}\frac{i\beta}{k_d}\frac{A}{2}[\exp(ik_d x + i\beta z) + \exp(-ik_d x + i\beta z)] \tag{3b}$$

$$Z\mathbf{H}_y = -\hat{y}\frac{ik}{k_d}\frac{A}{2}[\exp(ik_d x + i\beta z) + \exp(-ik_d x + i\beta z)] \tag{3c}$$

The total electric and magnetic fields in region $0 < x < t$ are

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2, \tag{4a}$$

$$Z\mathbf{H} = Z\mathbf{H}_1 + Z\mathbf{H}_2 \tag{4b}$$

where $(\mathbf{E}_1, \mathbf{H}_1)$ represents the electric and magnetic fields related with one plane wave and are given below

$$\mathbf{E}_1 = \frac{A}{2}\left[-i\hat{z} + \frac{i\beta}{k_d}\hat{x}\right]\exp(ik_d x + i\beta z) \tag{5a}$$

$$Z\mathbf{H}_1 = -\frac{A}{2}\frac{k}{k_d}i\hat{y}\exp(ik_d x + i\beta z) \tag{5b}$$

The electric and magnetic fields related to other plane wave are represented by $(\mathbf{E}_2, \mathbf{H}_2)$ as below

$$\mathbf{E}_2 = \frac{A}{2}\left[i\hat{z} + \frac{i\beta}{k_d}\hat{x}\right]\exp(-ik_d x + i\beta z) \tag{6a}$$

$$Z\mathbf{H}_2 = -\frac{A}{2}\frac{k}{k_d}i\hat{y}\exp(-ik_d x + i\beta z) \tag{6b}$$

It is of interest to find the solutions to the Maxwell equations which may be regarded as intermediate steps of the solutions $(\mathbf{E}, Z\mathbf{H})$ and dual to the solutions $(\mathbf{E}, Z\mathbf{H})$. These fractional solutions have been termed as fractional dual solutions to Maxwell equations. Fractional dual solutions may be obtained by fractionalizing curl operator in Maxwell equations or using fractional cross product

$(\mathbf{k}_i\times)^\alpha$ where $i = 1, 2$. For this purpose, we need eigenvectors and eigenvalues of the $(\mathbf{k}_i\times)$, where $\mathbf{k}_i$ are the wave vectors of the two plane wave bouncing back and forth. To determine the eigenvectors and eigenvalues of $(\mathbf{k}_1\times)$, we re-arrange Eq. (5) as follow

$$\mathbf{E}_1 = -\frac{Ak}{2k_d}\left[i\hat{z}\frac{k_d}{k} - \frac{i\beta}{k}\hat{x}\right]\exp(ik_dx + i\beta z) \tag{7a}$$

$$Z\mathbf{H}_1 = -\frac{Ak}{2k_d}i\hat{y}\exp(ik_dx + i\beta z) \tag{7b}$$

The eigen vectors and eigen values of the operator $\mathbf{k}_1\times$ are

$$\begin{aligned}
\mathbf{A}_1 &= \frac{1}{\sqrt{2}}\left[-\frac{i\beta}{k}\hat{x} + \hat{y} + \frac{ik_d}{k}\hat{z}\right], \qquad a_1 = i \\
\mathbf{A}_2 &= \frac{1}{\sqrt{2}}\left[\frac{i\beta}{k}\hat{x} + \hat{y} - \frac{ik_d}{k}\hat{z}\right], \qquad a_2 = -i \\
\mathbf{A}_3 &= -\frac{ik_d}{k}\hat{x} - \frac{i\beta}{k}\hat{z}, \qquad a_3 = 0
\end{aligned}$$

Field $\mathbf{E}_1$ may be expressed in terms of the eigenvectors of $\mathbf{k}_1\times$ as

$$\mathbf{E}_1 = [P_1\mathbf{A}_1 + Q_1\mathbf{A}_2 + R_1\mathbf{A}_3]\exp(ik_dx + i\beta z) \tag{8}$$

where the coefficients are

$$\begin{aligned}
P_1 &= \frac{-Ak}{2\sqrt{2}k_d} \\
Q_1 &= \frac{Ak}{2\sqrt{2}k_d} \\
R_1 &= 0
\end{aligned}$$

