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UNDERCOMPLETE DICTIONARY-BASED FEATURE EXTRACTION FOR RADAR TARGET IDENTIFICATION

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Abstract—Feature extraction is a challenging problem in radar target identification. In this paper we attempt to exploit the sparse property of the scattering signature with a undercomplete dictionary for target identification, and establish a feature extraction scheme based on the undercomplete dictionary. Furthermore, as an application, we present a feature vector, named as the atom dictionary feature, which is extracted from the scattering signatures over a wide-angle sector. Numerical simulation results show that the proposed atom dictionary feature can improve the performance of radar target identification due to the exploitation of the sparse property of the scattering signature.

1. INTRODUCTION

Radar target identification has been an active research area for over half a century. Usually, a feature extraction process plays an essential role for the success of target identification. Due to highly aspect-dependent nature of the scattered signature, feature extraction for target identification is an especially difficult problem. The complex natural resonance frequency (CNRF), also called as target pole, is only one aspect independent feature. It is theoretically well known that shape, size and material properties of a target determine the values of CNRFs and the target is uniquely characterized by the complete set of its CNRFs. CNRFs-based target identification has been a highly

attractive research area and many efforts have been attempted in the last decade [1, 2]. However, the precise extraction of these CNRFs from scattered data is not usually an easy process, especially in the presence of noise [3, 4]. Therefore, some resonance features which are indirectly related to the pole values of the target are extracted from the natural response components of scattering signals in alternative ways. Recently, Turhan-Sayan [5] attempted to use resonance mechanisms to reduce the aspect angle sensitivity of the extracted feature. It is partly proved that the late-time partitioned energy density vectors constructed by computing the spectral distribution of scattering signature energy of some non-overlapping subsequent time bands can significantly reduce aspect sensitivity. However, although these two features were proven useful under some situations, it is well recognized that the resonance feature-based identification alone will not be sufficient for most targets of interest due to the low resonance energy problem [6].

Different from the resonance features, scattering center feature, contained in the early-time response of target scattering signature, is aspect-dependent and can yield insight into geometrical and physical characters of the target. Extraction method of scattering centers was investigated by several groups, such as Kim et al. [7] and Choi et al. [8]. However, scattering centers are not robust to noise and aspect change, which restricts the scattering center-based target identification performance. For this reason, distinct frequency dependencies of scattering centers were considered [9, 10]. Although the sensitivities to noise and aspect variations can be reduced to some extent by theirs investigations, these approaches suffer from the particular models. Considering the deficiencies, feature extraction without the model restriction has received recently more intensive attention in the radar target identification community [11–18]. The one-dimensional scattering signature [11, 12] and its all kinds of transformations [13–18] were used to identify target. However, although the particular model restriction is avoided, the extracted feature is still highly aspect-dependent. Thus, for target identification, the scattering signatures of each target at many aspect angles have to be stored, which amounts to a significant storage retrieval problem.

In fact, except the electromagnetic features mentioned above, the target scattering signature holds another property — sparse property. Usually, the information underlying in the received high-dimension discrete scattering signature disperses only over some low-dimension space, not over whole high-dimension data space, and the scattering signature can be represented with a few basis elements in the particular transform domain. For example, the late-time and early-

time components of scattering signals can be accurately reconstructed by several main poles and stable scattering centers respectively, and the scattering information over a wide-angle sector can be represented by the scattering signatures from several observation angles. Since traditional signal analysis tools, such as Fourier or wavelet transform [19, 20], always try to represent the signal with same-property basis elements and cant be necessary to exploit the sparse diversity of the scattering signatures from different targets. Therefore, in order to extract the sparse diversity for target identification, the special signal representation method must be adopted. Actually, the signal representation can be classified into two categories according to the atom number of the dictionary and the dimension of the signal vector, i.e. complete, overcomplete and undercomplete dictionary cases. In recent years, there has been great deal of interest in obtaining sparse representation of the signal with complete and overcomplete dictionaries [20, 21]. Although a complete or overcomplete dictionary is used nominally, only several atoms with the strongest relative to the signal components are usually used to represent the signal. That is to say, a undercomplete dictionary is used in essence. Actually, a undercomplete dictionary can provide signals in signal space with two quite distinct representations, sparseness and no sparseness, which implies that the undercomplete dictionary is with the classification ability to signals in signal space. The capability of the undercomplete dictionary will be theoretically explained in the paper. Therefore, compared with the complete and overcomplete dictionary, the undercomplete dictionary is more suitable to the feature extraction and classification for the sparse property exploitation of the scattering signature. Inspired by the idea, in this paper we focus our attention to the feature extraction for the exploitation of the sparse property with the undercomplete dictionary. For this purpose, we define firstly the signal representation with the undercomplete atom dictionary, and explain theoretically the classification capability of the undercomplete dictionary to signal space and give two important properties of the undercomplete dictionary. Then, we derive the residual energy bound of signal decomposition by matching pursuits (MP) with a undercomplete dictionary, and present the solution procedure of the undercomplete dictionary for signal classification. Furthermore, as an application, a feature vector named as atom dictionary feature and its construction algorithm are proposed for radar target identification.

