

NUMERICAL MODELING OF ACTIVE DEVICES CHARACTERIZED BY MEASURED S-PARAMETERS IN FDTD

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Abstract—A new FDTD modeling approach for active devices characterized by measured S-parameters is presented. This approach applies vector fitting technique and piecewise linear recursive convolution (PLRC) technique to complete modeling process, and does not need to know the equivalent circuits of active devices. It preserves the explicit nature of the traditional FDTD method, and a general updated formula is derived. Furthermore, the main data-processing procedure is directly handled over the frequency band of interest, which avoids the time-domain non-causal error in traditional techniques.

1. INTRODUCTION

Over the last decade, it is very popular to analyze hybrid microwave circuits including distributed and lumped devices by using electromagnetic simulators, such as ADS, Ansoft HFSS, IE3D and so on. However, not all these simulators integrate the lumped element and device equations with the EM field directly, and the electromagnetic interactions between individual devices are neglected, so they do not accurately represent the physical structure of the circuit. Thus these simulators are not suitable for analyzing high frequency integrated circuits where the interactions between the distributed and lumped components play a vital role.

The finite-difference time-domain (FDTD) method seems to be attractive for this purpose [1]. Over recent fifteen years, much work has been reported for extending the original FDTD method to incorporate lumped components [2–6]. Results indicate that these extended FDTD approaches can successfully and accurately consider the electromagnetic interactions between distributed and lumped

components. However, all these approaches are based on equivalent circuit models of lumped devices. Unfortunately, in the case of active devices, only the measured S-parameters of the device under certain biasing conditions are provided by most manufacturers. Generally, we are not easy to obtain equivalent circuit of the device," replaced by " It is very difficult to obtain the equivalent circuits of practical complex active devices, especially for the three-terminal devices. Moreover, some classes of device are not even described reliably by the previous techniques. Therefore, the direct incorporation of active devices characterized by measured S-parameters into FDTD formulations is of great interest.

Little research has been reported in this field. In available papers [7, 8], the measured network parameters of active devices in frequency-domain are first transformed into the time-domain by directly using the inverse Fourier transform (IFT) technique. Then, the resulting time-domain parameters are inputted into the FDTD formulations by convolution products. In this technique, the transformation must ensure the system to satisfy the time-domain causality, but direct application of IFT technique usually brings non-causal time domain data.

In this paper, a novel approach is developed to incorporate active device characterized by measured S-parameters into FDTD algorithm by means of the vector fitting technique [9] and piecewise linear recursive convolution (PLRC) technique [10, 11]. It consists of three main steps: First, the measured S-parameters are converted into Y-parameters. Then each entry of which is fitted into a rational function by using vector fitting technique. Finally, the coefficients of the rational functions are directly incorporated into FDTD equations by means of PLRC technique. In this proposed approach, the main data-processing procedure is directly handled in frequency-domain, which avoids the time-domain non-causality. In addition, applying the PLRC technique, we can obtain a general FDTD updated formula, which preserves the explicit nature of the traditional FDTD method [12–20] and improve the computation efficiency.

To show the validity of this proposed approach, it has been applied to calculate the S-parameters of a microwave FET amplifier.

2. THEORY ANALYSIS

For the sake of conciseness, the main focus is placed on the two-port active devices. Assuming the two ports are associated to the two electric field component E_{z1} and E_{z2} . At nodes E_{z1} and E_{z2} , the

FDTD updated equation is expressed in following discrete form as

$$\begin{cases} \varepsilon \frac{E_{z1}^{n+1} - E_{z1}^n}{\Delta t} = \left[\nabla \times \vec{H} \right]_{z1}^{n+1/2} - J_1^{n+1/2} \\ \varepsilon \frac{E_{z2}^{n+1} - E_{z2}^n}{\Delta t} = \left[\nabla \times \vec{H} \right]_{z2}^{n+1/2} - J_2^{n+1/2} \end{cases} \quad (1)$$

where ε is the permittivity of substrate, Δt is the time increment, and J_1, J_2 denote the device current through the two ports, respectively. The device can be described by its Y-parameters matrix in the Laplace domain as follows

$$\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} Y_{11}(s) & Y_{12}(s) \\ Y_{21}(s) & Y_{22}(s) \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} \quad (2)$$

where V_p and I_p ($p = 1, 2$) denote the voltage and current at the two ports.

