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FULL WAVE ANALYSIS OF TWO LAYERED ORTHOGONAL MICROSTRIP TRANSMISSION LINES BASED ON SPECTRAL DOMAIN METHOD

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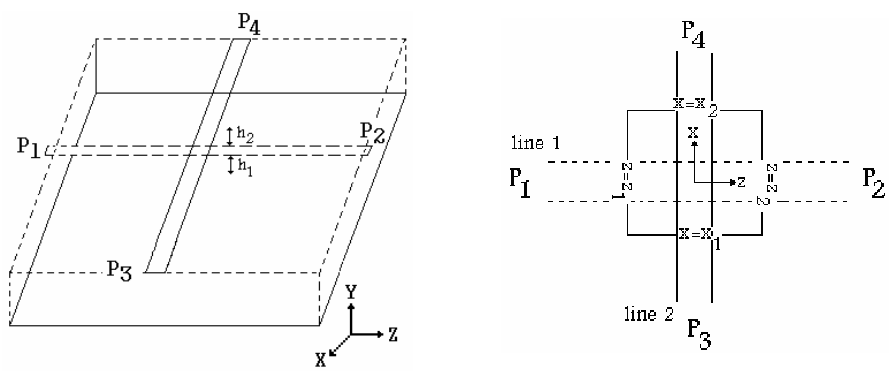
Abstract—In this paper, using moment method in the spectral domain, full wave analysis of two orthogonal microstrip transmission lines in two layered PCB board is presented. First, using 1-dimensional spectral domain method, the propagation constants and the currents on each line are obtained, without considering of the other line. Then, using Galerkine's method in 2 dimensional spectral domain, the scattering parameters of the structure are calculated. The results of our analysis are compared by the quasi-TEM approach. Comparisons show good agreement between our results and quasi-TEM approach.

1. INTRODUCTION

In many microwave circuits, multilayer multi-transmission lines are used as signal propagation tracks in PCB boards [1–4]. To decrease coupling between the lines, they usually designed in such a way that transmission lines of one layer are orthogonal with transmission lines on the two adjacent layers. Considering TEM mode for each line, coupling should not be achieved between the lines. In [2], the equivalent circuit of the structure is obtained based on quasi-TEM approximation. So, by increasing the frequency, the accuracy of the model is decreased. For this reason, we should consider the effect of high order modes on the responses. In this paper, a full wave analysis of the problem using the moment method in the spectral domain is presented. In Section 2, the method of analysis is described. First, using one dimensional spectral domain, the propagation constants and transverse currents on the uniform and infinitely transmission lines are obtained and then, using Galerkine's method in two dimensional spectral domain, the scattering parameters of the structure are calculated. Afterwards, in Section 3, the results of our analysis are compared with the quasi-TEM approach in [2].

2. THE METHOD OF ANALYSIS

Figure 1 shows the schematic view of the structure. Assuming that waves are launched from $z = -\infty$ at port P_1, fields and currents are propagated from the source by the propagation constant equals β_1 until they arrive at coupling region. As a result of this coupling phenomenon, currents are flowed into the line 2. At distances far from the coupling region, the currents on line 2 are propagated with the propagation constant equals β_2. But in the coupling region, the currents on each line are affected by the other line. It is clear that currents in this region are not uniform. First, we should obtain β_1, β_2 and transverse current components of each line. The thicknesses of the microstrip lines are neglected in this analysis because of their insignificant effect as discussed in [5, 6]. Standard spectral domain method [7] or the immittance approach [8] can be utilized to compute the propagation constant and current distributions of an infinitely long and uniform microstrip line depicted in Figure 2. As the result, the following matrix equation is obtained in the spectral domain at the



(a) (b)

Figure 1. (a) Schematic view of the two orthogonal microstrip transmission lines that located at distances h_1 and $h_1 + h_2$ from the ground. (b) Top view of the structure.

