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THE RELATION BETWEEN LUMPED-ELEMENT CIRCUIT MODELS FOR CYLINDRICAL DIELECTRIC RESONATOR AND ANTENNA PARAMETERS USING MBPE

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Abstract—Finite-Difference Time Domain (FDTD) is used to calculate the input impedance of the cylindrical dielectric resonator (DRA) antenna with different dimensions. A lumped- element circuit model for the input impedance calculation is proposed. The genetic algorithm is used to calculate the elements of the equivalent circuit. The Model-Based Parameter Estimation (MBPE) technique is used to find the variation of each element in the equivalent circuit with varying the physical dimensions of the antenna. The use of this method reduces the time required for calculating the input impedance of the cylindrical DRA in each variation of the antenna dimension.

1. INTRODUCTION

Dielectric resonator antennas (DRAs) have been considered for many applications at millimeter-wave frequency, since the systematic experimental investigations on DRAs were carried out by Long et al. [1]. DRAs share many of the advantages of microstrip antennas, including small size, light weight, ease of coupling to many types of transmission lines, and ease of integration with other active or passive microwave components. In addition, DRAs exhibit a relatively large bandwidth (≈10%), while the patch antenna has a typical bandwidth of only 1–3%. This requires the characterization of the dielectric resonator for its feed-point impedance at various frequencies in the band of operation.

The dielectric resonator antenna can be analyzed using various methods such as finite-difference time-domain (FDTD), finite element method, and method of moments (MoM) [2]. Though these methods give accurate results for feed-point impedance, they are complex and require large computational time. Such cumbersome computations can be avoided by drawing the equivalent circuit of the dielectric resonator and calculating its passive elements as discussed in [3–9]. Miller [10–12] describes the Model-Based Parameter Estimation (MBPE) technique and gives many examples to which it may be applied. The modeling, sampling, and solution of MBPE problem for both spatial and frequency domain problems, including the use of matrix inversion techniques to solve for the interpolation coefficients, are described in [13].

In this paper, input impedance of the cylindrical DRA is carried out on the base of FDTD method. The input impedance of dielectric resonator can be represented by a lumped-element equivalent circuit that has L, C, and R elements whose values are only related to the physical dimensions of the antenna such as height of the cylindrical DRA antenna, radius, feed probe length and dielectric permittivity of the dielectric resonator. The genetic algorithm is used to optimize the values of the elements.

A hybrid method for combining a rational function fitting models with a polynomial of physical dimensions of the antenna fitting model for each element of the equivalent circuit is introduced in this chapter. This hybrid method allows calculating the values of the elements of the equivalent circuit according to physical dimensions of the antenna then the input impedance as a function of frequency can be calculated at these dimensions of the antenna.

The paper is organized as follows; Section 2 reports the calculations of the equivalent circuit elements of the cylindrical dielectric resonator antenna using the genetic algorithm. The model based parameter estimation technique is used to construct the variations of each element of the equivalent circuit with the physical dimensions of the antenna. Section 3 illustrates and discusses some numerical results for input impedance of the cylindrical dielectric resonator antenna. Finally, conclusions are collected in Section 4.

2. THEORY

Consider cylindrical DRA located above an infinite size ground plane as shown in Fig. 1. The antenna has height "H", radius "a" and relative dielectric constant "ε_r". A coaxial probe of length "h" and radius "r" at a distance "d_f" from the center of the antenna excites the radiating element. The FDTD method [14] is used to determine the input impedance of the cylindrical DRA. Stair-casing is used in

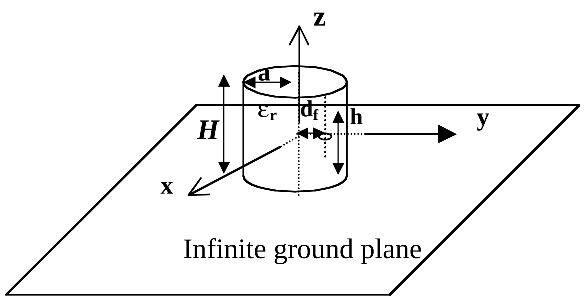

Figure 1. Geometry of the cylindrical dielectric resonator antenna $H = 3$ mm, mm $a = 4.2$ mm, $d_f = 3.5$ mm, $\varepsilon_r = 12$, and $h = 2.4$ mm.

modeling the curved structure of the antenna surface and Gaussian pulse with "T" $= 6.8$ ps is used as the excitation signal.

