Progress In Electromagnetics Research Letters, Vol. 1, 205–210, 2008

NUMERICAL METHOD OF SIMULATION OF MATERIAL INFLUENCES IN MR TOMOGRAPHY

M. Steinbauer and R. Kubasek

Faculty of Electrical Engineering and Communications
Brno University of Technology
Czech Republic

K. Bartusek

Institute of Scientific Instruments
Academy of Sciences of the Czech Republic
Czech Republic

Abstract—Generally all Magnetic Resonance Imaging (MRI) techniques are affected by magnetic and electric properties of measured materials, resulting in errors in MR image. Using numerical simulation we can solve the effect of changes in homogeneity of static and RF magnetic fields caused by specimen made from conductive and/or magnetic material in MR tomograph. This paper deals with numerical simulation of material susceptibility influence to magnetic field.

1. INTRODUCTION

In MR tomography, strong magnetic field is used (above 1 T). Because MR is very sensitive to inhomogeneity of this field, even such weak induced magnetic field as from para- or diamagnetic material is significant. Second mechanism, which affects field homogeneity, is magnetic field off eddy current induced by RF impulse in conductive material. Aim of this paper is in simulation of weakly magnetic material influence to static magnetic field.

For numerical simulation, two approaches can be used: normal Finite Element Method (FEM) and calculation of reaction field considering the induced polarity in the region. These approaches have been discussed in [6] and [7], where the accuracy consideration and possible reduction of numerical method errors can be found. Using FEM for calculation of magnetic field in MR tomography requires double precision arithmetic and sufficiently fine mesh, because change of the magnetic field in vicinity of slightly magnetic materials is weak in comparison to basic static magnetic field. Second approach may use single precision arithmetic, because only reactional field (induced own field of magnetic material) is computed.

2. 2D ANALYTIC SOLUTION

Let's have specimen with susceptibility χ_{m1} surrounded by reference medium with known susceptibility χ_{m2} and placed into static primal magnetic field with magnetic intensity vector $\mathbf{H}_0$ oriented in $\mathbf{u}_z$ direction — see Fig. 1 left. We have to determine magnetic intensity $\mathbf{H}$ of incurred field, which is superposition of primal and reaction field $\mathbf{H}_\mathrm{r}$ (effect of specimen magnetization).

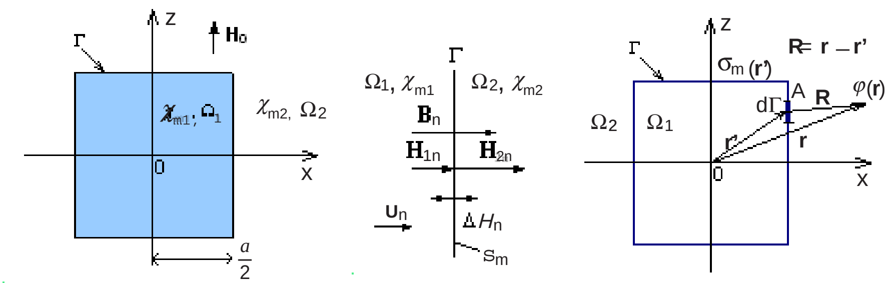

Figure 1. 2D analytic model for rectangular specimen (left), replacement of specimen magnetization effect by surface magnetic charge at the area boundaries (right) and boundary detail (middle).

Because there are not variable currents in whole area, magnetic field is irrotational (rot H = 0) and we can use scalar magnetic potential

$$\mathbf{H} = -\mathrm{grad}\varphi_\mathrm{m}. \tag{1}$$

Magnetic potential of primal field of intensity $\mathbf{H}_0$ is

$$\varphi_{\mathrm{m0}} = -\int \mathbf{H}_0 \cdot \mathbf{u}_z \mathrm{d}z = -\mathbf{H}_0 z. \tag{2}$$

Incidence of magnetized specimen from Fig. 1 left we can replace with effect of field of surface magnetic charge with density σ_m on boundary of areas Ω_1 a Ω_2 — see Fig. 1 right, whereas susceptibility of areas is now zero. First we have to compute magnetic charge density distribution on bound Γ and consequently the intensity of reaction field