2.1. Fitting Data with Vector Fitting Technique

In this section, the data-fitting procedure is discussed. According to the vector fitting technique, each entry of Y-matrix can be approximated by the following rational function.

$$Y_{pq}(s) = \sum_{i=1}^{N^{(p,q)}} \frac{c_i^{(p,q)}}{s - a_i^{(p,q)}} + g^{(p,q)} + sh^{(p,q)}, \quad (p, q = 1, 2) \quad (3)$$

The residues $c_i^{(p,q)}$ and poles $a_i^{(p,q)}$ are either real quantities or come in complex conjugate pairs, while $g^{(p,q)}$ and $h^{(p,q)}$ are real. In general, $c_i^{(p,q)}$, $a_i^{(p,q)}$, $g^{(p,q)}$ and $h^{(p,q)}$ are different for each $Y_{pq}(s)$. $N^{(p,q)}$ is the order of rational function. We are to estimate all coefficients in (3) so that a least squares approximation of $Y_{pq}(s)$ is obtained over a given frequency interval. To simplify notation, the indexes p, q will be ignored in the following discussion [21].

We assume that the S-parameters of the active device have been measured at M frequency points $s_k = j2\pi f_k$ ($k = 1, \dots, M$) over the frequency band of interest $[f_{\min}, f_{\max}]$. By use of standard network-theory expressions, these measured S-parameters are transformed into Y-parameters $Y(s_k)$. Now our goal is to get the coefficients of the rational function in (3), and a rational fitting technique must be selected. In the paper [14], a comparison of different fitting techniques is made, and vector fitting technique shows to be robust, accurate and quickly, so this technique is adopted in this paper.

In this fitting process, it is worth noting that to avoid leading to an ill-conditioned set of equations, we carry out the fitting procedure over the normalized frequency band $[f_{\min}, f_{\max}]$, where the normalized frequency is defined as $\bar{s}_k = j2\pi\bar{f}_k$, $\bar{f}_k = f_k/f_{ave}$, with $f_{ave} = \frac{1}{2}(f_{\min} + f_{\max})$. The main fitting procedures are described as follows:

Step 1: Specify a set of starting poles a_i , which are distributed over the frequency range of interest, and introduce a weight function $\sigma(\bar{s})$. Multiply $Y(\bar{s})$ with the weight function and introduce a rational approximation.

$$\sigma(\bar{s}) = 1 + \sum_{i=1}^N \frac{\tilde{c}_i}{\bar{s} - \bar{a}_i} \quad (4)$$

$$\sigma(\bar{s})Y(\bar{s}) \approx \sum_{i=1}^N \frac{\bar{c}_i}{\bar{s} - \bar{a}_i} + g + \bar{s}\bar{h} \quad (5)$$

where $\bar{a}_i = a_i/f_{ave}$, $\bar{c}_i = c_i/f_{ave}$ and $\bar{h} = h \cdot f_{ave}$. Furthermore, it is noted that the two equations have the same poles, and Equation (4) shows that the poles of $Y(\bar{s})$ become equal to the zeros of $\sigma(\bar{s})$.

Step 2: Substituting (4) into (5) yields the following equations:

$$\left(1 + \sum_{i=1}^N \frac{\tilde{c}_i}{\bar{s} - \bar{a}_i}\right)Y(\bar{s}) \approx \sum_{i=1}^N \frac{\bar{c}_i}{\bar{s} - \bar{a}_i} + g + \bar{s}\bar{h} \quad (6)$$

A set of measured $Y(\bar{s}_k)$ for a given frequency point \bar{s}_k ($k = 1, 2, \dots, M$) is replaced into (6), which leads to an over-determined linear problem in the unknowns \bar{c}_i , g , \bar{h} and \tilde{c}_i :

$$A_k x = b_k \quad (k = 1, 2, \dots, M) \quad (7)$$

where

$$A_k = \left[\frac{1}{\bar{s}_k - \bar{a}_1} \cdots \frac{1}{\bar{s}_k - \bar{a}_N}, 1, \bar{s}_k, \frac{-Y(\bar{s}_k)}{\bar{s}_k - \bar{a}_1} \cdots \frac{-Y(\bar{s}_k)}{\bar{s}_k - \bar{a}_N} \right]$$

$$x = [\bar{c}_1 \cdots \bar{c}_N, g, \bar{h}, \tilde{c}_1 \cdots \tilde{c}_N]^T, \quad b_k = Y(\bar{s}_k)$$

Step 3: Solve (7) via linear least squares for the unknown $\{\tilde{c}_i\}$, and calculate the zeros of (4), then replace new poles \bar{a}_i by the zeros found. Repeating the above procedure, we can obtain the approximate poles a_i of the origin problem (4) in few iterations.