The main steps for evaluating the equivalent circuit of cylindrical DRA are as follows;

1. Obtaining the input impedance data from FDTD for cylindrical DRA with fixed dimensions and varying the cylinder height as well as the relative permittivity of the cylinder.
2. Obtaining the equivalent lumped- element circuit model at each variation step by using GA with a fixed shape of equivalent circuit.
3. Using MBPE to obtain the relation between lumped-element circuit model for cylindrical dielectric resonator and Antenna Parameters.
4. Using this technique the values of the equivalent circuit model are obtained at a wide range of dimensions.

The lumped-element circuit model of the cylindrical DRA is shown in Fig. 2. The FDTD is used to determine the input impedance of cylindrical DRA for different heights "H", relative dielectric constant

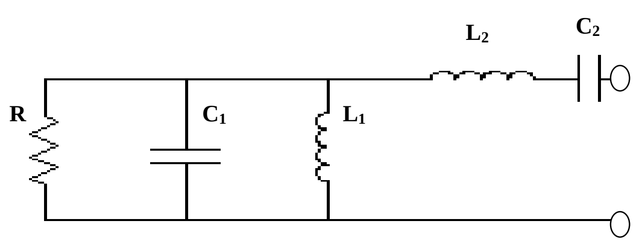

Figure 2. Lumped-element equivalent circuit for cylindrical DRA.

"ε_r" and keeping the other dimensions of the antenna with no change as in Fig. 1. The values for equivalent circuit elements are calculated by the use of a genetic algorithm for best impedance fidelity. This problem requires optimizing five variables, R, L_1, C_1, L_2, and C_2 for each length H and relative dielectric constant ε_r.

In the genetic algorithm (GA) each component parameter is represented by 16 bits binary coded gene. Lower and upper parameter limits for each component value are also set according to the resonance frequency and the Q-factor of FDTD calculation. The component parameter gene stung together end-to-end to form a single chromosome representing the antenna equivalent circuit model. In each optimization cycle or generation, the GA considers 100 different chromosomes (population size). Circuit analysis defines the performance of each candidate's solution. One form of the fitting model which is commonly employed in MBPE is represented by the rational function as

$$\begin{aligned} F(H,\varepsilon_r) &= \frac{N(H,\varepsilon_r)}{D(H,\varepsilon_r)} \\ &= \frac{N_0(H)+N_1(H)\varepsilon_r+N_2(H)\varepsilon_r^2+\cdots+N_n(H)\varepsilon_r^n}{D_0(H)+D_1(H)\varepsilon_r+D_2(H)\varepsilon_r^2+\cdots+D_{d-1}(H)\varepsilon_r^{d-1}+\varepsilon_r^d} \end{aligned} \quad (1)$$

where

$$\begin{aligned} N_0(H) &= N_0^0+N_0^1H+N_0^2H^2+\cdots+N_0^kH^k \\ N_1(H) &= N_1^0+N_1^1H+N_1^2H^2+\cdots+N_1^kH^k \\ &\vdots \\ N_n(H) &= N_n^0+N_n^1H+N_n^2H^2+\cdots+N_n^kH^k \end{aligned} \quad (2)$$

$$\begin{aligned} D_0(H) &= D_0^0+D_0^1H+D_0^2H^2+\cdots+D_0^kH^k \\ D_1(H) &= D_1^0+D_1^1H+D_1^2H^2+\cdots+D_1^kH^k \\ &\vdots \\ D_{d-1}(H) &= D_n^0+D_{d-1}^1H+D_{d-1}^2H^2+\cdots+D_{d-1}^kH^k \end{aligned} \quad (3)$$

where K represent the polynomial order. By sampling $F(H,\varepsilon_r)$ at N_H

and N_r points a matrix of the form $AX = B$ can be construed as

$$X = \begin{bmatrix} \overline{N}_0 \\ \overline{N}_1 \\ \cdots \\ \cdots \\ \overline{N}_n \\ \overline{D}_0 \\ \overline{D}_1 \\ \cdots \\ \cdots \\ \overline{D}_{d-1} \end{bmatrix} \tag{4}$$

$$B = \begin{bmatrix} F(H_1, \varepsilon_{r1})\varepsilon_{r1}^d \\ F(H_1, \varepsilon_{r2})\varepsilon_{r2}^d \\ \cdots \\ F(H_1, \varepsilon_{rNr})\varepsilon_{rNr}^d \\ F(H_2, \varepsilon_{r1})\varepsilon_{r1}^d \\ F(H_2, \varepsilon_{r2})\varepsilon_{r2}^d \\ \cdots \\ F(H_2, \varepsilon_{rNr})\varepsilon_{rNr}^d \\ \cdots \\ F(H_{NH}, \varepsilon_{r1})\varepsilon_{r1}^d \\ F(H_{NH}, \varepsilon_{r2})\varepsilon_{r2}^d \\ \cdots \\ F(H_{NH}, \varepsilon_{rNr})\varepsilon_{rNr}^d \end{bmatrix} \tag{5}$$

where

$$\overline{N}_i = \left[N_i^0 \ N_i^1 \ N_i^2 \ldots\ldots N_i^k\right]^T \qquad i = 0, 1, \ldots, n$$

$$\overline{D}_j = \left[D_j^0 \ D_j^1 \ D_j^2 \ldots\ldots D_j^k\right]^T \qquad j = 0, 1, \ldots, d-1$$