Using the fractional curl operator, fractional dual fields may be obtained using the following relations [2]

$$\mathbf{E}_{1fd} = [(ik)^{-1}\nabla\times]^\alpha\mathbf{E}_1 \tag{9a}$$

$$Z\mathbf{H}_{1fd} = [(ik)^{-1}\nabla\times]^\alpha(Z\mathbf{H}_1) \tag{9b}$$

It may be noted that $|\mathbf{k}_1| = |\mathbf{k}_2|$. This means that application of fractional cross product operator $(\mathbf{k}_1\times)^\alpha$ on vectors $(\mathbf{E}_1, Z\mathbf{H}_1)$ in Eq. (8) gives the results, which may be regarded as the intermediate steps between the initial solution set $(\mathbf{E}_1, Z\mathbf{H}_1)$ and final solution set

$(Z\mathbf{H}_1,\ -\mathbf{E}_1)$ and are given as

$$\begin{aligned}\mathbf{E}_{1fd} &= (\mathbf{k}_1\times)^\alpha \mathbf{E}_1\\ &= -\frac{Ak}{2k_d}\left[-\frac{i\beta}{k}\cos\left(\frac{\alpha\pi}{2}\right)\hat{x} + i\sin\left(\frac{\alpha\pi}{2}\right)\hat{y}\right.\\ &\quad\left.+\frac{ik_d}{k}\cos\left(\frac{\alpha\pi}{2}\right)\hat{z}\right]\exp(ik_d x + i\beta z) \qquad (10a)\\ Z\mathbf{H}_{1fd} &= (\mathbf{k}_1\times)^\alpha Z\mathbf{H}_1\\ &= -\frac{Ak}{2k_d}\left[\frac{i\beta}{k}\sin\left(\frac{\alpha\pi}{2}\right)\hat{x} + i\cos\left(\frac{\alpha\pi}{2}\right)\hat{y}\right.\\ &\quad\left.-\frac{ik_d}{k}\sin\left(\frac{\alpha\pi}{2}\right)\hat{z}\right]\exp(ik_d x + i\beta z) \qquad (10b)\end{aligned}$$

The behavior of fractional electric and magnetic field represent a counterclockwise rotation by an angle $\alpha\pi/2$.

To define the eigenvectors and eigenvalues of the cross product operator $(\mathbf{k}_2\times)$ Eq. (6) can be rearranged as follow

$$\mathbf{E}_2 = \frac{-kA}{2k_d}\left[-\frac{ik_d}{k}\hat{z} - \frac{i\beta}{k}\hat{x}\right]\exp(-ik_d x + i\beta z) \qquad (11a)$$

$$Z\mathbf{H}_2 = -\frac{Ak}{2k_d}i\hat{y}\exp(-ik_d x + i\beta z) \qquad (11b)$$

Eigenvectors and eigenvalues of the operator $(\mathbf{k}_2\times)$ are

$$\begin{aligned}\mathbf{A}_1 &= \frac{1}{\sqrt{2}}\left[-\frac{i\beta}{k}\hat{x} + \hat{y} - \frac{ik_d}{k}\hat{z}\right], & a_1 &= i\\ \mathbf{A}_2 &= \frac{1}{\sqrt{2}}\left[\frac{i\beta}{k}\hat{x} + \hat{y} + \frac{ik_d}{k}\hat{z}\right], & a_2 &= -i\\ \mathbf{A}_3 &= \frac{ik_d}{k}\hat{x} - \frac{i\beta}{k}\hat{z}, & a_3 &= 0\end{aligned}$$

Vector $\mathbf{E}_2$ may be expressed as linear combination of eigenvectors of $\mathbf{k}_2\times$

$$\mathbf{E}_2 = [P_2\mathbf{A}_1 + Q_2\mathbf{A}_2 + R_2\mathbf{A}_3]\exp(-ik_d x + i\beta z) \qquad (12)$$

where the coefficients are

$$\begin{aligned}P_2 &= \frac{-Ak}{2\sqrt{2}k_d}\\ Q_2 &= \frac{Ak}{2\sqrt{2}k_d}\\ R_2 &= 0\end{aligned}$$