Step 4: Finally, the rest unknowns $c_i g$ and h can be solved according to the poles found and $Y(s_k)$.

To perform a stable FDTD simulation, it is worth noting that none of the poles of $Y_{pq}(s)$ in (3) should be placed on the right-hand side of the s -plane. Hence, a verification and correction procedure (if necessary) must be applied at step 3 to turn the poles \bar{a}_i on the right-hand side of the s -plane (if any) into the left-hand side.

Furthermore, vector fitting technique can also be applied directly to vector functions with the assumption that all elements in the vector have identical poles. Hence, considering the Y-matrix of a two-port device, we can express it as a vector function:

$$Y(s) = [Y_{11}(s), Y_{12}(s), Y_{21}(s), Y_{22}(s)]^T \quad (8)$$

Then, applying an identical weight function $\sigma(s)$, we can obtain four rational functions of Y-parameters after running only one time.

2.2. Deriving FDTD Formula with PLRC Technique

In this section, we discuss the incorporation of the resulting rational functions into FDTD algorithm by using the PLRC technique in detail.

Assume a convolution operation

$$I^n = \int_0^{n\Delta t} V(n\Delta t - \tau) Y(\tau) d\tau \quad (9)$$

According to the PLRC technique, we define two variables as:

$$\chi_m = \int_{m\Delta t}^{(m+1)\Delta t} Y(\tau) d\tau \quad (10)$$

$$\xi_m = \frac{1}{\Delta t} \int_{m\Delta t}^{(m+1)\Delta t} (\tau - m\Delta t) Y(\tau) d\tau \quad (11)$$

If the variables meet the following relationship

$$\rho = \frac{\chi_m}{\chi_{m-1}} = \frac{\xi_m}{\xi_{m-1}} \quad (12)$$

the time-domain current I_z^n can be updated using recursive equation:

$$I_z^{n+1} = (\chi_0 - \xi_0) V_z^{n+1} + \xi_0 V_z^n + \rho I_z^n \quad (13)$$

We introduce four auxiliary current intensities $I_{pq}(s)$ ($p, q = 1, 2$), and (2) can be expressed as

$$I_p(s) = \sum_{q=1,2} I_{pq}(s) \quad p = 1, 2 \quad (14)$$

where

$$I_{pq}(s) = Y_{pq}(s)V_q(s) \quad (15)$$

Then, divide the known rational function of each Y-parameter into two parts:

$$Y_{pq}(s) = \sum_{i=1}^{N(p,q)} \frac{c_i^{(p,q)}}{s - a_i^{(p,q)}} + g^{(p,q)} + sh^{(p,q)} = \sum_{i=1}^{N(p,q)} Y_{pq,i}(s) + Y_{pq,o}(s) \quad (16)$$

Thus, the time-domain current $I_{pq}^{n+1/2}$ can be expressed as:

$$I_{pq}^{n+1/2} = \sum_{i=0}^{N(p,q)} I_{pq,i}^{n+1/2} + I_{pq,o}^{n+1/2} \quad (17)$$

We discuss the two right terms of the above formulation in detail.

1. If the residues $c_i^{(p,q)}$ and poles $a_i^{(p,q)}$ are real quantities, $Y_{pq,i}(s) = \frac{c_i^{(p,q)}}{s - a_i^{(p,q)}}$ is converted to the time-domain by inverse Fourier transform technique

$$Y_{pq,i}(t) = c_i^{(p,q)} e^{a_i^{(p,q)} \Delta t} u(t) \quad (18)$$

$u(t)$ is the unit step function. By introducing (18) into (10) and (11), we can get:

$$\chi_{m,i} = -\frac{c_i}{a_i} (1 - e^{a_i \Delta t}) e^{ma_i \Delta t} \quad (19)$$

$$\xi_{m,i} = -\frac{c_i}{a_i^2 \Delta t} [(1 - a_i \Delta t) e^{a_i \Delta t} - 1] e^{ma_i \Delta t} \quad (20)$$