Solving this matrix equation yields the set of numerator and denominator coefficients required by equation (1). In this paper $F(H, \varepsilon_r)$ is represented as the values of $R, L_1, C_1, L_2,$ and C_2, for different values of "H", and "ε_r". The unknown complex "X", is calculated by inverting the matrix A and multiplying it by the column B as

$$X = A^{-1}B \tag{6}$$

The matrix A can be written as

$$A=\left[\begin{array}{ccccccccc}
1 & H_1 & \cdots & H_1^k & \varepsilon_{r1} & H_1\varepsilon_{r1} & \cdots & H_1^k\varepsilon_{r1} & \cdots \\
1 & H_1 & \cdots & H_1^k & \varepsilon_{r2} & H_1\varepsilon_{r2} & \cdots & H_1^k\varepsilon_{r2} & \cdots \\
. & & & & & & . & & \\
. & & & & & & . & & \\
1 & H_1 & \cdots & H_1^k & \varepsilon_{rNr} & H_1\varepsilon_{rNr} & \cdots & H_1^k\varepsilon_{rNr} & \cdots \\
1 & H_2 & \cdots & H_2^k & \varepsilon_{r1} & H_2\varepsilon_{r1} & \cdots & H_2^k\varepsilon_{r1} & \cdots \\
1 & H_2 & \cdots & H_2^k & \varepsilon_{r2} & H_2\varepsilon_{r2} & \cdots & H_2^k\varepsilon_{r2} & \cdots \\
. & & & & & & . & & \\
. & & & & & & . & & \\
1 & H_2 & \cdots & H_2^k & \varepsilon_{rNr} & H_2\varepsilon_{rNr} & \cdots & H_2^k\varepsilon_{rNr} & \cdots \\
. & & & & & & . & & \\
. & & & & & & . & & \\
1 & H_{NH} & \cdots & H_{NH}^k & \varepsilon_{r1} & H_{NH}\varepsilon_{r1} & \cdots & H_{NH}^k\varepsilon_{r1} & \cdots \\
1 & H_{NH} & \cdots & H_{NH}^k & \varepsilon_{r2} & H_{NH}\varepsilon_{r2} & \cdots & H_{NH}^k\varepsilon_{r2} & \cdots \\
. & & & & & & . & & \\
. & & & & & & . & & \\
1 & H_{NH} & \cdots & H_{NH}^k & \varepsilon_{rNr} & H_{NH}\varepsilon_{rNr} & \cdots & H_{NH}^k\varepsilon_{rNr} & \cdots
\end{array}\right.$$

$$\begin{array}{cccccc}
\varepsilon_{r1}^n & H_1\varepsilon_{r1}^n & \cdots & H_1^k\varepsilon_{r1}^n & -F(H_1,\varepsilon_{r1}) & \cdots \\
\varepsilon_{r2}^n & H_1\varepsilon_{r2}^n & \cdots & H_1^k\varepsilon_{r2}^n & -F(H_1,\varepsilon_{r2}) & \cdots \\
 & & . & & & \\
 & & . & & & \\
\varepsilon_{rNr}^n & H_1\varepsilon_{rNr}^n & \cdots & H_1^k\varepsilon_{rNr}^n & -F(H_1,\varepsilon_{rNr}) & \cdots \\
\varepsilon_{r1}^n & H_2\varepsilon_{r1}^n & \cdots & H_2^k\varepsilon_{r1}^n & -F(H_2,\varepsilon_{r1}) & \cdots \\
\varepsilon_{r2}^n & H_2\varepsilon_{r2}^n & \cdots & H_2^k\varepsilon_{r2}^n & -F(H_2,\varepsilon_{r2}) & \cdots \\
 & & . & & & \\
 & & . & & & \\
\varepsilon_{rNr}^n & H_2\varepsilon_{rNr}^n & \cdots & H_2^k\varepsilon_{rNr}^n & -F(H_2,\varepsilon_{rNr}) & \cdots \\
 & & . & & & \\
 & & . & & & \\
\varepsilon_{r1}^n & H_{NH}\varepsilon_{r1}^n & \cdots & H_{NH}^k\varepsilon_{r1}^n & -F(H_{NH},\varepsilon_{r1}) & \cdots \\
\varepsilon_{r2}^n & H_{NH}\varepsilon_{r2}^n & \cdots & H_{NH}^k\varepsilon_{r2}^n & -F(H_{NH},\varepsilon_{r2}) & \cdots \\
 & & . & & & \\
 & & . & & & \\
\varepsilon_{rNr}^n & H_{NH}\varepsilon_{rNr}^n & \cdots & H_{NH}^k\varepsilon_{rNr}^n & -F(H_{NH},\varepsilon_{rNr}) & \cdots
\end{array}$$