$\Delta\mathbf{H} = \mathbf{H} - \mathbf{H}_0$

$$\Delta\mathbf{H}(\mathbf{r}) = \frac{1}{2\pi}\oint_{\Gamma} \sigma_{\mathrm{m}}(\mathbf{r}')\frac{\mathbf{u}_{\mathrm{r}}}{R(\mathbf{r},\,\mathbf{r}')}\mathrm{d}\Gamma. \tag{3}$$

Surface magnetic charge invokes scalar magnetic potential [5]

$$\varphi_{\mathrm{mr}}(\mathbf{r}) = -\frac{1}{2\pi}\oint_{\Gamma} \sigma_{\mathrm{m}}(\mathbf{r}')\ln R(\mathbf{r},\,\mathbf{r}')\mathrm{d}\Gamma. \tag{4}$$

Total scalar magnetic potential at point $\mathbf{r}$ is superposition of static primal field intensity (2) and contribution from charged bound (4)

$$\varphi_{\mathrm{m}}(\mathbf{r}) = -Hz - \frac{1}{2\pi}\oint_{\Gamma} \sigma_{\mathrm{m}}(\mathbf{r}')\ln R(\mathbf{r},\,\mathbf{r}')\mathrm{d}\Gamma. \tag{5}$$

Using condition of magnetic flux $\mathbf{B}_{\mathrm{n}} = B_{\mathrm{n}}\mathbf{u}_{\mathrm{n}}$ we obtain an integral formula for surface magnetic charge density normal component conjunction on bound Γ (see Fig. 1 middle)

$$B_{\mathrm{n}} = \mu_0(1+\chi_{\mathrm{m1}})H_{1\mathrm{n}} = \mu_0(1+\chi_{\mathrm{m2}})H_{2\mathrm{n}} \tag{6}$$

Analogically to the Gauss theorem causes magnetic charge of density σ_{m} at point A magnetic field of intensity

$$\Delta H_{\mathrm{n}} = \pm\frac{\sigma_{\mathrm{m}}(\mathrm{A})}{2}. \tag{7}$$

Using (5) and (1) we have the normal components of magnetic field intensity at point A (Fig. 1 middle)

$$H_{1\mathrm{n}} = H_0\mathbf{u}_z\mathbf{u}_{\mathrm{n}} + \frac{1}{2\pi}\mathrm{grad}\oint_{\Gamma,\,r\in\Omega_1} \sigma_{\mathrm{m}}\left(\mathbf{r}'\right)\ln R(\mathbf{r},\,\mathbf{r}')\mathrm{d}\Gamma\mathbf{u}_{\mathrm{n}}, \tag{8}$$

$$H_{2\mathrm{n}} = H_0\mathbf{u}_z\mathbf{u}_{\mathrm{n}} + \frac{1}{2\pi}\mathrm{grad}\oint_{\Gamma,\,r\in\Omega_1} \sigma_{\mathrm{m}}\left(\mathbf{r}'\right)\ln R(\mathbf{r},\,\mathbf{r}')\mathrm{d}\Gamma\mathbf{u}_{\mathrm{n}}. \tag{9}$$

Whenever $\mathrm{A}\in\Gamma$ and thus $\mathbf{r} \in \Gamma$, has integral in formulas (8) a (9) singularity at point A (where $\mathbf{r} = \mathbf{r}'$). We can remove this singularity omitting point $\mathbf{r} = \mathbf{r}'$ from integration and taking field contribution of this point using (7) instead. So we can write

$$H_{1\mathrm{n}} = H_0\mathbf{u}_z\cdot\mathbf{u}_{\mathrm{n}} + \frac{1}{2\pi}\oint_{\substack{\Gamma\\ \mathrm{r}\neq\mathrm{r}'}} \sigma_{\mathrm{m}}\left(\mathbf{r}'\right)\frac{1}{R(\mathbf{r},\,\mathbf{r}')}\mathrm{d}\Gamma\mathbf{u}_R\cdot\mathbf{u}_{\mathrm{n}} - \frac{\sigma_{\mathrm{m}}(A)}{2}, \tag{10}$$