The required fractional solutions of Eq. (12) may be considered as the intermediate step between the solution set ($\mathbf{E}_2$, $Z\mathbf{H}_2$) and the dual solution set ($Z\mathbf{H}_2$, $-\mathbf{E}_2$) and are given by

$$\begin{aligned}
\mathbf{E}_{2fd} &= (\mathbf{k}_2\times)^\alpha \mathbf{E}_2 \\
&= -\frac{Ak}{2k_d}\exp(-i\alpha\pi)\left[-\frac{i\beta}{k}\cos\left(\frac{\alpha\pi}{2}\right)\hat{x} - i\sin\left(\frac{\alpha\pi}{2}\right)\hat{y}\right. \\
&\quad \left. -\frac{ik_d}{k}\cos\left(\frac{\alpha\pi}{2}\right)\hat{z}\right]\exp(-ik_d x + i\beta z) \qquad (13a)
\end{aligned}$$

$$\begin{aligned}
Z\mathbf{H}_{2fd} &= (\mathbf{k}\times)^\alpha Z\mathbf{H}_1 \\
&= -\frac{Ak}{2k_d}\exp(-i\alpha\pi)\left[-\frac{i\beta}{k}\sin\left(\frac{\alpha\pi}{2}\right)\hat{x} + i\cos\left(\frac{\alpha\pi}{2}\right)\hat{y}\right. \\
&\quad \left. -\frac{ik_d}{k}\sin\left(\frac{\alpha\pi}{2}\right)\hat{z}\right]\exp(-ik_d x + i\beta z) \qquad (13b)
\end{aligned}$$

If we summarize the fractional fields in the above equations, they show rotation by an angle of $\alpha\pi/2$ in counterclockwise direction. Therefore the total fractional fields for the region $0 \leq x \leq t$ are considered to be the fractional intermediate steps between ($\mathbf{E}$, $Z\mathbf{H}$) and ($Z\mathbf{H}, -\mathbf{E}$). That shows rotation by an angle of $\alpha\pi/2$ in counterclockwise direction. These are obtained by substituting the fractional results in Eq. (10) and Eq. (13) in Eq. (4)

$$\begin{aligned}
\mathbf{E}_{fd} =& -\frac{Ak}{k_d}\exp(-i\alpha\pi/2)\left[-\frac{i\beta}{k}\cos\left(\frac{\alpha\pi}{2}\right)\cos\left(k_d x + \frac{\alpha\pi}{2}\right)\hat{x}\right. \\
& \left. -\sin\left(\frac{\alpha\pi}{2}\right)\sin\left(k_d x + \frac{\alpha\pi}{2}\right)\hat{y} - \frac{k_d}{k}\cos\left(\frac{\alpha\pi}{2}\right)\sin\left(k_d x + \frac{\alpha\pi}{2}\right)\hat{z}\right]\exp(i\beta z) \\
& \qquad (14a)
\end{aligned}$$

$$\begin{aligned}
Z\mathbf{H}_{fd} =& -\frac{Ak}{k_d}\exp(-i\alpha\pi/2)\left[-\frac{\beta}{k}\sin\left(\frac{\alpha\pi}{2}\right)\sin\left(k_d x + \frac{\alpha\pi}{2}\right)\hat{x}\right. \\
& \left. +i\cos\left(\frac{\alpha\pi}{2}\right)\cos\left(k_d x + \frac{\alpha\pi}{2}\right)\hat{y} - \frac{ik_d}{k}\sin\left(\frac{\alpha\pi}{2}\right)\cos\left(k_d x + \frac{\alpha\pi}{2}\right)\hat{z}\right]\exp(i\beta z) \\
& \qquad (14b)
\end{aligned}$$

To derive the fractional fields of the E-waves in free space $x > t$, rearrange Eq. (2)

$$\mathbf{E} = -\frac{Ak_0}{h}\sin(k_d t)\exp(ht)\left[-\frac{i\beta}{k_0}\hat{x} - \frac{h}{k_0}\hat{z}\right]\exp(-hx + i\beta z) \qquad (15a)$$

$$Z_0\mathbf{H} = -\frac{Ak_0}{h}\sin(k_d t) i\hat{y}\exp(ht)\exp(-hx + i\beta z) \qquad (15b)$$