Obviously, (12) can be met

$$\rho_i^{(p,q)} = \frac{\chi_{m,i}^{(p,q)}}{\chi_{m-1,i}^{(p,q)}} = \frac{\xi_{m,i}^{(p,q)}}{\xi_{m-1,i}^{(p,q)}} = e^{a_i^{(p,q)} \Delta t} \quad (21)$$

In this case, the time-domain current $I_{pq,i}^n$ can be updated using recursive equation:

$$I_{pq,i}^{n+1} = \left(\chi_{0,i}^{(p,q)} - \xi_{0,i}^{(p,q)} \right) V_q^{n+1} + \xi_{0,i}^{(p,q)} V_q^n + \rho_i^{(p,q)} I_{pq,i}^n \quad (22)$$

2. If $c_i^{(p,q)}$ and $c_{i+1}^{(p,q)}$, $a_i^{(p,q)}$ and $a_{i+1}^{(p,q)}$ are complex conjugate pairs, the Equations (18)–(22) are yet satisfied, but they can be simplified. From the definition of (10)–(12), we can prove that $\chi_{m,i}^{(p,q)}$ and $\chi_{m,i+1}^{(p,q)}$, $\xi_{m,i}^{(p,q)}$ and $\xi_{m,i+1}^{(p,q)}$, $\rho_i^{(p,q)}$ and $\rho_{i+1}^{(p,q)}$ are all complex conjugate pairs, so $I_{pq,i}^{n+1}$ and $I_{pq,i+1}^{n+1}$ are also complex conjugate pairs and their sum is a real quantity.

$$\begin{aligned} I_{pq,i}^{n+1} + I_{pq,i+1}^{n+1} &= 2\text{Re} \left(I_{pq,i}^{n+1} \right) = 2 \left\{ \text{Re} \left(\chi_{0,i}^{(p,q)} - \xi_{0,i}^{(p,q)} \right) \cdot V_q^{n+1} \right. \\ &\quad \left. + \text{Re} \left(\xi_{0,i}^{(p,q)} \right) \cdot V_q^n + \text{Re} \left(\rho_i I_{pq,i}^n \right) \right\} \end{aligned} \quad (23)$$

So only one complex of a complex conjugate pair is needed. In other words, if $a_i^{(p,q)}$ and $a_i^{(p,q)}$ contain $2N_g^{(p,q)}$ conjugate complex, only $N_g^{(p,q)}$ variables need to be saved.

3. Substituting $Y_{pq,o}(s) = g^{(p,q)} + sh^{(p,q)}$ into (15), we can obtain $I_{pq,o}(s) = (g^{(p,q)} + sh^{(p,q)}) V_q(s)$. Applied the transformation of $s \rightarrow \partial/\partial t$, the following central-difference formulation can be achieved

$$I_{pq,o}^{n+1/2} = \left(g^{(p,q)}/2 + h^{(p,q)}/\Delta t \right) V_q^{n+1} + \left(g^{(p,q)}/2 - h^{(p,q)}/\Delta t \right) V_q^n \quad (24)$$

Finally, substituting (22)–(23) into (17) and assuming $a_i^{(p,q)}$ and $a_i^{(p,q)}$ are composed of $N_r^{(p,q)}$ real quantities and $N_g^{(p,q)}$ pairs of conjugate complex (total number $N^{(p,q)} = N_r^{(p,q)} + 2N_g^{(p,q)}$), we can write the total current as:

$$I_{pq}^{n+1/2} = A_{pq} V_q^{n+1} + B_{pq} V_q^n + I_{pq,t}^n \quad (25)$$

where

$$A_{pq} = \left(\chi_{0,t}^{(p,q)} - \xi_{0,t}^{(p,q)} \right) + g^{(p,q)}/2 + h^{(p,q)}/\Delta t \quad (26)$$

$$B_{pq} = \left(\xi_{0,t}^{(p,q)} + g^{(p,q)}/2 - h^{(p,q)}/\Delta t \right) \quad (27)$$

