SCATTERING OF ELECTROMAGNETIC PLANE WAVE BY A PERFECTLY CONDUCTING SLIT AND A PEMC PARALLEL CYLINDER

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Abstract—Diffraction of a plane wave from a geometry which contains an infinite slit in a perfect electric conducting (PEC) plane and a perfectly electromagnetic conductor (PEMC) cylinder is presented. The method is based on the extension of Clemmow, Karp and Russek solution for the diffraction by a wide slit. The results are compared with the published work and agreement is fairly good.

1. INTRODUCTION

Geometrical theory of diffraction (GTD), developed by Keller and his associates [1–7], is one of the most powerful high frequency technique for evaluating the acoustic and electromagnetic waves diffracted by obstacles with or without edges. To derive the solutions for the fields diffracted by edged obstacles, we need the solution of the canonical problem, that is, field diffracted by a wedge, to which half plane is a special case. The multiple objects scattering problem was investigated by many researchers [8–22]. Moreover, the diffraction of electromagnetic plane wave by a slit in a conducting screen has been studied extensively using various kinds of analytical methods such as Mathieu function expansion and Kobayashi potential [23, 24]. In most cases, however, the problem has been limited to ideal cases in which the screen is made of a conducting plane with zero thickness, and the surrounding medium is isotropic and homogeneous. A possible technique is to use fictitious line sources, located according to the

geometry of each scatterers. This technique was used by Clemmow [25] for the diffraction by a wide slit and extended by Elsherbeni and Hamid [26, 27]. In this paper we have applied the technique to three scatterers, i.e., two parallel perfect electric conducting (PEC) half planes and a perfectly electromagnetic conductor cylinder (PEMC).

The concept of PEMC as the generalization of the perfect electric conductor (PEC) and a perfect magnetic conductor (PMC) [28] has been studied recently. It has attracted the attention of many researchers [29–32]. The PEMC boundary conditions are of the general form

$$\vec{n} \times (\vec{H} + M\vec{E}) = 0 \qquad \vec{n} \cdot (\vec{D} - M\vec{B}) = 0$$

where M denotes the admittance of the PEMC boundary. Here, PMC corresponds to $M = 0$, while PEC corresponds to $M = \pm\infty$.

In this paper, the interaction fields between the PEMC cylinder and a PEC slit are presented by using the known solutions for the scattered field by an isolated half plane and an isolated cylinder due to a plane wave incidence and a line source excitation.

2. FORMULATION OF THE PROBLEM

A slit may be viewed as composed of two coplanar half-planes, with zero wedge angles, separated by a slit width $2d$. The geometry and co-ordinates of the problem are shown in Fig. 1(a). The ray optical technique, comprising of geometrical optics and its extension, provides a simple and physical approach to the description of the diffraction of an electromagnetic wave by an object, as it contains only trigonometric functions. This is a great advantage of the GTD over other conventional methods. In this section first we will present scattering of plane wave from PEC half plane, PEC slit and an isolated PEMC cylinder.
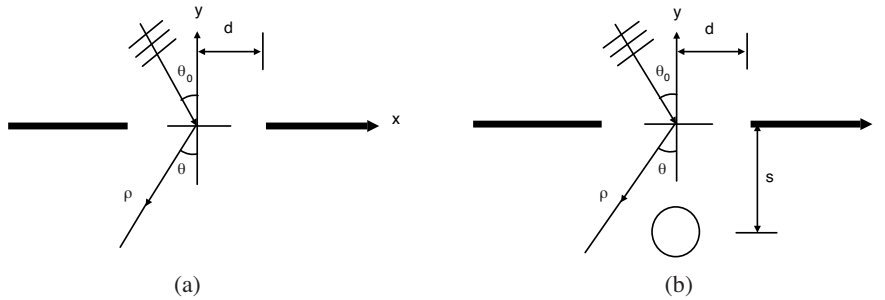

Figure 1. (a) Unloaded slit. (b) Slit loaded by a PEMC cylinder.