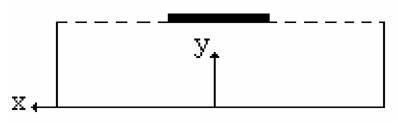


Figure 2. Cross section of the uniform microstrip transmission line.

dielectric-air interface of the structure in Figure 2:

$$\begin{bmatrix} \tilde{Z}_{xx} & \tilde{Z}_{xz} \\ \tilde{Z}_{zx} & \tilde{Z}_{zz} \end{bmatrix} \begin{bmatrix} \tilde{J}_x(k_x,\, \beta) \\ \tilde{J}_z(k_x,\, \beta) \end{bmatrix} = \begin{bmatrix} \tilde{E}_x(k_x,\, \beta) \\ \tilde{E}_z(k_x,\, \beta) \end{bmatrix} \tag{1}$$

where

$$\begin{aligned}
\tilde{Z}_{xx} &= \frac{-1}{k_x^2+\beta^2}\left[\beta^2\cdot\tilde{Z}_h + k_x^2\cdot\tilde{Z}_e\right]\\
\tilde{Z}_{xz} &= \tilde{Z}_{zx} = \frac{-k_x\beta}{k_x^2+\beta^2}\left[\tilde{Z}_e - \tilde{Z}_h\right]\\
\tilde{Z}_{zz} &= \frac{-1}{k_x^2+\beta^2}\left[k_x^2\cdot\tilde{Z}_h + \beta^2\cdot\tilde{Z}_e\right]\\
\tilde{Z}_e &= \frac{j\omega\mu}{\gamma_2+\gamma_1\coth\gamma_1 h}\\
\tilde{Z}_h &= -\frac{j\gamma_1}{\omega\varepsilon\times\left(\dfrac{\varepsilon_0\gamma_1}{\varepsilon_r\gamma_2}+\coth\left(\gamma_1 h_1\right)\right)}\\
\gamma_1 &= \sqrt{k_x^2+\beta^2-\omega^2\mu\varepsilon_r}\\
\gamma_2 &= \sqrt{k_x^2+\beta^2-\omega^2\mu\varepsilon_0}
\end{aligned}$$

It should be noted that (1) is the Fourier transform of the electric field integral equation (EFIE) in the conventional space domain analysis. To obtain the unknown current distributions, the first step is to expand them in terms of known Maxwellian basis functions $f_{x,\,z}(x)$ [7]:

$$\tilde{J}_x(k_x) = j\sum_{m=1}^{M} a_{x_m}\cdot\tilde{F}_{x_m}(k_x) \tag{2a}$$

$$\tilde{J}_z(k_x) = j\sum_{n=1}^{N} a_{z_n}\cdot\tilde{F}_{z_n}(k_x) \tag{2b}$$

To obtain the current distributions on the line, the Galerkine's method in the spectral domain has been used. By utilizing Parseval's theorem as described in [9] we arrive at:

$$j\sum_{m=1}^{M} a_{x_m}\int_{-\infty}^{+\infty}\tilde{F}^*_{x_p}(k_x)\cdot\tilde{Z}_{xx}(k_x,\,\beta)\cdot\tilde{F}_{x_m}(k_x)dk_x$$
$$+\sum_{n=1}^{N} a_{z_n}\int_{-\infty}^{+\infty}\tilde{F}^*_{x_p}(k_x)\cdot\tilde{Z}_{xz}(k_x,\,\beta)\cdot\tilde{F}_{z_n}(k_x)dk_x=0 \tag{3a}$$
$$j\sum_{m=1}^{M} a_{x_m}\int_{-\infty}^{+\infty}\tilde{F}^*_{x_q}(k_x)\cdot\tilde{Z}_{zx}(k_x,\,\beta)\cdot\tilde{F}_{x_m}(k_x)dk_x$$
$$+\sum_{n=1}^{N} a_{z_n}\int_{-\infty}^{+\infty}\tilde{F}^*_{z_q}(k_x)\cdot\tilde{Z}_{zz}(k_x,\,\beta)\cdot\tilde{F}_{z_n}(k_x)dk_x=0 \tag{3b}$$

where $p = 1, 2, \ldots, M$ and $q = 1, 2, \ldots, N$, because of the linearity of the above equations, they can be written as the following matrix formulation:

$$[A]_{(M+N)\times(M+N)}\begin{bmatrix}[a_x]_{M\times 1}\\ [a_z]_{N\times 1}\end{bmatrix}=[0]_{(M+N)\times 1} \tag{4}$$

The simultaneous equations in (4) can be solved for the propagation constant, by setting the determinant of the coefficient matrix equal to zero and by seeking the root of the resulting characteristic equation. The same procedure that has been described, can be used to calculate propagation constant and current distributions on the line that located in x direction. By calculating the propagation constant and current distributions on the uniform line, we can write the currents on the lines in Figure 1 as bellow:

$$J^{(1)}_{x,\,z}(x,\,z)=\begin{cases} f_{x,\,z}(x)\left(e^{-j\beta_1(z-z_1)}\pm\Gamma_1 e^{j\beta_1(z-z_1)}\right) & z<z_1\\ f_{x,\,z}(x)\sum\limits_{n=1}^{N}a_n^{x,\,z}g_n^{(1)}(z) & z_1<z<z_2\\ T_1\cdot f_{x,\,z}(x)\cdot e^{-j\beta_1(z-z_2)} & z_2<z\end{cases} \tag{5}$$