2. CLASSIFICATION WITH UNDERCOMPLETE DICTIONARY

2.1. Definition and Property of Undercomplete Dictionary

The signal sparse representation with a complete or overcomplete dictionary has been an active research area for numerous pattern classification problems. However, to the author's knowledge, very little theoretical work has been put forward on a undercomplete dictionary for the signal representation and classification. In this subsection, we focus our attention on the signal representation and classification with the undercomplete dictionary. In the following, the definition of the signal representation with a undercomplete dictionary is firstly given.

Definition 1: Let $\mathbb{R}^M$ be a M dimension signal space, we define a undercomplete dictionary $\mathbf{\Phi}$ as a family including N atoms in $\mathbb{R}^M (N < M)$ and a undercomplete dictionary-based representation of signal $\mathbf{s} \in \mathbb{R}^M$ as

$$\mathbf{s} = \mathbf{\Phi}\boldsymbol{\alpha} = \sum_{i=1}^{N} \alpha_i \boldsymbol{\varphi}_i \tag{1}$$

where the coefficient vector $\boldsymbol{\alpha} \in \mathbb{R}^N$. If $\|\boldsymbol{\alpha}\|_0 < N$ where $\|\boldsymbol{\alpha}\|_0 = \text{Card}\{j : |\alpha_j| \neq 0\}$ and $\text{Card}(\cdot)$ denote the cardinal number of set, we say the representation is sparse. Furthermore, if $\tilde{\boldsymbol{\alpha}} = \arg\min \|\boldsymbol{\alpha}\|_0$, the expansion supported by the coefficient vector $\tilde{\boldsymbol{\alpha}}$ is the sparsest representation of the signal $\mathbf{s}$.

According to the above definition, if let $Span\mathbf{\Phi}$ be the closed linear span of atoms in $\mathbf{\Phi}$, the space $Span\mathbf{\Phi}$ must be a subspace of the signal space$\mathbb{R}^M$, which can be written as

$$Span\mathbf{\Phi} = \{\alpha_1\boldsymbol{\varphi}_1 + \alpha_2\boldsymbol{\varphi_2} \cdots + \alpha_N\boldsymbol{\varphi}_N | \alpha_n \in \Bbbk, \boldsymbol{\varphi}_n \in \mathbf{\Phi}\} \subset \mathbb{R}^M \tag{2}$$

where $\Bbbk$ denotes complex set.

Obviously, $Span\mathbf{\Phi}$ is a complete subspace in signal space$\mathbb{R}^M$, which means that any $\mathbf{s} \in Span\mathbf{\Phi} \subset \mathbb{R}^M$ could be sparsely represented by the dictionary $\mathbf{\Phi}$; however, any $\mathbf{s} \in \overline{Span\mathbf{\Phi}}$ couldn't be done. Consequently, we can obtain two properties about the undercomplete dictionary.

Property 1: Let $\mathbf{\Phi} = [\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \cdots, \boldsymbol{\varphi}_N] \in \mathbb{R}^{M\times N}$ be a undercomplete dictionary. If $\mathbf{G} = \{\boldsymbol{\varphi}_{\gamma 1}, \boldsymbol{\varphi}_{\gamma 2}, \cdots \boldsymbol{\varphi}_{\gamma m}\}$ is the subspace composed by any m atoms in $\mathbf{\Phi}$, then

$$Span\mathbf{G} \subseteq Span\mathbf{\Phi} \tag{3}$$

Property 2: Let $\Phi = [\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \cdots, \boldsymbol{\varphi}_N] \in \mathbb{R}^{M\times N}$ be a undercomplete dictionary, for any $\mathbf{s} \in Span\boldsymbol{\Phi} \subset \mathbb{R}^M$ there exists the desired precision ε such that

$$\|\mathbf{e}\|^2 < \varepsilon, \quad \text{s.t. } \boldsymbol{\Phi}\boldsymbol{\alpha} + \mathbf{e} = \mathbf{s} \tag{4}$$

however, if $\mathbf{s} \in \overline{Span\boldsymbol{\Phi}}$, Equation (4) can't be held.

Clearly, Property 1 claims the relationship between subspaces supported by the undercomplete dictionary with different dictionary size. According to the property, it is concluded that with the increasing of the atom number the signal subspace represented sparsely by the undercomplete dictionary will be enlarged. Property 2 shows the classification ability of the undercomplete dictionary to signal space. From the property, we can conclude that the undercomplete dictionary divides the signal space $\mathbb{R}^M$ into two subspaces, i.e., $Span\boldsymbol{\Phi}$ and $\overline{Span\boldsymbol{\Phi}}$. For these two subspaces, different representations, such as sparseness and no sparseness, are presented by the undercomplete dictionary. And furthermore, the difference in the representation can be illuminated by the residue energy $\|\mathbf{e}\|^2$. Obviously, the signal representation with a complete or overcomplete dictionary discussed by Mallat and Zhang in [21] is a particular case of the representation with a undercomplete dictionary.

2.2. Matching Pursuits with Undercomplete Dictionary

The MP algorithm proposed by Mallat and Zhang [21] is a nonlinear iteration procedure for the linear decomposition. In general, an overcomplete dictionary isn't always necessary to the decomposition procedure. The algorithm is also suitable to the linear representation of signals with a undercomplete dictionary, but with some different properties.