2.1. Wedge Excited by Plane Wave

The problem is two dimensional since all fields are uniform in the z-direction. The incident field is given as

$$\begin{pmatrix} E_z^i \\ H_z^i \end{pmatrix} = \begin{pmatrix} E_0 \\ H_0 \end{pmatrix} \exp[jk(x\cos\phi_0 + y\sin\phi_0)] \tag{1}$$

The diffracted field is derived due to an incident plane wave plus a fictitious line source located at the edge of opposite wedge/half plane.

The uniform expression for the field diffracted from wedge has the form:

$$\begin{pmatrix} E_z^d \\ H_z^d \end{pmatrix} = \frac{\exp[-j(k\rho)]}{\sqrt{\rho}} D_{\substack{s \\ h}}(\rho, \phi, \phi_0; n) E^i \tag{2}$$

where

$$D_{\substack{s \\ h}}(\rho, \phi, \phi_0, n) =$$

$$-\sqrt{\rho}\left\{ -sgn\left(\sin\left(\frac{\pi+\phi-\phi_0}{2n}\right)\right)\cos\left(\frac{\pi+\phi-\phi_0}{2n}\right)\right.$$

$$F\left[\sqrt{2k\rho}n\left|\sin\left(\frac{\pi+\phi-\phi_0}{2n}\right)\right|\right]$$

$$-sgn\left(\sin\left(\frac{\pi-(\phi-\phi_0)}{2n}\right)\right)\cos\left(\frac{\pi-(\phi-\phi_0)}{2n}\right)$$

$$F\left[\sqrt{2k\rho}n\left|\sin\left(\frac{\pi-(\phi-\phi_0)}{2n}\right)\right|\right]$$

$$\pm sgn\left(\sin\left(\frac{\pi+\phi+\phi_0}{2n}\right)\right)\cos\left(\frac{\pi+\phi+\phi_0}{2n}\right)$$

$$F\left[\sqrt{2k\rho}n\left|\sin\left(\frac{\pi+\phi+\phi_0}{2n}\right)\right|\right]$$

$$\pm sgn\left(\sin\left(\frac{\pi-(\phi+\phi_0)}{2n}\right)\right)\cos\left(\frac{\pi-(\phi+\phi_0)}{2n}\right)$$

$$F\left.\left[\sqrt{2k\rho}n\left|\sin\left(\frac{\pi-(\phi+\phi_0)}{2n}\right)\right|\right]\right\} \tag{3}$$

D_s and D_h are the diffraction coefficients of E- and H-polarization respectively. Here $p = \frac{1}{n} = \frac{\pi}{\phi_\omega}$ and for half plane $n = 2$, $\phi_\omega = 2\pi$. Function $F(x)$ is the Fresnel integral defined as

$$F(x) = \frac{1}{\pi}\exp\left(jx^2 + j\frac{\pi}{4}\right)\int_x^\infty \exp\left(-j\mu^2\right)d\mu \tag{4}$$

The diffraction co-efficient for the half plane is

$$D_{\frac{s}{h}}(\rho, \phi, \phi_0) =$$

$$-\sqrt{\rho}\left\{-sgn\left(\sin\left(\frac{\pi + \phi - \phi_0}{4}\right)\right)\cos\left(\frac{\pi + \phi - \phi_0}{4}\right)\right.$$

$$F\left[\sqrt{8k\rho}\left|\sin\left(\frac{\pi + \phi - \phi_0}{4}\right)\right|\right]$$

$$-sgn\left(\sin\left(\frac{\pi - (\phi - \phi_0)}{4}\right)\right)\cos\left(\frac{\pi - (\phi - \phi_0)}{4}\right)$$

$$F\left[\sqrt{8k\rho}\left|\sin\left(\frac{\pi - (\phi - \phi_0)}{4}\right)\right|\right]$$