(-) is for J_x component. In (5), Γ_1 and T_1 are scattering parameters of line 1. The currents on the line 2 can be written as:

$$J^{(2)}_{x,\,z}(x,\,z)=\begin{cases} \Gamma_2\cdot h_{x,\,z}(z)\cdot e^{j\beta_2(x-x_1)} & x<x_1\\ h_{x,\,z}(z)\sum\limits_{m=1}^{M}b_m^{x,\,z}g_m^{(2)}(x) & x_1<x<x_2\\ T_2\cdot h_{x,\,z}(z)\cdot e^{-j\beta_1(x-x_1)} & x_2<x\end{cases} \tag{6}$$

$g_n^{(1)}$ and $g_m^{(2)}$ are PSW-P basis functions which have been used for the currents on the lines in the coupling region [10], and f, h are current distributions in uniform and infinitely long lines which were obtained by 1-dimensional spectral domain. The currents must be continuous at the boundaries $x = x_1,\, x_2$ and $z = z_1,\, z_2$, so:

$$\begin{aligned} 1 - \Gamma_1 &= a_1^x \\ 1 + \Gamma_1 &= a_1^z \\ T_1 = a_N^x &= a_N^z \\ \Gamma_2 = b_1^x &= b_1^z \\ T_2 = b_M^x &= b_M^z \end{aligned} \tag{7}$$

By applying 2-dimensional Fourier transform into the currents in (5) and (6) and considering (7) we have:

$$\begin{aligned} \tilde{J}^{(1)}_{x,\,z}(k_x,\, k_z) \;=\; \tilde{F}_x\left(k_x\right) \Bigg[& \left(\frac{e^{jk_z z_1}}{j(k_z - \beta_1)} + \tilde{G}_1^{(1)}(k_z) \right) \\ & \pm \Gamma_1 \cdot \left(\frac{e^{jk_z z_1}}{j(k_z + \beta_1)} + \tilde{G}_1^{(1)}(k_z) \right) + \sum_{n=2}^{N-1} a_n^{x,\,z} \tilde{G}_1^{(1)}\left(k_z\right) \\ & + T_1 \left(\frac{e^{jk_z z_2}}{j(\beta_1 - k_z)} + \tilde{G}_N^{(1)}(k_z) \right) \Bigg] \end{aligned} \tag{8a}$$

$$\begin{aligned} \tilde{J}^{(2)}_{x,\,z}(k_x,\, k_z) = \tilde{H}_z(k_z) \Bigg[& \Gamma_2 \cdot \left(\frac{e^{jk_x z_1}}{j(k_x + \beta_2)} + \tilde{G}_1^{(2)}(k_z) \right) \\ & + T_2 \cdot \left(\frac{e^{jk_x x_2}}{j(\beta_2 - k_x)} + \tilde{G}_M^{(2)}(k_x) \right) + \sum_{m=2}^{M-1} b_m^{x,\,z} \tilde{G}_m^{(2)}(k_x) \Bigg] \end{aligned} \tag{8b}$$

$\tilde{G}_n^{(1)}$ and $\tilde{G}_m^{(2)}$ are Fourier transforms of basis functions. Unknown parameters in (8) are $\Gamma_{1,\,2}$, $T_{1,\,2}$, $a_n^{x,\,z}$, $b_m^{x,\,z}$. To obtain these unknowns, 2-dimensional spectral domain approach has been utilized for the structure shown in Figure 1. The continuity conditions of tangential E- and H-fields are applied in the spectral domain at interfaces $y = h_1$ and $y = h_1 + h_2$ to generate a set of equations which are the Fourier transforms of the mixed potential integral equations (MPIE). This set of equations is very similar to (1), except that the right-hand side is no longer zero due to source excitation. It can be written as:

$$\left[\tilde{M}\right] \begin{bmatrix} [\Gamma,\, T] \\ [a_n^{x,\,z}] \\ [b_m^{x,\,z}] \end{bmatrix} = \left[\tilde{V}\right] \tag{9}$$

In calculating $[\tilde{M}]$ and $[\tilde{V}]$, there exist some singularities in denominator of their components. These poles are from two points. Some of them are related to the dyadic green's functions that gives the transverse resonance condition, namely the surface wave propagation constants [8, 12]. Other poles are at $k_z = \pm\beta_1$ and $k_x = \pm\beta_2$. These poles indicate propagation constants in non-coupling region along the lines. Utilizing residue theorem, we can remove these singularities [11, 12].