The fractional dual fields in free space region $x > t$ become as follow

$$\begin{aligned}\mathbf{E}_{fd} &= -\frac{Ak_0}{h}\sin(k_d t)\left[-\frac{i\beta}{k_0}\cos\left(\frac{\alpha\pi}{2}\right)\hat{x}\right.\\ &\left.+i\sin\left(\frac{\alpha\pi}{2}\right)\hat{y}-\frac{h}{k_0}\cos\left(\frac{\alpha\pi}{2}\right)\hat{z}\right]\exp(-h(x-t)+i\beta z) \quad (16a)\\ Z_0\mathbf{H}_{fd} &= -\frac{Ak_0}{h}\sin(k_d t)\left[\frac{i\beta}{k_0}\sin\left(\frac{\alpha\pi}{2}\right)\hat{x}\right.\\ &\left.+i\cos\left(\frac{\alpha\pi}{2}\right)\hat{y}+\frac{h}{k_0}\sin\left(\frac{\alpha\pi}{2}\right)\hat{z}\right]\exp(-h(x-t)+i\beta z) \quad (16b)\end{aligned}$$

For $\alpha = 0$, we get the original solutions in both regions and for $\alpha = 1$ the fractional fields in both region rotated by an angle $\pi/2$ in counterclockwise direction, results in TE surface wave mode from TM surface wave mode.

The surface impedance of the fractional surface waves in dielectric region of thickness "t" obtained from Eq. (14a) and Eq. (14b) is

$$Z_d = \frac{E_{zfd}}{H_{yfd}} = i\frac{k_d}{k}z\tan\left(k_d x + \frac{\alpha\pi}{2}\right) \tag{17}$$

and that in the free space ($x > t$) is

$$Z_{free} = \frac{E_{zfd}}{H_{yfd}} = i\frac{h}{k_0}z_0 \tag{18}$$

According to the transverse-resonance technique [9], the transverse resonance at $x = t$ requires that the sum of impedances seen looking toward the shot-circuit(in this case the PEC at $x = 0$) and that at the input to the infinite line (in this case $x > t$) vanish. Therefore equating Eq. (17) and Eq. (18) gives the required fractional eigenvalue equation

$$ht = \frac{k_d t}{\kappa}\tan\left(k_d t + \frac{\alpha\pi}{2}\right) \tag{19}$$

where $\frac{z}{k} = \frac{z_0}{\kappa k_0}$. We note that for $\alpha = 0$, Eq. (19) reduces to

$$ht = \frac{k_d t}{\kappa}\tan(k_d t) \tag{20}$$

which represent the eigenvalue equation of TM mode. But for $\alpha = 1$, Eq. (19) gives the eigenvalue equation of TE mode that is