According to the definition in [21], let $\boldsymbol{\Phi} = (\varphi_r(t))_{r\in\Gamma}$ denote a dictionary, where $r$denotes a generalized index from the set Γ. The inner product between the signal $s(t)$ and atom $\varphi_r(t)$ is defined as

$$\langle s, \varphi_r \rangle = \int_{-\infty}^{+\infty} s(t)\varphi_r^*(t)dt \tag{5}$$

where superscript $*$ denotes complex conjugate. Assuming the dictionary $\boldsymbol{\Phi}$ includes Q atoms and any atom $\varphi_r(t)$ (i.e., for each $r \in \Gamma_Q$) satisfies $\|\varphi_r\|^2 = \langle \varphi_r, \varphi_r \rangle = 1$, the MP algorithm involves the following iterative procedure.

For any signal $s(t)$, we project it onto each element $\varphi_r(t)$ of the dictionary $\mathbf{\Phi}$. $\varphi_{r0}(t)$ is selected when

$$|\langle s, \varphi_{r0}\rangle| \geq |\langle s, \varphi_r\rangle|, \quad \forall r \in \Gamma_Q \tag{6}$$

and obtain the decomposition

$$s(t) = \langle s, \varphi_{r0}\rangle \varphi_{r0} + R^1 s \tag{7}$$

where $R^1 s$ denotes the residual after the first decomposition. Then the atom $\varphi_{r0}(t)$ is wiped off from the atom dictionary, and $R^1 s$ is respectively projected onto the remaining $Q-1$ elements of the dictionary. Repeating the projection process, after Q iterative, $s(t)$ can be decomposed into

$$s(t) = \sum_{n=0}^{Q-1} \langle R^n s, \varphi_{rn}\rangle \varphi_{rn} + R^Q s \tag{8}$$

where $R^0 s = s(t)$, $|\langle R^n s, \varphi_{rn}\rangle| \geq |\langle R^n s, \varphi_r\rangle|$, $\forall r \in \Gamma_{Q-n}$.

Mallat has proven that if $\mathbf{\Phi}$ is a complete or overcomplete dictionary of the signal space

$$s(t) = \lim_{Q\to\infty} \sum_{n=0}^{Q-1} \langle R^n s, \varphi_{rn}\rangle \varphi_{rn} \tag{9}$$

If one utilizes a complete or overcomplete dictionary, the MP algorithm can be used to represent any signal in the signal space with any desired precision. Thus, the MP with a complete or overcomplete dictionary provides all signals in signal space with a consistent representation. However, as explained above, for signals in signal space two different representations are presented by a undercomplete dictionary in the space.

We define a undercomplete dictionary $\mathbf{\Phi} = [\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \cdots, \boldsymbol{\varphi}_N] \in \mathbb{R}^{M\times N}(N < M)$, where Γ denotes the index set of atoms in $\mathbf{\Phi}$. For any signal $\mathbf{s} \in \Omega \subset \mathbb{R}^M$, two correlation ratios of signal $\mathbf{s}$ with respect to $\mathbf{\Phi}$ are defined respectively as

$$\lambda_{\mathbf{\Phi}}^{\sup}(\mathbf{s}) = \sup_{r\in\Gamma} \frac{|\langle \mathbf{s}, \varphi_r\rangle|}{\|\mathbf{s}\|} \tag{10}$$

$$\lambda_{\mathbf{\Phi}}^{\inf}(\mathbf{s}) = \inf_{r\in\Gamma} \frac{\left|\left\langle R^{N-1}\mathbf{s}, \varphi_r\right\rangle\right|}{\|R^{N-1}\mathbf{s}\|} \tag{11}$$

and two dictionary distribution ratios are defined respectively as

$$B_{\inf}(\mathbf{\Phi}) = \inf_{\|\mathbf{s}\|=1} \left[\lambda_{\mathbf{\Phi}}^{\inf}(\mathbf{s})\right] \tag{12}$$

$$B_{\sup}(\mathbf{\Phi}) = \sup_{\|\mathbf{s}\|=1} \left[\lambda_{\mathbf{\Phi}}^{\sup}(\mathbf{s})\right] \tag{13}$$

where $B_{\inf}(\mathbf{\Phi})$ and $B_{\sup}(\mathbf{\Phi})$ represents the lower and upper bound of the undercomplete dictionary distribution ratio for subspace Ω respectively.

So, for signal $\mathbf{s} \in \Omega \subset \mathbb{R}^M$, the decay of the residual through m decompositions ($m \leq N$) is satisfied with

$$\|\mathbf{s}\| \left(1 - B_{\sup}(\mathbf{\Phi})^2\right)^{m/2} \leq \|R^m\mathbf{s}\| \leq \|\mathbf{s}\| \left(1 - B_{\inf}(\mathbf{\Phi})^2\right)^{m/2} \tag{14}$$

The proof to (14) is given in the following.