3. RESULTS AND DISCOSSION

To demonstrate the accuracy of the method, the results of our analyses are compared by the quasi-TEM approach [2] in Figure 3. The dimensions of the structure shown in Figure 1 are as bellow:

Substrate permeability (ε_r) = 4.4

$h_1 = 0.635$ mm

h_2=1.27 mm

W_1 (Width of line1) =1.174 mm

W_2 (Width of line2) =2.3495 mm

Figure 3 shows the plot of propagation constant of the lines versus frequency. In these frequencies, the results of Quasi-TEM method and our full-wave analysis are very close. But by increasing the frequency of operation, the difference between the results of two approaches is sensible. The scattering parameters of the structure are indicated in Figure 4.

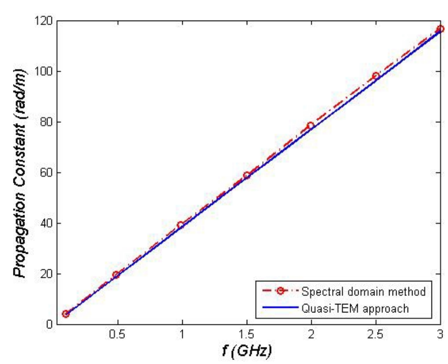


Figure 3. Propagation constants of the lines versus frequency.

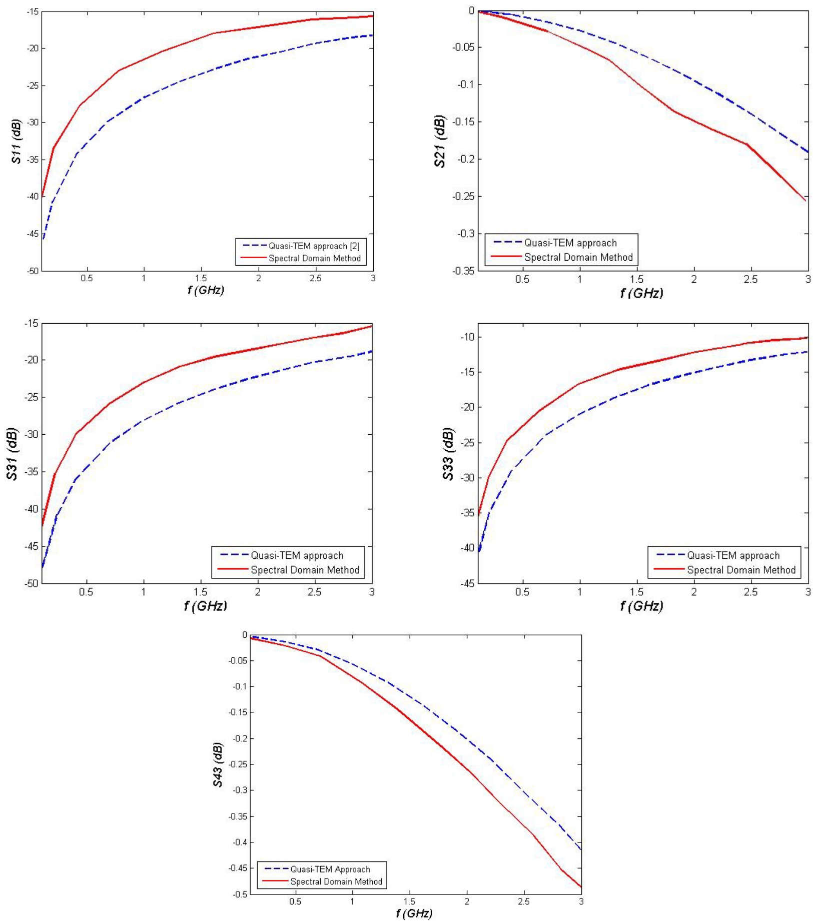


Figure 4. Scattering parameters of the structure.

4. CONCLUSION

In this paper, using moment method in the spectral domain, full wave analysis of two orthogonal microstrip transmission lines has been presented. As the frequency is increased, the coupling between two lines is increased. So, this structure can be useful in low frequency operations. Noticeable points in our analysis compared to HFSS and other microwave software packages, in addition to a vanishingly short execution time, are the ideas and views of which this analysis provides for the designers. In other word, using moment method in spectral domain, we could obtain closed form relations of the solution.

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