$$\chi_{0,t}^{(p,q)} = \frac{1}{2} \sum_{i=1}^{N_r^{(p,q)}} \chi_{0,i}^{(p,q)} + \sum_{i=N_r^{(p,q)}+1}^{N_r^{(p,q)}+N_g^{(p,q)}} \text{Re} \left(\chi_{0,i}^{(p,q)} \right) \quad (28)$$

$$\xi_{0,t}^{(p,q)} = \frac{1}{2} \sum_{i=1}^{N_r^{(p,q)}} \xi_{0,i}^{(p,q)} + \sum_{i=N_r^{(p,q)}+1}^{N_r^{(p,q)}+N_g^{(p,q)}} \operatorname{Re} \left(\xi_{0,i}^{(p,q)} \right) \quad (29)$$

$$I_{pq,t}^n = \frac{1}{2} \sum_{i=1}^{N_r^{(p,q)}} \left(\rho_i^{(p,q)} + 1 \right) I_{pq,i}^n + \sum_{i=N_r^{(p,q)}+1}^{N_r^{(p,q)}+N_g^{(p,q)}} \operatorname{Re} \left[\left(\rho_i^{(p,q)} + 1 \right) I_{pq,i}^n \right] \quad (30)$$

$$I_{pq,i}^{n+1} = \left(\chi_{0,i}^{(p,q)} - \xi_{0,i}^{(p,q)} \right) V_q^{n+1} + \xi_{0,i}^{(p,q)} V_q^n + \rho_i^{(p,q)} I_{pq,i}^n \quad (31)$$

The relationships of the voltage with the electric field and the current intensity with the current density are approximately expressed as

$$V_z^n = \int E_z^n dz \approx E_z^n \Delta z \quad (32)$$

$$I_z^{n+1/2} = \iint J_z^{n+1/2} dx dy \approx J_z^{n+1/2} \Delta x \Delta y \quad (33)$$

From (14), the time-domain current at each port can be expressed as

$$I_p^{n+1/2} = \sum_{q=1,2} I_{pq}^{n+1/2} \quad p = 1, 2 \quad (34)$$

Finally, substituting (32)–(34) and (25) into (1), we can obtain the FDTD formulation of the electric field at each port

$$\begin{bmatrix} E_{z1}^{n+1} \\ E_{z2}^{n+1} \end{bmatrix} = \frac{1}{\alpha} \begin{bmatrix} A_{11} + \varepsilon/\alpha \Delta t & A_{12} \\ A_{21} & A_{22} + \varepsilon/\alpha \Delta t \end{bmatrix}^{-1} \begin{bmatrix} T_1^n \\ T_2^n \end{bmatrix} \quad (35)$$

where

$$\alpha = \frac{\Delta z}{\Delta x \Delta y},$$

$$T_p^n = \frac{\varepsilon}{\Delta t} E_{zp}^n + \left[\nabla \times \vec{H} \right]_{zp}^{n+1/2} - \sum_{q=1,2} \left(\alpha B_{pq} E_{zq}^n + \frac{1}{\Delta x \Delta y} I_{pq,t}^n \right) \quad (36)$$

From (35), we can find that this approach preserves the full explicit nature of the standard FDTD method.

3. NUMERICAL EXAMPLE

To illustrate the validity and accuracy of the proposed approach described above, we apply this approach to analyze a microwave

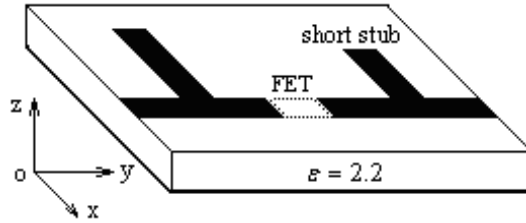


Figure 1. Structure of microwave FET amplifier.

FET amplifier circuit, which structure is shown in Figure 1. The S-parameters of this amplifier are calculated.