$$\pm sgn\left(\sin\left(\frac{\pi + \phi + \phi_0}{4}\right)\right)\cos\left(\frac{\pi + \phi + \phi_0}{4}\right)$$

$$F\left[\sqrt{8k\rho}\left|\sin\left(\frac{\pi + \phi + \phi_0}{4}\right)\right|\right]$$

$$\pm sgn\left(\sin\left(\frac{\pi - (\phi + \phi_0)}{4}\right)\right)\cos\left(\frac{\pi - (\phi + \phi_0)}{4}\right)$$

$$\left.F\left[\sqrt{8k\rho}\left|\sin\left(\frac{\pi - (\phi + \phi_0)}{4}\right)\right|\right]\right\} \tag{5}$$

It is assumed that point of observation is far from the slit. For large argument approximation, Fresnel integral simplifies as

$$F(x) \approx \frac{1}{2\sqrt{\pi x}}\exp\left[-j\frac{\pi}{4}\right]$$

Hence (5) simplifies to

$$D(\phi, \phi_0) \approx -\frac{1}{\sqrt{8\pi k}}\exp\left[-j\frac{\pi}{4}\right]\left[\sec\frac{\phi - \phi_0}{2} \mp \sec\frac{\phi + \phi_0}{2}\right] \tag{6}$$

The angles between the incident and diffracted rays and normal to the screen are ϕ and ϕ_0 respectively.

2.2. A Slit Excited by Plane Wave

In this sub-section the field from two isolated half planes, placed parallel to each other, is determined. These two half planes have been considered to form a slit.

2.2.1. Field Diffracted from Right Half Plane

It is assumed that the slit is wide, so field diffracted by the slit may be considered as the sum of field diffracted by each isolated half plane. The singly diffracted field at an observation point is the sum of two contributions due to each isolated half plane. The field diffracted from right edge is

$$E_r^d(\rho_1, \phi_1) = -\frac{1}{\sqrt{8\pi k}} \rho_1^{-\frac{1}{2}} \exp(-jk\rho_1) \exp\left[-j\frac{\pi}{4}\right]$$
$$\left[\sec\frac{\phi_1 - \phi_{01}}{2} \mp \sec\frac{\phi_1 + \phi_{01}}{2}\right] \exp[jkd\cos\phi_{01}] \quad (7)$$

In the far field of the slit $(\rho \gg a)$:

$$\rho_1 = \rho - d\sin\theta, \qquad \phi_1 = \frac{3\pi}{2} + \theta, \qquad \phi_{01} = \frac{\pi}{2} + \theta_0$$

This far-field substitution for ρ_1 is used in the exponential term; in the amplitude term $\rho_1 = \rho$

$$E_r^d = \frac{\exp\left[-j\left(k\rho + \frac{\pi}{4}\right)\right]}{\sqrt{8\pi k\rho}} \exp[jkd(\sin\theta - \sin\theta_0)]\left[\csc\frac{\theta - \theta_0}{2} \mp \sec\frac{\theta + \theta_0}{2}\right]$$
$$(8)$$

2.2.2. Field Diffracted from Left Half Plane

The field diffracted from left edge is

$$E_l^d(\rho_2, \phi_2) = -\frac{1}{\sqrt{8\pi k}} \rho_2^{-\frac{1}{2}} \exp(-jk\rho_2) \exp\left[-j\frac{\pi}{4}\right]$$
$$\left[\sec\frac{\phi_2 - \phi_{02}}{2} \mp \sec\frac{\phi_2 + \phi_{02}}{2}\right] \exp[jkd\cos\phi_{02}] \quad (9)$$

In the far field of the slit $(\rho \gg a)$:

$$\rho_2 = \rho + d\sin\theta, \qquad \phi_2 = \frac{3\pi}{2} - \theta, \qquad \phi_{02} = \frac{\pi}{2} - \theta_0$$