$$H_{2\mathrm{n}} = H_0 \mathbf{u}_z \cdot \mathbf{u}_\mathrm{n} + \frac{1}{2\pi} \oint\limits_{\substack{\Gamma \\ \mathrm{r} \neq \mathrm{r}'}} \sigma_\mathrm{m}\left(\mathbf{r}'\right) \frac{1}{R(\mathbf{r},\, \mathbf{r}')} \mathrm{d}\Gamma \mathbf{u}_R \cdot \mathbf{u}_\mathrm{n} + \frac{\sigma_\mathrm{m}(A)}{2}, \quad (11)$$

where was used

$$\operatorname{grad} \ln R(\mathbf{r},\, \mathbf{r}') = \frac{1}{R(\mathbf{r}, \mathbf{r}')} \mathbf{u}_R. \quad (12)$$

Substituting from (10) and (11) into (6) we have after some rearrangement

$$\frac{\chi_\Delta}{2\pi} \oint\limits_{\substack{\Gamma \\ \mathrm{r} \in \Gamma,\, \mathrm{r} \neq \mathrm{r}'}} \frac{\sigma_\mathrm{m}\left(\mathbf{r}'\right)}{R\left(\mathbf{r},\, \mathbf{r}'\right)} \mathrm{d}\Gamma \mathbf{u}_R \cdot \mathbf{u}_\mathrm{n} + \frac{\sigma_\mathrm{m}(\mathbf{r})}{2} = -\chi_\Delta H_0 \mathbf{u}_z \cdot \mathbf{u}_\mathrm{n}, \quad (13)$$

where differential susceptibility was introduced

$$\chi_\Delta = \frac{\chi_{\mathrm{m}1} - \chi_{\mathrm{m}2}}{\chi_{\mathrm{m}1} + \chi_{\mathrm{m}2} + 2}. \quad (14)$$

Formula (13) is not analytically solvable, thus we solve it numerically by mean of boundary element method. After solution of (13) using collocation method described in [5] we have obtained results, shown in next figure. Shape of magnetic flux density is in the Fig. 2. In this simulation the aluminium specimen (Ω_1) was considered with $\chi_{\mathrm{m}1} = 22 \cdot 10^{-6}$, length of specimen $z = 20\,\mathrm{mm}$, thickness $a = (3,\ 5 \text{ and } 7)\,\mathrm{mm}$. Specimen was immersed into the water with $\chi_{\mathrm{m}2} = -9 \cdot 10^{-6} - (\Omega_2)$.

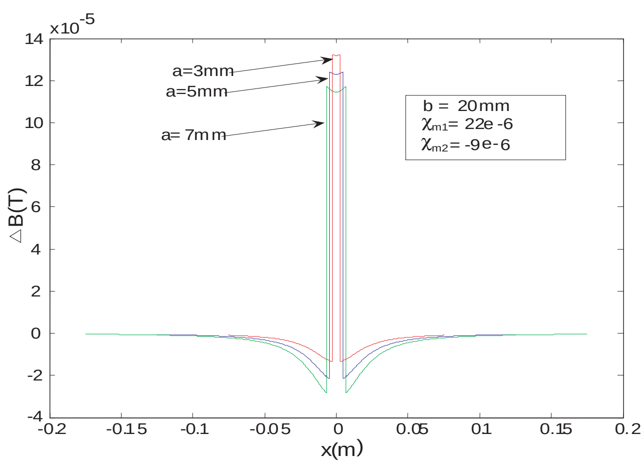

Figure 2. Result of numerical solution-magnetic flux density obtained by collocation method.

3. 3D NUMERICAL SOLUTION

Three dimensional numerical modeling was provided using FEM and Ansys software. The scalar magnetic potential was computed by solving of Laplace's equation

$$\Delta\varphi_{\mathrm{m}} = \mathrm{div}\mu\left(-\mathrm{grad}\varphi_{\mathrm{m}}\right) = 0. \tag{15}$$

One of used model is in Fig. 3. Here weakly paramagnetic specimen is surrounded by diamagnetic reference substance. The model was meshed with Solid96 element type. Boundary conditions were set up to achieve induction $B_0 = 4{,}700\,\mathrm{T}$ in z-axes direction: $\varphi_{\mathrm{m}} = const.$ on the surfaces Γ_1, Γ_2, $\frac{\partial\varphi_{\mathrm{m}}}{\partial \mathrm{n}} = 0$ on the shell surface Γ_3.