Proof: According to the MP procedure, we obtain the mth decomposition which satisfies

$$\left|\left\langle R^{m-1}\mathbf{s}, \boldsymbol{\varphi}_{r(m-1)}\right\rangle\right| = \sup_{\gamma\in\Gamma} \left|\left\langle R^{m-1}\mathbf{s}, \boldsymbol{\varphi}_{r}\right\rangle\right| = \lambda_{\mathbf{\Phi}}^{\sup}(R^{m-1}\mathbf{s}) \left\|R^{m-1}\mathbf{s}\right\| \tag{15}$$

since

$$\begin{aligned} \|R^m\mathbf{s}\|^2 &= \left\|R^{m-1}\mathbf{s}\right\|^2 - \left|\left\langle R^{m-1}\mathbf{s}, \boldsymbol{\varphi}_{r(m-1)}\right\rangle\right|^2 \\ &= \left\|R^{m-1}\mathbf{s}\right\|^2 - [\lambda_{\mathbf{\Phi}}^{\sup}(R^{m-1}\mathbf{s})]^2 \left\|R^{m-1}\mathbf{s}\right\|^2 \\ &= (1 - [\lambda_{\mathbf{\Phi}}^{\sup}(R^{m-1}\mathbf{s})]^2) \left\|R^{m-1}\mathbf{s}\right\|^2 \end{aligned} \tag{16}$$

and hence, according to the definition of $B_{\sup}(\mathbf{\Phi})$ and $B_{\inf}(\mathbf{\Phi})$, for $N \geq m > 0$ we can derive the relationship

$$\begin{gathered} [1-B_{\sup}(\mathbf{\Phi})^2]^{1/2} \left\|R^{m-1}\mathbf{s}\right\| \leq \|R^m\mathbf{s}\| \leq [1-B_{\inf}(\mathbf{\Phi})^2]^{1/2} \left\|R^{m-1}\mathbf{s}\right\| \\ [1-B_{\sup}(\mathbf{\Phi})^2]^{m/2} \|\mathbf{s}\| \leq \|R^m\mathbf{s}\| \leq [1 - B_{\inf}(\mathbf{\Phi})^2]^{m/2} \|\mathbf{s}\| \end{gathered} \tag{17}$$

According to previous conclusion in this paper, different representations, such as sparseness and no sparseness, are presented by the undercomplete dictionary $\mathbf{\Phi}$ to signal space$\mathbb{R}^M$. Therefore, the norm $\left\|R^N\mathbf{s}\right\|$ of signals in $\overline{Span\mathbf{\Phi}}$ is very different from that of signals in $Span\mathbf{\Phi}$. In general, the former is higher than the latter after N decompositions. The fact is intensely implies that undercomplete dictionary with MP has an inherent ability to classify signals, which has been applied by authors to radar target identification [14] as one of numerous pattern classification problems.

3. SOLUTION OF UNDERCOMPLETE DICTIONARY FOR SIGNAL CLASSIFICATION

As shown above, undercomplete dictionary has an inherent ability to classify signals, and the norm $\left\|R^N\mathbf{s}\right\|$ of signals in $\overline{Span\mathbf{\Phi}}$ is usually higher than that of signals in $Span\mathbf{\Phi}$ after N decompositions. Therefore, for a practical classification problem, the key issue is that undercomplete dictionaries, as features, are established beforehand to characterize the signals belonging to the different categories. According to the classification procedure mentioned above, the solution of the undercomplete dictionary for signal classification can be generalized as: Let $\mathbf{\Phi} = [\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \cdots, \boldsymbol{\varphi}_N] \in \mathbb{R}^{M\times N}$ be a undercomplete dictionary, for a training signal set $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_P] \in \mathbb{R}^{M\times P}$ the undercomplete dictionary can be found by

$$\begin{aligned} &\min_{\Phi\in\mathbb{R}^{M\times N}} \|\mathbf{E}\|^2 \\ &\text{s.t. } \Phi\mathbf{A} + \mathbf{E} = \mathbf{S} \end{aligned} \tag{18}$$

Usually, the optimization in (18) is a NP-hard problem and the global search is very time-consuming. However, it is worth emphasizing that the expansion coefficient matrix $\mathbf{A}$ is entirely dependent on the undercomplete dictionary Φ. Once Φ is known, the coefficient matrix $\mathbf{A}$ can be found by matching pursuits [21]. Hereby, a two-step algorithm, in a way similar text in [22], is presented for the optimization problem in (18), which is generalized as follows:

1) Φ and $\mathbf{S}$ are known. Find a sparse coefficient matrix $\mathbf{A}$;

2) $\mathbf{A}$ and $\mathbf{S}$ are known. Find the best undercomplete dictionary Φ.

Clearly, the coefficient matrix $\mathbf{A}$ isn't necessarily sparsest in a single iteration and the found undercomplete dictionary Φ isn't optimal in the global search. Hence, in order to obtain the optimal undercomplete dictionary Φ, the above two-step algorithm must be repeated until a stop-criterion is satisfied. Furthermore, from the above two-step algorithm it is observed that the first part in the algorithm is a simple decomposition process, which can be easily realized by MP algorithm, but the second part is a complicated high-dimension minimum process yet. The key of the solution for the optimization undercomplete dictionary is to solve the minimum problem in the second part.

