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THE SYNTHESIS OF COMPLEX-ANGLE ZEROS FOR ON-BOARD ANTENNA ARRAYS

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Abstract—Shelkunoff circle synthesis techniques are only effective for equispaced antenna arrays. These techniques are based on reorganizing the zeros on the Shelkunoff circle. We propose a technique based on locating complex-direction zeros to synthesize arbitrary on-board antenna arrays. It is based on the analytical continuation of Green's functions and on a new representation of the complex plane (analogous to the Shelkunoff circle).

1. INTRODUCTION

Some of the most efficient techniques for synthesizing equispaced linear antenna arrays are based in the main properties of the unit circle in Shelkunoff theory [1]. Examples are those of Woodward-Lawson [2] and Orchard-Elliot [3], and various improved versions [4–7]. The application of modern search techniques based on the perturbation of zeros on the Shelkunoff unit circle has enabled further advances to be made in these techniques and others [8–13], and indeed these search algorithms for optimizing radiation patterns have proven to be particularly efficient. The use of the Shelkunoff circle is not possible, however, for non-uniform arrays, and the main search techniques have been based on perturbing the excitation coefficients, which is considerably less efficient than perturbation of zeros.

In recent years, a number of techniques have been put forward for the synthesis of on-board antenna arrays [14–21]. These procedures take into account scattering by the platform supporting the antenna and mutual coupling. These techniques are based on the synthesis of array excitations instead of Shelkunoff unit circle theory (which is inapplicable to non-uniform and on-board antenna arrays).

Other related techniques for on-board antenna array are based on minimization of the electromagnetic scattering [22–24].

The use of global search algorithms for on-board antenna arrays requires iterative methods of numerical modeling that are laborious to develop and computationally expensive. A method based on the perturbation of zeros would improve the speed of convergence and the quality of the solution.

We here present a new technique based on the analysis of complex-angle zeros for the synthesis of non-uniform and on-board antenna arrays. It is based on the analytical continuation of the real radiation pattern to complex angles. Genetic algorithms used to perturb the zeros of this complex-angle pattern yield good results. On-board antenna arrays are modeled using a method-of-moments technique to take the mutual electromagnetic coupling and the electromagnetic coupling with the platform into account.

2. ANTENNA ARRAY MODELED BY THE METHOD OF MOMENTS

The method of moments (MoM) [25] can be used to model the electromagnetic behaviour of the on-board antenna array, taking into account the coupling between antennas and the coupling of the array with the supporting structure.

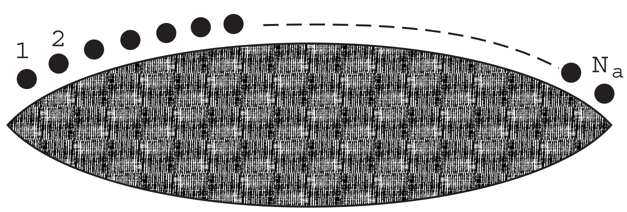

Figure 1. Scheme of an antenna array on board a complex structure.

Assume an on-board array of N_a antennas as illustrated schematically in Figure 1. In the MoM, the structure and antennas are modeled by N basis functions using the relationship

$$I = ZV. \tag{1}$$

The dimensions of the matrices are $N \times 1$ for I and V, and $N \times N$ for Z.

The representation includes special basis functions for the feed-points. In particular, there are N_a basis functions representing the feed-points of the antenna array and N_s basis functions representing the rest of the antennas and the structure (in a typical problem $N_s \gg N_a$). For the sake of simplicity, we assume that the feed-points are ordered at the beginning of the list of basis functions, so that (1) can be written as

$$\begin{pmatrix} V_a \\ V_s \end{pmatrix} = \begin{pmatrix} Z_{aa} & Z_{as} \\ Z_{sa} & Z_{ss} \end{pmatrix} \begin{pmatrix} I_a \\ I_s \end{pmatrix} \tag{2}$$

If one assumes a transmitting antenna array, the excitations are only at the feed-points, i.e.,

$$V_s = 0 \tag{3}$$

so that V_a represents the excitations of the array.