This far-field substitution for ρ_2 is used in the exponential term; in the amplitude term $\rho_2 = \rho$

$$E_l^d = \frac{\exp\left[-j\left(k\rho + \frac{\pi}{4}\right)\right]}{\sqrt{8\pi k\rho}} \exp[-jkd(\sin\theta - \sin\theta_0)]$$
$$\left[-\csc\frac{\theta - \theta_0}{2} \mp \sec\frac{\theta + \theta_0}{2}\right] \qquad (10)$$

Thus the singly diffracted field due to perfectly conducting slit at an observation point is:

$$E^d = E_r^d + E_l^d$$

$$E^d = \sqrt{\frac{k}{2\pi\rho}} \exp\left[-jk\rho - j\frac{\pi}{4}\right] f^{(1)} \qquad (11)$$

where

$$f^{(1)} = j\frac{\sin kd(\sin\theta - \sin\theta_0)}{k\sin\dfrac{\theta - \theta_0}{2}} \mp \frac{\cos kd(\sin\theta - \sin\theta_0)}{k\cos\dfrac{\theta + \theta_0}{2}}$$

Numerical results shown in Fig. 2 are in good agreement with the Elsherbeni's results [26]. It may be noted that these results are also valid for all incident angles.

2.3. Perfect Electromagnetic Conductor Cylinder Excited by Plane Wave

A circular cylinder is defined by the surface $\rho = $ a, while its axis coincides with the z-axis. The scattered field due to plane wave incident on circular cylinder [34] is:

$$E_p^C = \frac{\pi}{2j} H_0(k\rho) G_p(\phi, \phi_0, a) \qquad (12)$$

where

$$G_p(\phi, \phi_0, a) = -\frac{2j}{\pi} \sum_{n=0}^{\infty} \epsilon_n (-1)^n T_n \cos[n(\phi - \phi_0)] \qquad (13)$$

T_n is the transmission co-efficient for perfect electromagnetic conductor cylinder [35–37] given as:

$$T_n = \begin{cases} \dfrac{H_n^{(2)}(ka)J_n'(ka) + M^2\eta_0^2 J_n(ka)H_n^{(2)'}(ka)}{\left(1 + M^2\eta_0^2\right)H_n^{(2)}(ka)H_n^{(2)'}(ka)} & \text{Co-polarized} \\[4mm] \dfrac{2M\eta_0}{\pi ka\left(1 + M^2\eta_0^2\right)H_n^{(2)}(ka)H_n^{(2)'}(ka)} & \text{Cross polarized} \end{cases} \qquad (14)$$

In above equations the Neumann number $\epsilon_n = 1$ for $n = 0$ and 2 for $n > 0$, $J_n(x)$ is the Bessel function of argument x and order n and $H_n(x)$ is the Hankel function of the second kind of order n and argument x.

Perfectly conducting slit, kd = 4, $\theta_0 = 0$
- - - Elsherbeni
___ Present work

Perfectly conducting slit, kd = 8, $\theta_0 = 0$
- - - Elsherbeni
___ Present work

(a)