Assuming that $\Phi = \{\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \cdots, \boldsymbol{\varphi}_N\} \in \mathbb{R}^{M\times N}$ represents a undercomplete dictionary, we obtain a expression of the training signal

set $\mathbf{S}$, which can be expressed as

$$\tilde{\mathbf{S}} \stackrel{\Delta}{=} \Phi\mathbf{A} \tag{19}$$

The corresponding residual is given

$$\mathbf{E} = \mathbf{S} - \tilde{\mathbf{S}} \tag{20}$$

In order to minimize the residual energy for the given coefficient matrix $\mathbf{A}$, in a way similar to the Method of Optimal Directions in [22], we denote by $\Delta\Phi$ the adjustment of the undercomplete dictionary Φ:

$$\tilde{\Phi} = \Phi + \Delta\Phi \tag{21}$$

and the new residual due to the residual $\mathbf{E}$ is given

$$\tilde{\mathbf{E}} = \mathbf{E} - \Delta\Phi\mathbf{A} \tag{22}$$

So the residual energy after the adjustment is

$$\begin{aligned} \left\|\tilde{\mathbf{E}}\right\|^2 &= \|\mathbf{E} - \Delta\Phi\mathbf{A}\|^2 \\ &= \mathbf{E}^T\mathbf{E} - \mathbf{A}^T\Delta\Phi^T\mathbf{E} - \mathbf{E}^T\Delta\Phi\mathbf{A} + \mathbf{A}^T\Delta\Phi^T\Delta\Phi\mathbf{A} \end{aligned} \tag{23}$$

Obviously, the minimization of the residual energy $\left\|\tilde{\mathbf{E}}\right\|^2$ is equivalent to the maximization of the expression

$$J = \mathbf{A}^T\Delta\Phi^T\mathbf{E} + \mathbf{E}^T\Delta\Phi\mathbf{A} - \mathbf{A}^T\Delta\Phi^T\Delta\Phi\mathbf{A} \tag{24}$$

After the differential operation, under the least mean square (LMS) rule we get

$$\Delta\Phi = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{E} \tag{25}$$

Usually, for a given coefficient matrix $\mathbf{A}$ the adjustment $\Delta\Phi$ of the undercomplete dictionary Φ is only an approximate solution under the LMS rule, and the adjusted undercomplete dictionary $\tilde{\Phi}$ is not guaranteed to be optimal for the given signals $\mathbf{S}$. Therefore, the solution procedure must be repeated.

4. FEATURE EXTRACTION BASED ON UNDERCOMPLETE DICTIONARY

As an application in pattern recognition problem, in this section we present the feature extraction method with the undercomplete dictionary for the sparse property exploitation of target scattering signature. For completeness of the explanation, the 1-D range profile feature and classification criterion are firstly analyzed.

4.1. Range Profile Feature and Matching Score Classification Criterion

The 1-D range profile is the scattering distribution of a target along the radial distance and provides information about the position and scattering strength of the target's scattering centers at that aspect. The range profile as feature vector for radar target identification has been an active research area and many efforts have been attempted in the last decade [11, 12, 15, 16]. The matching score classification criterion (MSCC) [11] is a fundamental criterion for the range profile-based target recognition, with which a class of test data is determined according to the maximum correlation coefficient between the received range profile and all templates. Assuming that the waveform $f(x)$ is a measured range profile of an unknown target, $g_{ij}(x)$ is the restored range profile feature template at aspect angle j of ith target, and $x \in [a, b]$, the matching score can be generalized as

$$C(f, g_{ij}) = \frac{\left| \int_a^b f(x) g_{ij}^*(x) dx \right|}{\left| \int_a^b |f(x)|^2 dx \int_a^b |g_{ij}(x)|^2 dx \right|^{1/2}} \tag{26}$$

If the range profile $f(x)$ and range profile template $g_{ij}(x)$ are normalized to unit energy, a linear relation between the signature $f(x)$ and template $g_{ij}(x)$ is then given by

$$f(x) = B_{ij} g_{ij}(x) + e_{ij} \tag{27}$$

where e_{ij} denotes the decomposition residual,$B_{ij} = \int_a^b f(x) g_{ij}^*(x) dx = \langle f(x), g_{ij}(x) \rangle$ is the projection of $f(x)$ onto $g_{ij}(x)$. Obviously, the linear relation in (27) can be thought as a single-stage decomposition expression of $f(x)$ on the undercomplete dictionary with the template $g_{ij}(x)$ as atom, which means that MSCC might be thought as an extreme case of the classification procedure of the undercomplete dictionary with multiple atoms.

4.2. Feature Extraction Based on Undercomplete Dictionary with Multiple Atoms

Actually, the classification based on a undercomplete dictionary with multiple atoms is a MSCC-generalized procedure. The atoms in the undercomplete dictionary can be the different feature vectors, such as CNRFs, scattering centers, 1-D range profiles and its all kinds of transformation, or other feature vectors. For the purpose to demonstrate the effectiveness of the feature extraction based on

undercomplete dictionary, in this paper the 1-D range profile as an example is used to the feature extraction based on undercomplete dictionary. The extracted feature is named as the atom dictionary feature.