$$
\begin{array}{cccc}
-F(H_1,\varepsilon_{r1})H_1^k & -F(H_1,\varepsilon_{r1})\varepsilon_{r1} & \cdots & -F(H_1,\varepsilon_{r1})H_1^k\varepsilon_{r1} \\
-F(H_1,\varepsilon_{r2})H_1^k & -F(H_1,\varepsilon_{r2})\varepsilon_{r2} & \cdots & -F(H_1,\varepsilon_{r2})H_1^k\varepsilon_{r2} \\
\vdots & & \vdots & \\
-F(H_1,\varepsilon_{rNr})H_1^k & -F(H_1,\varepsilon_{rNr})\varepsilon_{rNr} & \cdots & -F(H_1,\varepsilon_{rNr})H_1^k\varepsilon_{rNr} \\
-F(H_2,\varepsilon_{r1})H_2^k & -F(H_2,\varepsilon_{r1})\varepsilon_{r1} & \cdots & -F(H_2,\varepsilon_{r1})H_2^k\varepsilon_{r1} \\
-F(H_2,\varepsilon_{r2})H_2^k & -F(H_2,\varepsilon_{r2})\varepsilon_{r2} & \cdots & -F(H_2,\varepsilon_{r2})H_2^k\varepsilon_{r2} \\
\vdots & & \vdots & \\
-F(H_2,\varepsilon_{rNr})H_2^k & -F(H_2,\varepsilon_{rNr})\varepsilon_{rNr} & \cdots & -F(H_2,\varepsilon_{rNr})H_2^k\varepsilon_{rNr} \\
\vdots & & \vdots & \\
-F(H_{NH},\varepsilon_{r1})H_{NH}^k & -F(H_{NH},\varepsilon_{r1})\varepsilon_{r1} & \cdots & -F(H_{NH},\varepsilon_{r1})H_{NH}^k\varepsilon_{r1} \\
-F(H_{NH},\varepsilon_{r2})H_{NH}^k & -F(H_{NH},\varepsilon_{r2})\varepsilon_{r2} & \cdots & -F(H_{NH},\varepsilon_{r2})H_{NH}^k\varepsilon_{r2} \\
\vdots & & \vdots & \\
-F(H_{NH},\varepsilon_{rNr})H_{NH}^k & -F(H_{NH},\varepsilon_{rNr})\varepsilon_{rNr} & \cdots & -F(H_{NH},\varepsilon_{rNr})H_{NH}^k\varepsilon_{rNr}
\end{array}
$$

$$
\left.
\begin{array}{cccc}
\cdots & -F(H_1,\varepsilon_{r1})\varepsilon_{r1}^{d-1} & \cdots & -F(H_1,\varepsilon_{r1})H_1^k\varepsilon_{r1}^{d-1} \\
\cdots & -F(H_1,\varepsilon_{r2})\varepsilon_{r2}^{d-1} & \cdots & -F(H_1,\varepsilon_{r2})H_1^k\varepsilon_{r2}^{d-1} \\
\vdots & & & \\
\cdots & -F(H_1,\varepsilon_{rNr})\varepsilon_{rNr}^{d-1} & \cdots & -F(H_1,\varepsilon_{rNr})H_1^k\varepsilon_{rNr}^{d-1} \\
\cdots & -F(H_2,\varepsilon_{r1})\varepsilon_{r1}^{d-1} & \cdots & -F(H_2,\varepsilon_{r1})H_2^k\varepsilon_{r1}^{d-1} \\
\cdots & -F(H_2,\varepsilon_{r2})\varepsilon_{r2}^{d-1} & \cdots & -F(H_2,\varepsilon_{r2})H_2^k\varepsilon_{r2}^{d-1} \\
\vdots & & & \\
\cdots & -F(H_2,\varepsilon_{rNr})\varepsilon_{rNr}^{d-1} & \cdots & -F(H_2,\varepsilon_{rNr})H_2^k\varepsilon_{rNr}^{d-1} \\
\vdots & & & \\
\cdots & -F(H_{NH},\varepsilon_{r1})\varepsilon_{r1}^{d-1} & \cdots & -F(H_{NH},\varepsilon_{r1})H_{NH}^k\varepsilon_{r1}^{d-1} \\
\cdots & -F(H_{NH},\varepsilon_{r2})\varepsilon_{r2}^{d-1} & \cdots & -F(H_{NH},\varepsilon_{r2})H_{NH}^k\varepsilon_{r2}^{d-1} \\
\vdots & & & \\
\cdots & -F(H_{NH},\varepsilon_{rNr})\varepsilon_{rNr}^{d-1} & \cdots & -F(H_{NH},\varepsilon_{rNr})H_{NH}^k\varepsilon_{rNr}^{d-1}
\end{array}
\right]
\qquad (7)
$$