The parameters of the microstrip line used for our computations are thickness of the substrate ($H = 0.795 \text{ mm}$), width of the metal strip ($W = 2.4 \text{ mm}$), dielectric constant of the substrate ($\epsilon = 2.2$), and thickness of the metal strip (zero), which corresponds to the characteristic impedance of the microstrip line (50Ω). In FDTD simulation, the space steps are $\Delta x = 0.4 \text{ mm}$, $\Delta y = 0.4 \text{ mm}$ and $\Delta z = 0.265 \text{ mm}$. The total mesh dimensions are $44\Delta x \times 102\Delta y \times 20\Delta z$. The dimension of all metal strips can be referred to literature [6]. The first-order Mur's absorbing boundary condition is adopted to truncate the computing region, except the boundary of $z = 0$. A Gaussian pulse source with an internal resistor (50Ω) is used to excite the circuit at the 1st port, and a matched load (50Ω) is connected to the 2nd port. The time step is $\Delta t = 0.53 \text{ ps}$.

A JS8851-AS FET is used in this amplifier circuit, as shown in Figure 1. The S-parameters of FET with 41 frequency points linearly distributed over a frequency-band ($4 \text{ GHz} \sim 8 \text{ GHz}$) are obtained, which are illustrated in Figure 2. According to the proposed

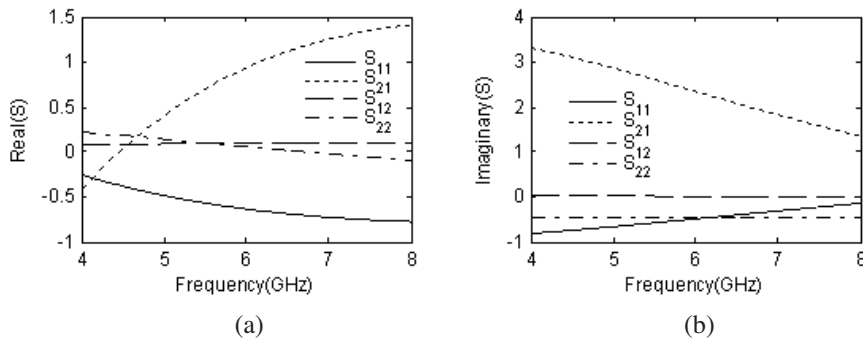


Figure 2. S-parameters of FET (a) real part (b) imaginary part.

approach, the S-parameters over a finite-band are first converted into the Y-parameters, which can be expressed as a vector form, i.e., $Y(s) = [Y_{11}(s), Y_{12}(s), Y_{21}(s), Y_{22}(s)]$. Applying the iterative procedure of vector fitting technique and defining the starting poles of $(4e9, 5.33e9, 6.67e9, 8e9)$, then we obtain the fourth order rational function approximations of Y-parameters with an identical set of poles. Convergence is reached in two iterations. Finally, the resulting coefficients of the rational functions are directly incorporated into the FDTD updated equations by using the PLRC technique. The simulation is performed for 1600 time steps. We calculate the S-parameters of this amplifier circuit, which are compared with those in reference [6]. The compared results are shown in Figure 3. It can be seen from Figure 3 that good agreement is achieved over the frequency range of interest. It demonstrates the accuracy and validity of the proposed approaches.

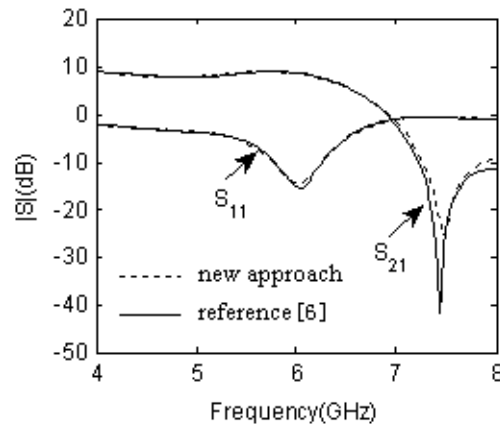


Figure 3. S-parameters of the microwave FET amplifier circuit.

4. CONCLUSION

Most of the extended FDTD methods are based on equivalent circuit models of lumped devices, but equivalent circuits of most practical active devices are not easy to obtain. To overcome this problem, in this paper, we propose a novel approach for incorporating lumped device into the FDTD simulator. The vector fitting technique and PLRC technique are adopted to complete the modeling task. Some details in the fitting process are discussed to ensure the stable FDTD simulation, and a full-explicit FDTD updated formulation is derived. Finally, a numerical example is analyzed to illustrate the validity of

this proposed method. It is useful for designer to accurately analyze hybrid microwave circuits including practical active devices.

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