Perfectly conducting slit, kd = 4, $\theta_0 = 0$

Perfectly conducting slit, kd = 4, $\theta_0 = 30$

Perfectly conducting slit, kd = 8, $\theta_0 = 0$

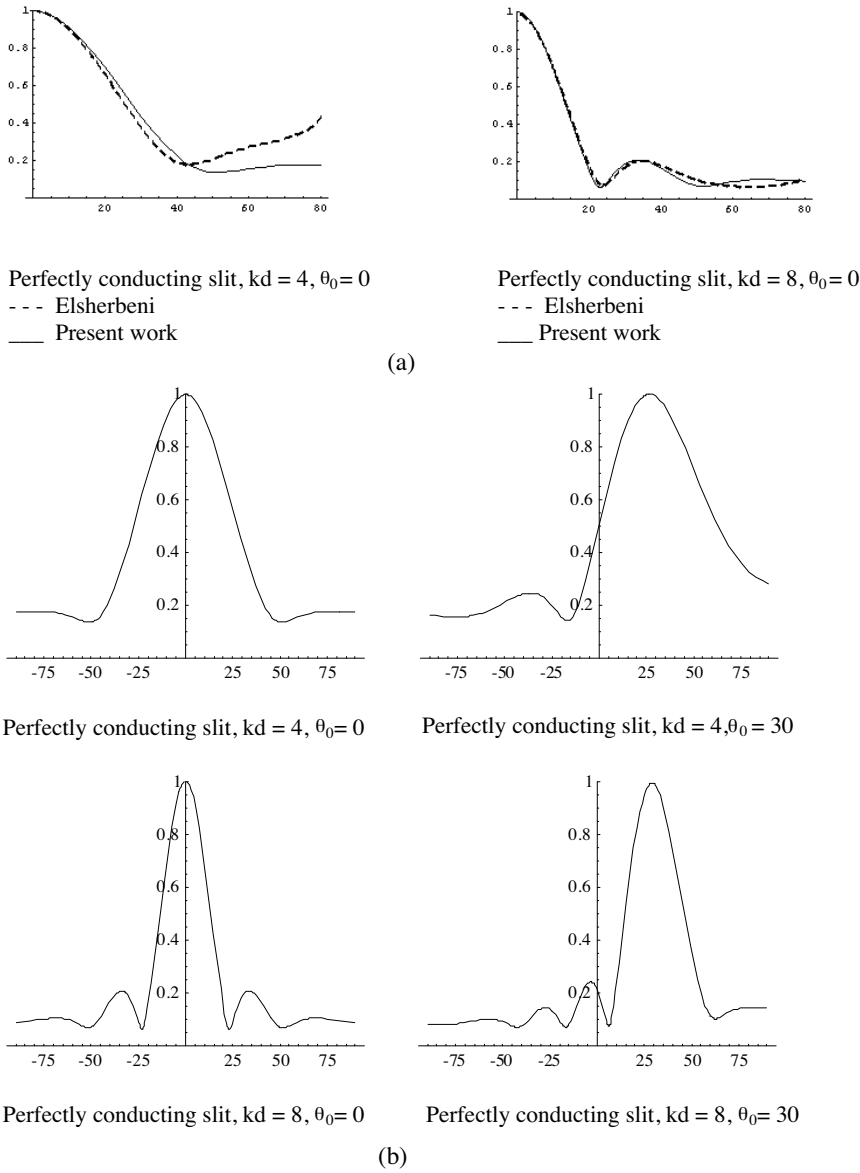Perfectly conducting slit, kd = 8, $\theta_0 = 30$

(b)

Figure 2. (a) Comparison with Elsherbeni's results. (b) Perfectly conducting slit for $kd = 4$ and $kd = 8$ at incident angles $\theta_0 = 0$ and $\theta_0 = 30$.

3. CYLINDRICAL WAVE INCIDENT

In this section, the interaction fields between the PEMC cylinder and the two parallel PEC half planes are presented by using the known solutions for the scattered field by the half plane alone and a cylinder alone due to line source excitation.

3.1. Isolated Half Plane Excited by a Line Source

For a line source of unit amplitude at (ρ_0, ϕ_0) and parallel to the z-axis, the total field in the presence of the half plane is the incident field plus the scattered field. The incident field is given by:

$$E^i = \frac{\pi}{2j} H_0(kR)$$

where R is the distance between the line source and the field point, k is the wave number, and $H_0(x)$ is the Hankel function of the second kind of order zero and argument x. Using the exact series solution of the total field due to a line source near a conducting wedge/half plane [33], is found to be

$$E_l = \frac{\pi}{2j} H_0(k\rho) F(\phi, \rho_0, \phi_0) \tag{15}$$

where

$$
\begin{aligned}
F(\phi, \rho_0, \phi_0) = & -\exp[jk\rho_0 \cos(\phi - \phi_0)] \\
& + 2 \sum_{n=1}^{\infty} j^{\frac{n}{2}} J_{\frac{n}{2}}(k\rho_0) \sin\left(\frac{n\phi}{2}\right) \sin\left(\frac{n\phi_0}{2}\right)
\end{aligned} \tag{16}
$$