3. NUMERICAL RESULTS

To verify the analysis, the FDTD results for the input impedance of cylindrical DRA are compared with the results in [15]. The dimensions of the structure are $H = 3\,\text{mm}$, $a = 4.2\,\text{mm}$, $d = 3.5\,\text{mm}$, $h = 2.4\,\text{mm}$, $\varepsilon_r = 12$ and the probe radius 0.1 mm. The input impedance and the corresponding $|S_{11}|$ of the cylindrical DRA are shown in Fig. 3 and Fig. 4, respectively. Good agreement between the FDTD results and the published ones is obtained.

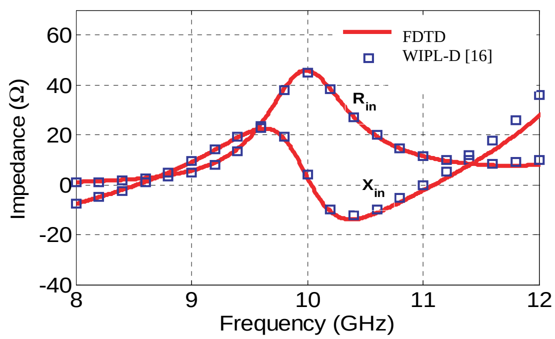

Figure 3. The input impedance of the CDRA vs. frequency.

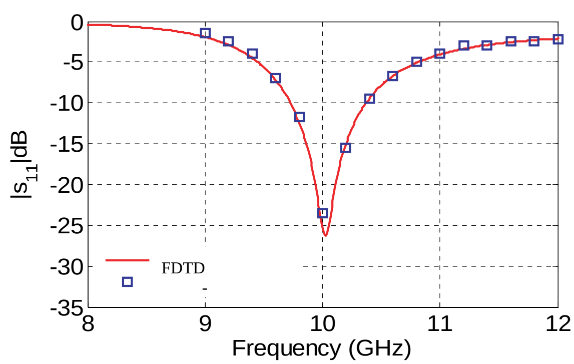

Figure 4. The amplitude of S_{11} of the CDRA vs. frequency.

The input impedance of the cylindrical DRA with different height H and the relative dielectric constant ε_r. is calculated. The lumped-element equivalent circuit for cylindrical DRA is shown in Fig. 2. The Genetic algorithm is used to calculate the values of the elements of the equivalent circuit for each H and ε_r. These values are shown in Tables 1–4.

Table 1. The equivalent circuit elements for $\varepsilon_r = 20$.

Height H(*mm*)	R(*Ohms*)	L_1(*pH*)	C_1(*pF*)	L_2(*pH*)	C_2(*pF*)
3.5	65.171	61.61	6.1847	830	0.54
4.5	64.655	80.0	7.05	600	0.788
5.5	60.88	83.44	7.525	500	0.988
6.5	52.554	84.6	7.94	330	1.188
7.5	44.077	71.2	9.73	130	1.308

Table 2. The equivalent circuit elements for $\varepsilon_r = 16$.

Height H(*mm*)	R(*Ohms*)	L_1(*pH*)	C_1(*pF*)	L_2(*pH*)	C_2(*pF*)
3.5	55.448	66.416	5.539	750	0.48
4.5	54.961	77	5.84	650	0.6
5.5	50.696	80.61	6.325	550	0.68
6.5	44.655	82.479	6.62	460	0.76
7.5	36.637	83.03	6.8	370	0.87

Table 3. The equivalent circuit elements for $\varepsilon_r = 9.8$.

Height H(*mm*)	R(*Ohms*)	L_1(*pH*)	C_1(*pF*)	L_2(*pH*)	C_2(*pF*)
3.5	40.406	76.4	3.12	879.3	0.26435
4.5	39.673	83.0	3.43	859.3	0.29359
5.5	35.864	88	3.63	828.3	0.315
6.5	31.532	90	3.82	780	0.335
7.5	25.482	91.5	3.89	740	0.355

Table 4. The equivalent circuit elements for $\varepsilon_r = 12$.