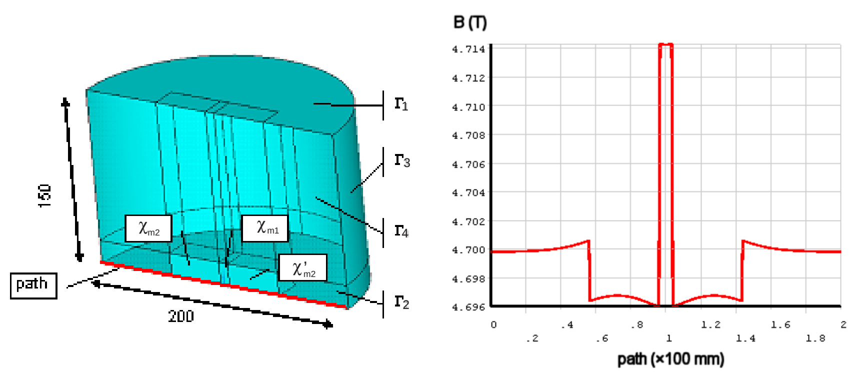

Figure 3. FEM model (left) and compute result (right) for magnetic flux density.

FEM modeling and solution is described in [8]. One of obtained results — the module of magnetic induction B along the "path" marked in Fig. 3 — is shown in the figure in the right.

4. CONCLUSIONS

Both method of numerical modeling of magnetic field deformation in MRI, caused by weakly magnetic specimen, which were described here, was compared with experimental results [3, 8]. Proximity of measured and numerically modeled data was good — see [5]. To enable comparison, simulation was adjusted to the same conditions as experiment: size of sample, susceptibilities and magnetic field of $\mathrm{B} = 4.7\,\mathrm{T}$.

Based on this simulation, MR measuring technique was founded [8], which is suitable for substances with no signal in MR tomography. The method uses Gradient Echo (GE) method and benefits from magnetic induction field shape in specimen vicinity, which is immersed in reference medium with measurable MR signal. After an optimization this method can be used for investigation of the materials used in MR tomography as well as of biological tissues affecting quality of MR images.

ACKNOWLEDGMENT

This work is supported by the grants GAAV B208130603 and GAAV B208130604.

REFERENCES

1. Ernst, R. R., G. Bodenhausen, and A. Wokaun, *Principles of NMR in One and Two Dimensions*, Oxford Science Publishing, 1987.
2. Sepúlveda, N. G., I. M. Thomas, and J. P. Wikswo, Jr., "Magnetic susceptibility tomography for three-dimensional imaging of diamagnetic and paramagnetic objects," *IEEE Transaction on Magnetics*, Vol. 30, No. 6, 5062–5069, 1994.
3. Kubasek, R., M. Steinbauer, and K. Bartusek, "Material influences in MR tomography, measurement and simulation," *Journal of Electrical Engineering*, Vol. 8, 58–61, Zilina, 2006.
4. Wang, Z. J., S. Li, and J. C. Hasselgrove, "Magnetic resonance imaging measurement of volume magnetic susceptibility using a boundary condition," *Journal of Magnetic Resonance*, Vol. 142, 477–481, 1999.
5. Hwang, S. N. and F. W. Wehrli, "Experimental evaluation of surface charge method for computing the induced magnetic field in trabecular bone," *Journal of Magnetic Resonance*, Vol. 139, 35–45, 1999.
6. Eastham, J. F., R. J. Hill-Cottingham, I. R. Young, and J. V. Hajnal, "A method of inverse calculation for regions of small susceptibility variations," *IEEE Transaction on Magnetics*, Vol. 33, No. 2, 1212–1215, 1997.
7. Eastham, J. F., R. J. Hill-Cottingham, I. R. Young, J. V. Hajnal, P. J. Leonard, and W. Lin, "Use of finite element analysis to calculate field changes in low susceptibility materials," *Proceedings of Compumag 95 International Conference*, 546–547, Berlin, Germany, 1995.
8. Steinbauer, M., "Magnetic susceptibility measurement using MRI methods," Ph.D. thesis, Brno University of Technology, 2005.