3.2. Perfect Electromagnetic Conductor Cylinder Excited by a Line Source

The scattered field due to cylindrical wave incident on circular cylinder [34] is:

$$E_l^C = \frac{\pi}{2j} H_0(k\rho) G_l(\phi, \phi_0, a) \tag{17}$$

where

$$G_l(\phi, \phi_0, a) = -\sum_{n=0}^{\infty} \epsilon_n j^n T_n H_n(k\rho_0) \cos[n(\phi - \phi_0)] \tag{18}$$

T_n is the transmission co-efficient for perfect electromagnetic conductor cylinder [35–37] given as:

$$
T_n = \begin{cases} -\dfrac{H_n^{(2)}(ka)J_n^{/}(ka)+M^2\eta_0^2 J_n(ka)H_n^{(2)/}(ka)}{\left(1+M^2\eta_0^2\right)H_n^{(2)}(ka)H_n^{(2)/}(ka)} & \text{Co-polarized} \\[4mm] \dfrac{2M\eta_0}{\pi ka\left(1+M^2\eta_0^2\right)H_n^{(2)}(ka)H_n^{(2)/}(ka)} & \text{Cross polarized} \end{cases} \tag{19}
$$

Here, again the Neumann number $\epsilon_n = 1$ for $n = 0$ and 2 for $n > 0$, $J_n(x)$ is the Bessel function of argument x and order n and $H_n(x)$ is the Hankel function of the second kind of order n and argument x.

4. INTERACTION CONTRIBUTION OF THE GEOMETRY

We have two conducting half planes separated by a distance $2d$, where $2kd \gg 1$ and a circular cylinder of radius a whose axis is parallel to the edges of two parallel half planes as shown in Fig. 1(b). We consider that all the three bodies are illuminated by a plane wave of unit amplitude. The field at any observation point is considered to be composed of the incident field plus a response field from each of the two half planes and the cylinder. The response field consists of scattered field by the three scatterers due to the original plane wave plus an interaction field which will be represented by three fictitious line sources located at the edges of half planes and at the cylinder axis in order to take into account multiple interaction between three objects. If the plane wave is restricted such that the incident field does not illuminate the lower faces of the half planes, the total field in the forward direction is given by [38]:

$$
E^t = E^i + E^s
$$

where

$$
E^s = E^{s1} + E^{s2} + E^{s3} \tag{20}
$$

$$
\begin{aligned}
E^{s1} &= \frac{\pi}{2j}H_0(k\rho_1)[\exp(-jkd\sin\theta_0)]D(\phi_1,\phi_{01}) \\
&\quad +c_3 F(\phi_1,s_1,\phi_{31})+c_2 F(\phi_1,2d,\phi_{21}) \quad\quad (21a) \\
E^{s2} &= \frac{\pi}{2j}H_0(k\rho_2)[\exp(+jkd\sin\theta_0)]D(\phi_2,\phi_{02}) \\
&\quad +c_3 F(\phi_2,s_2,\phi_{32})+c_1 F(\phi_2,2d,\phi_{12}) \quad\quad (21b) \\
E^{s3} &= \frac{\pi}{2j}H_0(k\rho_3)[\exp(-jks\sin\theta_0)]D(\phi_3,\phi_{03})
\end{aligned}
$$