Height H(*mm*)	R(*Ohms*)	L_1(*pH*)	C_1(*pF*)	L_2(*pH*)	C_2(*pF*)
3.5	46.622	71.0	4.1027	2959	0.096522
4.5	45.672	74.09	4.7	3330	0.0920
5.5	40.87	75.3	5.127	3795	0.08165
6.5	37.026	77.022	5.358	4395.9	0.0716522
7.5	29.32	78.02	5.5528	5019.5	0.058085

The MBPE technique is used to find the relation between the values of each element of the equivalent circuit with varying the height H, and the relative dielectric constant, ε_r. of the antenna. Consider $n = 2$ and $d = 1$ in equations (1) and (2). Each variable is expanded by a polynomial of order $K = 4$ (the total number of unknowns is 20). By varying the height H and relative dielectric constant ε_r, the elements of the equivalent circuit are used to calculate the input impedance of the antenna.

In general the rational function for any element in the equivalent circuit can be written as

$$F(H,\varepsilon_r) = \frac{N(H,\varepsilon_r)}{D(H,\varepsilon_r)} = \frac{N_0(H) + N_1(H)\varepsilon_r + N_2(H)\varepsilon_r^2}{D_0(H) + \varepsilon_r} \tag{8}$$

where

$$\begin{aligned} N_0(H) &= N_0^0 + N_0^1 H + N_0^2 H^2 + N_0^3 H^3 + N_0^4 H^4 \\ N_1(H) &= N_1^0 + N_1^1 H + N_1^2 H^2 + N_1^3 H^3 + N_1^4 H^4 \\ N_2(H) &= N_2^0 + N_2^1 H + N_2^2 H^2 + N_2^3 H^3 + N_2^4 H^4 \\ D_0(H) &= D_0^0 + D_0^1 H + D_0^2 H^2 + D_0^3 H^3 + D_0^4 H^4 \end{aligned} \tag{9}$$

Using equation (6) to calculate the unknown coefficients for each element of the equivalent circuit model,

For the resistor R

$$\begin{aligned} N_0^0 &= 3.5090\times10^4, \quad N_0^1 = -2.8082\times10^4, \quad N_0^2 = 0.8123\times10^4 \\ N_0^3 &= -0.1013\times10^4 \quad N_0^4 = 0.0046\times10^4, \quad N_1^0 = 0.1970\times10^4 \\ N_1^1 &= -0.1560\times10^4, \quad N_1^2 = 0.0445\times10^4, \quad N_1^3 = -0.0055\times10^4 \\ N_1^4 &= -0.1013\times10^4, \quad N_2^0 = 0.0081\times10^4, \quad N_2^1 = -0.0064\times10^4 \\ N_2^2 &= 0.0019\times10^4, \quad N_2^3 = -0.0002\times10^4, \quad N_2^4 = 0 \\ D_0^0 &= 0.1701\times10^4, \quad D_0^1 = -0.1360\times10^4, \quad D_0^2 = 0.0392\times10^4 \\ D_0^3 &= -0.0049\times10^4, \quad D_0^4 = -0.0002\times10^4 \end{aligned}$$

For the inductor L_1

$$\begin{aligned} N_0^0 &= -0.26\times10^{-7}, \quad N_0^1 = 0.1979\times10^{-7}, \quad N_0^2 = -0.0574\times10^{-7} \\ N_0^3 &= 0.0073\times10^{-7}, \quad N_0^4 = -0.0003\times10^{-7}, \quad N_1^0 = 0.338\times10^{-8} \\ N_1^1 &= -0.2523\times10^{-8}, \quad N_1^2 = 0.0715\times10^{-8}, \quad N_1^3 = -0.009\times10^{-8} \\ N_1^4 &= 0.0004\times10^{-8}, \quad N_2^0 = -0.1379\times10^{-9}, \quad N_2^1 = 0.1037\times10^{-9}, \\ N_2^2 &= -0.029\times10^{-9}, \quad N_2^3 = 0.0036\times10^{-9}, \quad N_2^4 = -0.0002\times10^{-9} \end{aligned}$$

$$D_0^0 = -77.1972, \quad D_0^1 = 55.2580, \quad D_0^2 = -16.6960, \quad D_0^3 = 2.1986,$$
$$D_0^4 = -0.1070$$