Assuming that matrix $\mathbf{Y}_m = [\mathbf{y}_{\theta_1}^m, \mathbf{y}_{\theta_2}^m, \cdots, \mathbf{y}_{\theta_L}^m]$ is made up of range profile vectors of the mth target from L different aspect sectors, and $\mathbf{y}_{\theta_k}^m = [\mathbf{y}_{\theta_k}^m(0), \mathbf{y}_{\theta_k}^m(1), \cdots, \mathbf{y}_{\theta_k}^m(N-1)]^T$ is the scattered vector at kth aspect angle. Considering that a computationally expensive search of all possible atom vectors must be performed completely for an optimal dictionary selection, in order to simplify the solution procedure the atom dictionary feature Φ_m of mth target is initialized by Q (less than L) range profile vectors using an equally spaced selection from $\mathbf{Y}_m$, and then p range profile vectors selected with equal angle interval from the remaining vectors are used to construct the training data matrix $\mathbf{S}_m$. In order to avoid the energy diversity of range profile vector due to different range and aspect, all training vectors are normalized to unit energy. Then, the atom dictionary feature Φ_m to characterize the mth target can be easily constructed after several iterations.

Furthermore, if the aspect angle sector supported by the atom dictionary feature Φ_m is equal to $[-\Theta/2, \Theta/2]$, any normalized range profile $\mathbf{y}_\theta$ ($\theta \in [-\Theta/2, \Theta/2]$) can be represented as

$$\mathbf{y}_\theta = \alpha_1^\theta \boldsymbol{\varphi}_1 + \cdots + \alpha_q^\theta \boldsymbol{\varphi}_q \cdots + \alpha_Q^\theta \boldsymbol{\varphi}_Q + \mathbf{e}_\theta^Q \tag{28}$$

and

$$\left\| \mathbf{e}_\theta^Q \right\|^2 \leq e_{\max}^2 \tag{29}$$

where α_q^θ is the projection coefficient of the range profile $\mathbf{y}_\theta$ on the atom $\boldsymbol{\phi}_q$, $\mathbf{e}_\theta^Q$ is the residual vector after the Q decompositions, $e_{\max}^2$ is the residual energy threshold value of all range profiles over $[-\Theta/2, \Theta/2]$ after the Q decompositions.

Obviously, the atom dictionary feature proposed in this subsection is essentially a undercomplete dictionary extracted adaptively cross the aspect angle, which could represent effectively scattering information over the given aspect sector. According to Property 1, we can conclude that the smallest signal space is supported by the undercomplete dictionary with single atom. Therefore, for MSCC-based target identification, a great number of templates from all target aspects must be established, which is impractical because of its huge computation burden and storage requirement. However, for the proposed atom dictionary feature, the represented signal space can be enlarged by multiple atoms, which implies that the wider target aspect sector

can be supported by the undercomplete dictionary with multiple atoms. Thus, if there exists an atom dictionary feature with multiple atoms which characterizes any range profile on aspect angle sector $[-\Theta/2, \Theta/2]$, the size of the reference feature database could be reduced to some degree.

5. SIMULATION RESULTS

To demonstrate the effectiveness of the feature extraction scheme based on the undercomplete dictionary, several simulations are conducted using simulated data, calculated scattering signatures of three thin wires through a time-domain electric field integral equation, as well as measured scattering signatures in a time-domain UWB chamber.

5.1. Simulations Using Calculated Scattering Signatures

In this experiment, three thin wire targets with different sizes are used. The thin wires have the same length-width-ratio of 1/200 but with different lengths (named as Target 1, 2, and 3 respectively). The far-field scattering signals of three targets at the incident aspect angles $\theta = 2.5^\circ, 5^\circ, \cdots, 180^\circ$ are calculated using time-domain electric field integral equation with 25 ps sampling interval. The truncated far-field scattering signatures with 128 valid sampling points are obtained in 72 aspect angles to build the raw database.

In order to test the effectiveness of the atom dictionary feature, atom dictionary feature $\mathbf{\Phi}_2$ of target 2 is initialized by its scattering signatures at angle index 20, 28 and 36, and scattering signatures at angle index 10 and 50 are used to build the training data matrix $\mathbf{S}_2$. Once the initialization is finished, $\mathbf{\Phi}_2$ is trained with 50 cycles by the corresponding data matrix $\mathbf{S}_2$ according to the algorithm listed in Table 1. Then, all of scattering signatures of three targets are decomposed on the extracted feature $\mathbf{\Phi}_2$ and the atom dictionary feature with the single scattering signature as atom. The decomposition residuals are respectively shown in Fig. 1 and Fig. 2 (indices 1–72, 73–144, 145–217, corresponding to 72 aspect angles of Target 1, Target 2, and Target 3 respectively). As shown in Fig. 1 and Fig. 2, compared with the atom dictionary with single scattering signature as atom, the extracted atom dictionary feature can support obviously the wider aspect sector. The atom dictionaries with single atom could support the scattering signatures only in adjacency aspect angle of the atom. However, the target scattering signatures over the aspect sector from 2.5° to 180° could be represented by the extracted atom dictionary feature with three atoms, which implies

a very important result that the template storage space for target identification could be reduced significantly by the proposed dictionary feature.