$$+c_1 G(\phi_3, s_1, \phi_{13}) + c_2 G(\phi_3, s_2, \phi_{23}) \tag{21c}$$

where c_1, c_2 and c_3 are the unknown strengths of the line sources at half plane edges and along the cylinder axis, respectively. We use well-known far field conditions in which $\phi_0 = \phi_{01} = \phi_{03} = \frac{\pi}{2} + \theta_0$, $\phi_{02} = \frac{\pi}{2} - \theta_0$, $\phi_1 = \phi_3 \simeq \frac{3\pi}{2} + \theta$, $\phi_2 \simeq \frac{3\pi}{2} - \theta$, $\phi_{12} = \phi_{21} \simeq \pi$, $\phi_{13} = \psi \simeq \tan^{-1}(\frac{s}{d})$, $\phi_{31} = \phi_{32} \simeq \pi + \psi$, $\phi_{23} \simeq \pi - \psi$, $\rho_1 \simeq \rho - d\sin\theta$, $\rho_2 \simeq \rho + d\sin\theta$, $\rho_3 \simeq \rho - s\sin\theta$, and s_1 and s_2 are the distances between the edges of the two half planes and the cylinder axis, respectively.

To determine c_1, c_2 and c_3, we can follow the analysis of Karp and Russek [12] by imposing the requirement that the fields scattered by the two half planes and the cylinder be consistent with one another. The same technique was used by Elsherbeni and Hamid [38].

$$
\begin{aligned}
& 2c_1 - c_2[F(\phi_{31}, 2d, \phi_{21}) + F(\phi_{21}, 2d, \phi_{21})] \\
& -c_3[F(\phi_{31}, s_1, \phi_{31}) + F(\phi_{21}, s_1, \phi_{31})] = \\
& \exp(-jkd\sin\theta_0)[D(\phi_{31}, \phi_{01}) + D(\phi_{21}, \phi_{01})]
\end{aligned} \tag{22a}
$$

$$
\begin{aligned}
& 2c_2 - c_1[F(\phi_{32}, 2d, \phi_{12}) + F(\phi_{12}, 2d, \phi_{12})] \\
& -c_3[F(\phi_{32}, s_2, \phi_{32}) + F(\phi_{12}, s_2, \phi_{32})] = \\
& \exp(jkd\sin\theta_0)[D(\phi_{32}, \phi_{02}) + D(\phi_{12}, \phi_{02})]
\end{aligned} \tag{22b}
$$

$$
\begin{aligned}
& 2c_3 - c_1[G(\phi_{13}, s_1, \phi_{13}) + G(\phi_{23}, s_1, \phi_{13})] \\
& -c_2[G(\phi_{13}, s_2, \phi_{23}) + G(\phi_{23}, s_2, \phi_{23})] = \\
& \exp(-jks\cos\theta_0)[D(\phi_{13}, \phi_{03}) + D(\phi_{23}, \phi_{03})]
\end{aligned} \tag{22c}
$$

We solve 22(a) $\sim$ 22(c) for c_1, c_2 and c_3, the scattered field is found and rewritten in the form

$$E^s = \frac{\exp(-jk\rho)}{\sqrt{\pi k\rho}} E(\theta, d, s) \tag{23}$$

where the scattered field pattern E is obtained from (20).