For the capacitor C_1

$$N_0^0 = -0.234 \times 10^{-7}, \quad N_0^1 = 0.1877 \times 10^{-7}, \quad N_0^2 = -0.0545 \times 10^{-7}$$
$$N_0^3 = 0.0068 \times 10^{-7}, \quad N_0^4 = -0.0003 \times 10^{-7}, \quad N_1^0 = -0.3023 \times 10^{-8}$$
$$N_1^1 = 0.2361 \times 10^{-8}, \quad N_1^2 = -0.0675 \times 10^{-8}, \quad N_1^3 = -0.0084 \times 10^{-8}$$
$$N_1^4 = -0.0004 \times 10^{-8}, \quad N_2^0 = 0.5312 \times 10^{-9}, \quad N_2^1 = -0.4215 \times 10^{-9}$$
$$N_2^2 = 0.1217 \times 10^{-9}, \quad N_2^3 = -0.0152 \times 10^{-9}, \quad N_2^4 = 0.0007 \times 10^{-9}$$
$$D_0^0 = -644.52, \quad D_0^1 = 446.832, \quad D_0^2 = -117.27, \quad D_0^3 = 13.6074$$
$$D_0^4 = -0.5894$$

For the inductor L_2

$$N_0^0 = 0.466 \times 10^{-7}, \quad N_0^1 = -0.4317 \times 10^{-7}, \quad N_0^2 = 0.1222 \times 10^{-7}$$
$$N_0^3 = -0.0158 \times 10^{-7}, \quad N_0^4 = 0.0008 \times 10^{-7}, \quad N_1^0 = -0.2381 \times 10^{-7}$$
$$N_1^1 = 0.1813 \times 10^{-7}, \quad N_1^2 = -0.0489 \times 10^{-7}, \quad N_1^3 = -0.0058 \times 10^{-7}$$
$$N_1^4 = -0.0003 \times 10^{-7}, \quad N_2^0 = 0.1056 \times 10^{-8}, \quad N_2^1 = -0.0770 \times 10^{-8}$$
$$N_2^2 = 0.0205 \times 10^{-8}, \quad N_2^3 = -0.0024 \times 10^{-8}, \quad N_2^4 = 0.0001 \times 10^{-8}$$
$$D_0^0 = -100.3385, \quad D_0^1 = 63.1455, \quad D_0^2 = -16.33, \quad D_0^3 = 1.7817$$
$$D_0^4 = -0.0690$$

For the capacitor C_2

$$N_0^0 = -0.6901 \times 10^{-9}, \quad N_0^1 = 0.5600 \times 10^{-9}, \quad N_0^2 = -0.1678 \times 10^{-9}$$
$$N_0^3 = 0.0219 \times 10^{-9}, \quad N_0^4 = -0.0011 \times 10^{-9}, \quad N_1^0 = 0.9181 \times 10^{-10}$$
$$N_1^1 = -0.7465 \times 10^{-10}, \quad N_1^2 = 0.2235 \times 10^{-10}, \quad N_1^3 = -0.0293 \times 10^{-10}$$
$$N_1^4 = 0.0014 \times 10^{-10}, \quad N_2^0 = -0.7456 \times 10^{-11}, \quad N_2^1 = 0.6077 \times 10^{-11}$$
$$N_2^2 = -0.1818 \times 10^{-11}, \quad N_2^3 = 0.0237 \times 10^{-11}, \quad N_2^4 = -0.0011$$
$$D_0^0 = -1.5458 \times 10^3, \quad D_0^1 = 1.2477 \times 10^3, \quad D_0^2 = -0.3727 \times 10^3$$
$$D_0^3 = 0.0482 \times 10^3, \quad D_0^4 = 0.0023 \times 10^3$$

To verify our analysis, Fig. 5 shows the input impedance of the cylindrical DRA antenna at $H = 3.5\,\text{mm}$, and $\varepsilon_r = 20$, these values are included in Table 1, compared with that calculated equation (8) (MBPE). Excellent agreement is obtained.

Figures 6 to 9 gives a comparison between the calculated input impedance by using the FDTD method and that obtained from MBPE

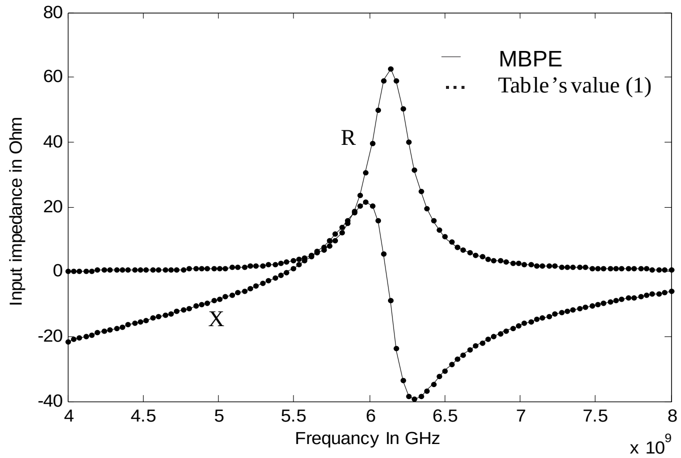

Figure 5. The input impedance for cylindrical DRA for $\varepsilon_r = 20$ and $H = 6.5\,\text{mm}$.