$$ht = -\frac{k_d t}{\kappa}\cot(k_d t) \tag{21}$$

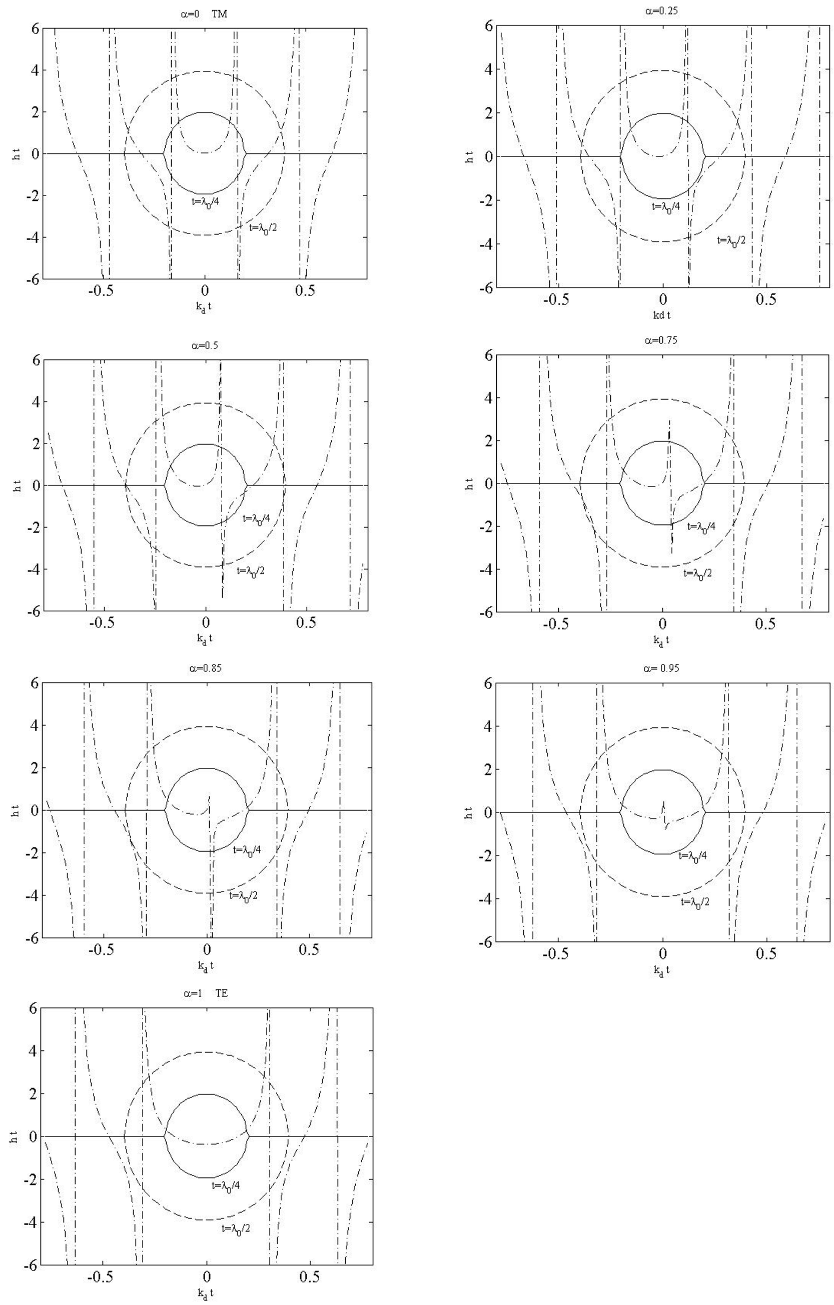

Figure 1. Plots of eigenvalue equations for various values of fractional parameter.

As the phase matching of tangential continuity at $x = t$ interface for all value of z is achieved by the following relation

$$(k_d t)^2 + (ht)^2 = (\kappa - 1)(k_0 t)^2 \quad (22)$$

which represents the equation of a circle of radius $\sqrt{\kappa - 1}(k_0 t)$.

We have carried out the simultaneous numerical solution of Eq. (19) and Eq. (22) for $t = \lambda_0/4$ and $t = \lambda_0/2$, where λ_0 is the free space wavelength, and $\kappa = 2.56$ as shown in Fig. 1. The two circles represented by solid line and dash line corresponded to $t = \lambda_0/4$ and $t = \lambda_0/2$ respectively. For $\alpha = 0$, circle corresponding to $t = \lambda_0/4$ results in one TM surface wave mode, while circle for $t = \lambda_0/2$ results in two TM surface wave modes. We have obtained plots for various values of fractional parameter α. From the plots shown in Fig. 1, we have observed the following informations. For $\alpha = 0, 0.25, 0.5$ or $0 \leq \alpha \leq 0.5$, there exist one surface wave mode for $t = \lambda_0/4$ and two surface wave modes for $t = \lambda_0/2$. But for $\alpha = 0.75, 0.85, 0.95, 1$ or $0.5 \leq \alpha \leq 1$, we can see only one surface wave mode. This is because the TE mode in surface waveguide does not propagate until the radius of the circle, $\sqrt{\kappa - 1}(k_0 t)$, becomes greater than $\pi/2$ [18].

3. CONCLUSIONS

We have noted that for $\alpha = 0$, we get the original TM fields in both dielectric and free space regions, but for $\alpha = 1$, the electric and magnetic fields of original TM fields are rotated by an angle $\pi/2$ in counterclockwise direction and become TE surface wave modes. If the fractional fields are evaluated for higher order values of fractional parameter α, then for even values of α, we obtain TM surface wave modes and while for odd values of α we get TE surface wave modes. The corresponding fractional order surface wave eigen modes are analyzed for various values of α that show an intermediate steps between TM surface wave eigen modes and TE surface wave eigen modes.

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