5. DISCUSSION

The solution for the diffraction of an incident plane wave by a slit has been studied in GTD regime. We have derived a simple and convenient expression for the field diffracted by an infinite slit in a PEC plane when the wavelength is greater than or equal to the slit width. The principal result is that this field can be accurately calculated everywhere by
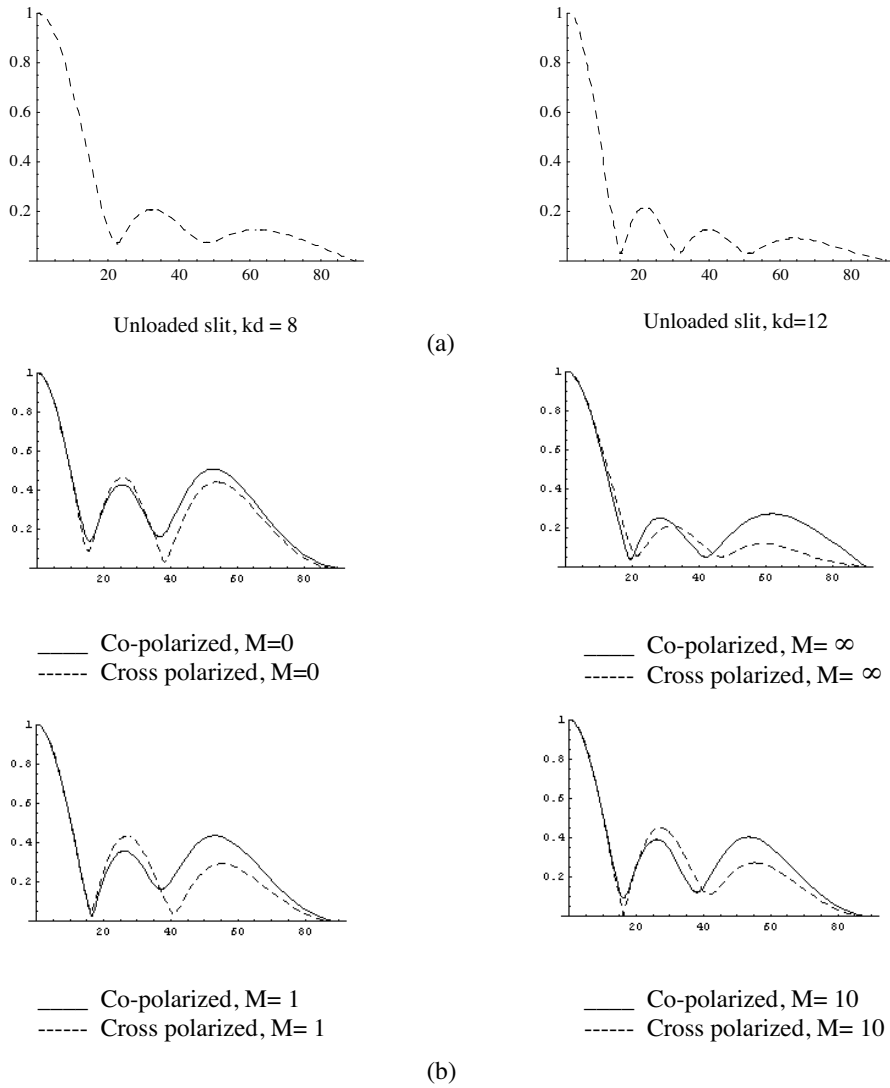
Unloaded slit, kd = 8   Unloaded slit, kd=12

(a)

_____ Co-polarized, M=0   _____ Co-polarized, M= ∞
------ Cross polarized, M=0   ------ Cross polarized, M= ∞

_____ Co-polarized, M= 1   _____ Co-polarized, M= 10
------ Cross polarized, M= 1   ------ Cross polarized, M= 10

(b)

Figure 3. (a) Unloaded slit with $kd = 8$ and $kd = 12$. (b) Scattering from PEC slit and PEMC cylinder with $ks = 0$ and $kd = 8$.

considering two half planes composing the slit and excited by the plane wave. Fig. 2(a) shows the normalized diffracted field from the slit with $kd = 4$ and $kd = 8$ compared with the normalized diffracted field by Elsherbeni [26]. Fig. 2(b) shows the results at different incident angles

with $kd = 4$ and $kd = 8$. Fig. 3(a) shows the diffraction pattern of an unloaded slit. The diffraction field patterns in the presence of a PEMC cylinder for both co-polarized and cross-polarized for $kd = 8$, $ka = 0.5$ and $\epsilon_r = 4$ are shown in Fig. 3(b). The plots for unloaded slit diffraction pattern are in good agreement with the corresponding patterns given by Keller [39].

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