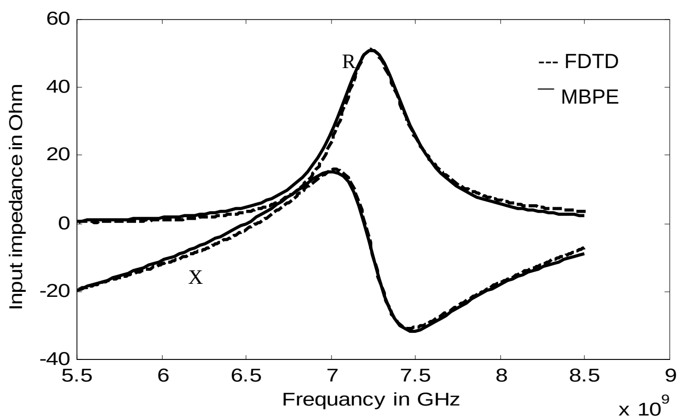

Figure 6. The input impedance for cylindrical DRA for $\varepsilon_r = 16$ and $H = 5\,\text{mm}$.

for different H and ε_r that are not included in Tables 1 to 4. Good agreement between the two methods can be found. By using the MBPE for the different elements of the equivalent circuit, the input impedance of the cylindrical DRA can be determined for different dimensions of H and ε_r in the ranges of Tables 1–4. There is no need to run the FDTD in the above cases. The input impedance of the DRA can be calculated directly from the equivalent circuit obtained by using MBPE technique.

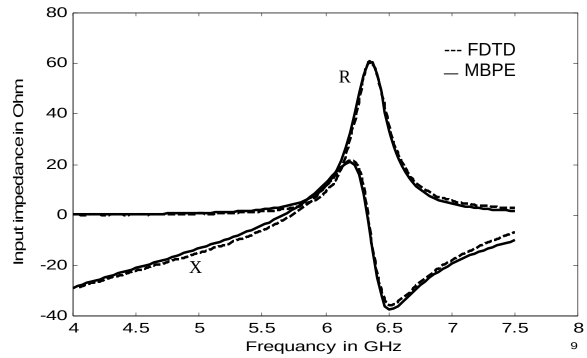

Figure 7. The input impedance for cylindrical DRA for $\varepsilon_r = 20$ and $H = 6\,\mathrm{mm}$.

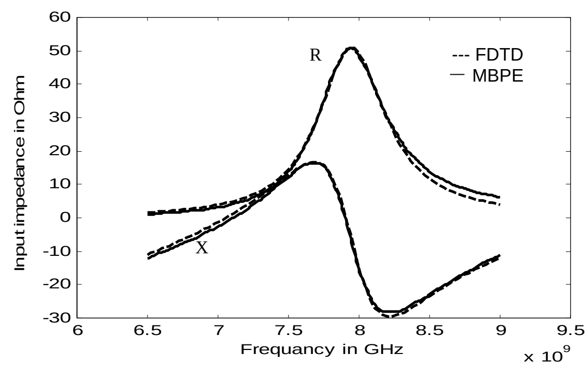

Figure 8. The input impedance for cylindrical DRA for $\varepsilon_r = 14$ and $H = 4.5\,\mathrm{mm}$.

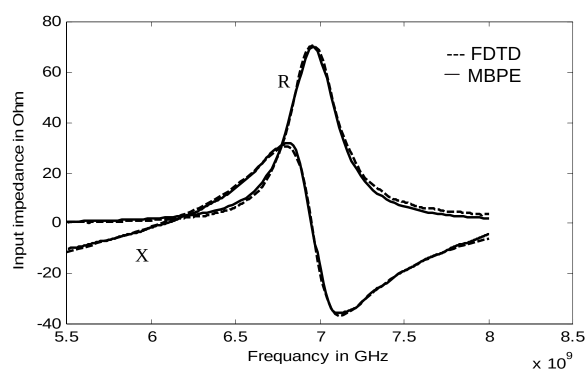

Figure 9. The input impedance for cylindrical DRA for $\varepsilon_r = 20$ and $H = 4\,\mathrm{mm}$.

4. CONCLUSIONS

A lumped element equivalent circuit of cylindrical DRA is calculated for different antenna dimensions. The FDTD method and the genetic algorithm are used to calculate the values of the elements of the equivalent circuit. MBPE is used to find the relation between each element in the equivalent circuit and the antenna dimensions. This method is used to reduce time required to calculate the input impedance over a wide frequency range.

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