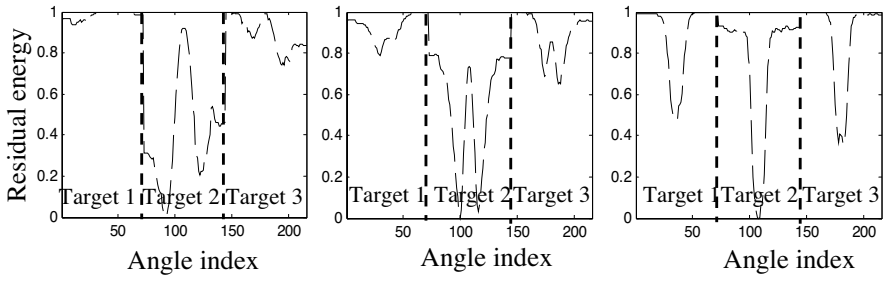

Figure 1. The decomposition residual by atom dictionary feature with the single scattering signature of Target 2 as atom (a) scattering signature at index 20 as atom, (b) scattering signature at index 28 as atom, (c) scattering signature at index 36 as atom.

Furthermore, in order to demonstrate the effectiveness of atom dictionaries to characterize targets further, atom dictionary features of another two targets are extracted according to the previous procedure. Test scattering signature is selected randomly from scattering signatures of Target 2 at 72 different aspects. Then the white Gaussian noise is added to the selected signal to product a noisy signal. The noisy signal is decomposed on the extracted atom dictionary features of three targets with the MP algorithm. The averaged energy of the decomposition residuals of 1000 samples are shown for different SNR in Fig. 3. In this figure, the averaged decomposition residual on the atom dictionary feature of Target 2 is obviously smaller than that corresponding to other two targets even at a very low SNR, such as -10 dB. From simulation results, we can observe that accurate target identification could be carried out using the atom dictionary feature in an almost aspect independent manner without any prior parameterization model hypothesis, and the aspect dependence of template for wide-angle target identification could be reduced effectively by using the atom dictionary feature.

5.2. Simulations Using Measured Scattering Signatures

In this experiment, the measured scattering signatures in a time domain UWB chamber system are used. The chamber used in this

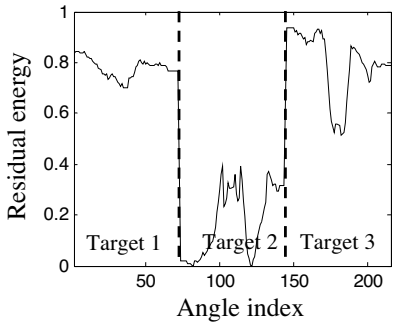

Figure 2. The decomposition residual by extracted atom dictionary feature of Target 2 with multiple atoms.

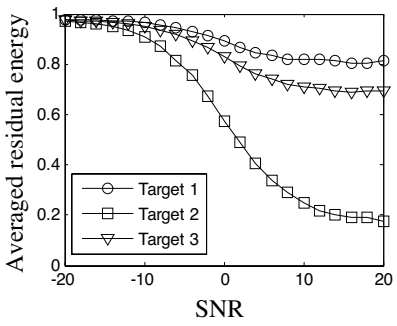

Figure 3. Averaged residual energy of sc(atte)ring signatures from Target 2 on atom dictionaries corresponding to three targets.

paper is 9.7-m long by 4.6-m wide by 3.3-m high and is lined with 46-cm pyramid absorbers. The generator provides a periodic sequence of ultrashort pulses of negative polarity with amplitude about 25 V and duration about 30 ps (for 0.5 levels) to a UWB coaxial horn antenna of the ridge type. The operating band of the antenna is $1.0 \sim 10.0$ GHz at the voltage standing wave ratio < 2.0. The scattering signature of a candidate target is received by an identical horn. A waveform processing oscilloscope with $0 \sim 50$ GHz operating band is used to acquire the received signature.

Targets used in simulations are shown in Fig. 4. Target 1 is a simple aluminum missile model with a 0.142 m length fuselage, Target 2 is an aluminum plan model with a same length body, and Target 3 is a plastic model with same shape as Target 2. The relative measurement aspect angle in the azimuth plane of each target is varied from 0° to 180° with a 6.2° increment. Four dictionary features for each target are extracted according to the initialization configuration in Fig. 5, where the scattering signatures signed by the "solid" line are used to initialize atom dictionary features, and the signatures by the "dashed" line are applied to the training data. As a result, four undercomplete dictionaries with four atoms, as features, are used to characterize each of three targets. Subsequently, all of measured scattering signatures of three targets are decomposed on four atom dictionary features of Target 1, and the decomposition residual energy $\left\|R^4\mathbf{s}\right\|^2$ is shown in Fig. 6. As seen in Fig. 6, atom dictionary features of Target 1 provide scattering signatures

of three targets with two distinctly different representations. After completing the decompositions, residual energy of scattering signatures from the region supported by every of atom dictionary features is distinctly lower than that of scattering signatures away from the supported region. Although for three target models the supported region of the atom dictionary feature cannot be enlarged as same as simulations using calculated scattering signatures due to the aspect-sensitive nature of measured scattering signatures, the discriminable ability to the different targets is enhanced by the fusion procedure of decisions on multiple atoms, which is seen clearly in Fig. 7. Fig. 7 presents the results using the residual energy to identify three targets, where "ADF" denotes the identification results of the extracted atom dictionary feature with multiple atoms, and "MSCC" denotes the results of the single scattering signature as atom with the matching score classification criterion (the scattering signature vectors over the measured angular region of 0°–180° with a 12.4° angular increment are used as feature templates). In this experiment, the candidate target is randomly selected from the target library with an equal probability of being present. Once the target has been selected, a test scattering signature is randomly selected from 28 measured signatures of the selected target with an equal probability and contaminated by additive white Gaussian noise with a specified level of signal to noise ratio (SNR). The identification results are conducted by 1000 identification experiments using 1000 independent noisy test sequences. As shown in Fig. 7, the "ADF" identification method outperforms markedly the "MSCC" identification method from −10 dB to 30 dB in our simulations, which demonstrates the effectiveness of the feature extraction scheme based on the undercomplete dictionary further.

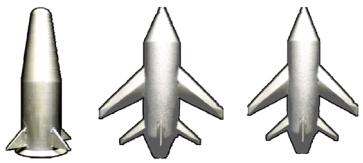

Figure 4. Target models used in simulations.

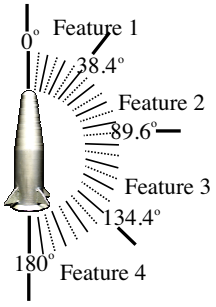

Figure 5. Initialization configuration for extracting atom dictionary features to characterize the target scattering.

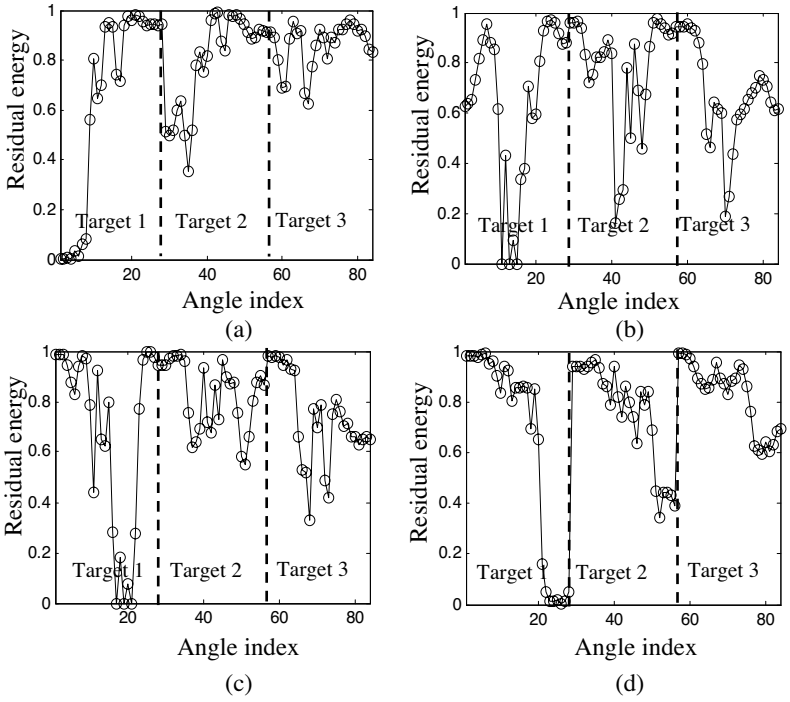

Figure 6. Residual energy after all scattering signatures of three targets are completely decomposed on four atom dictionary features of Target 1 respectively. (a) Feature 1, (b) Feature 2, (c) Feature 3, (d) Feature 4.

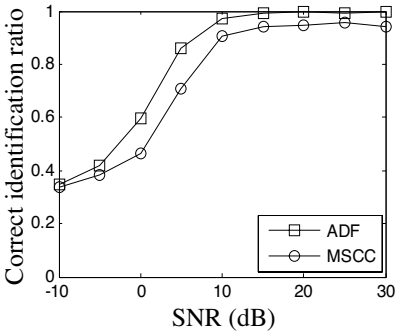

Figure 7. Correct identification results of the "ADF" and "MSCC" methods with a varied SNR.

6. CONCLUSION

In this paper, we carry out an in-depth study on the undercomplete dictionary for signal classification. We explain the classification ability of the undercomplete dictionary and establish a feature extraction scheme based on the undercomplete dictionary. Furthermore, as an application, we present a novel feature named as the atom dictionary feature for radar target identification, which is applied in [23]. Numerical simulation results show that the proposed atom dictionary feature can exploit sparse property in the scattering signature for target identification. Compared with the MSCC, the proposed feature extraction scheme can give a more promising identification due to its inherent decision-fusion procedure. In addition, it is worth paying attention that for some applications the complicated scattering behaveriors of radar target over a wide-angle sector could be represented by the proposed atom dictionary feature and the aspect dependence of the feature could be reduced effectively.

Moreover, it should be noted that the atom dictionary feature is extracted from the whole scattering signature without any prior parameterization model hypothesis. Therefore, as compared with the CNRF and scattering center features, the feature is with less limitation to the target identification applications. In addition, in this paper the time-domain scattering signatures are used to construct the atom dictionary feature. However, the frequency spectrum of the scattering signature can be also used to construct the dictionary, which can avoid the problems in HRRP-based target recognition schemes, such as the significant performance degeneration due to small time shift between the measured and stored HRRP feature templates. Future work needs to emphasize particularly on the problems faced by the atom dictionary feature in practical applications, such as the best size of the atom dictionary feature and design of the identification system.

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