Solving the electromagnetic problem is then equivalent to solving (2). Using the inverse operator, one has

$$\begin{pmatrix} I_a \\ I_s \end{pmatrix} = \begin{pmatrix} Z_{aa} & Z_{as} \\ Z_{sa} & Z_{ss} \end{pmatrix}^{-1} \begin{pmatrix} V_a \\ V_s \end{pmatrix} \tag{4}$$

and using (3)

$$\begin{pmatrix} I_a \\ I_s \end{pmatrix} = \begin{pmatrix} Z_{aa} & Z_{as} \\ Z_{sa} & Z_{ss} \end{pmatrix}^{-1} \begin{pmatrix} V_a \\ 0 \end{pmatrix} \tag{5}$$

The inverse of the impedance matrix is the admittance matrix $Y = Z^{-1}$, so that

$$\begin{pmatrix} I_a \\ I_s \end{pmatrix} = \begin{pmatrix} Y_{aa} & Y_{as} \\ Y_{sa} & Y_{ss} \end{pmatrix} \begin{pmatrix} V_a \\ 0 \end{pmatrix} \tag{6}$$

and the currents can be calculated straightforwardly. At the feed-points

$$I_a = Y_{aa} V_a \tag{7}$$

and for the rest of the structure

$$I_s = Y_{sa} V_a \tag{8}$$

Y_{aa} and Y_{sa} can be calculated from the inverse of Z or using the Schur complement of Z_{ss} as

$$Y_{aa} = \left(Z_{aa} - Z_{as} Z_{ss}^{-1} Z_{sa}\right)^{-1} \tag{9}$$

$$Y_{sa} = -Z_{ss}^{-1} Z_{sa} \left(Z_{aa} - Z_{as} Z_{ss}^{-1} Z_{sa}\right)^{-1} \tag{10}$$

3. RADIATED FIELDS

In the far-field region, the field radiated by the on-board antenna array is given by

$$\mathbf{E}(\hat{k}) = -j\frac{k\eta_0}{4\pi}\int_{S'}(\bar{\bar{\mathbf{I}}} - \hat{k}\hat{k})\mathbf{J}(\mathbf{r}')e^{j\hat{\mathbf{k}}\mathbf{r}'}ds'\left\{\frac{e^{-jkr}}{r}\right\} \tag{11}$$

which can be expressed in the 2D form:

$$\mathbf{E}(\hat{\rho}) = -\eta_0\sqrt{\frac{jk}{8\pi}}\int_{C'}(\bar{\bar{\mathbf{I}}} - \hat{\rho}\hat{\rho})\mathbf{J}(\mathbf{r}')e^{jk\hat{\rho}\mathbf{r}'}dc'\left\{\frac{e^{-jk\rho}}{\sqrt{\rho}}\right\} \tag{12}$$

These equations can be expanded in terms of the subsectional basis functions of the method of moments. We define S as the matrix that relates the radiation in a set of radiation directions $\hat{K} = (\hat{k}_0, \hat{k}_1, \ldots, \hat{k}_{N_k})$ with the basis functions. It can be calculated from (11) or (12).

Then radiation in the set of directions $\hat{K}$ (sampling points of the radiation pattern) is represented by the vector E:

$$E = SI = \begin{pmatrix} S_a & S_s \end{pmatrix}\begin{pmatrix} I_a \\ I_s \end{pmatrix} \tag{13}$$

where the submatrices S_a and S_s represent the contribution to S from the feed-points and from the antennas and supporting structure, respectively.

From (7) and (8), (13) can be written as

$$E = SI = S_aI_a + S_sI_s = S_aY_{aa}V_a + S_sY_{sa}V_a \tag{14}$$

Defining the matrix R_a as

$$R_a = S_aY_{aa} + S_sY_{sa} \tag{15}$$

one can represent the radiation in terms of the excitation vector only:

$$E = R_aV_a \tag{16}$$

The matrix R_a relates excitation of the elements of the antenna array with the radiation pattern at the sampling directions $\hat{K}$, taking into account the mutual coupling and the coupling with the structure. The main advantage in using the matrix R_a is that it is calculated only once, and then (16) can be used to calculate the radiation pattern for any arbitrary excitation straightforwardly and rapidly.

4. COMPLEX ANGLE RADIATION

One can calculate the radiation at a complex angle ϕ using the analytical continuation of the cos() and sin() functions in

$$\hat{\rho} = \cos(\phi)\hat{x} + \sin(\phi)\hat{y}, \tag{17}$$

$$\hat{k} = \sin(\theta)\cos(\phi)\hat{x} + \sin(\theta)\sin(\phi)\hat{y} + \cos(\theta)\hat{z} \tag{18}$$

in (12) and (11), respectively. While this in general involves a major modification and improvement of the MoM routines for computing the radiation in a complex direction. The computation of the matrix impedance is unaffected.

5. Z-PLANE REPRESENTATION

From a physical viewpoint, the classical methods of design based on the Shelkunoff circle employ the synthesis of zeros at complex angles. This concept may be hidden by the polynomial expressions that are used in these methods, but the zeros that do not lie on the Shelkunoff circle represent zeros at complex angles. This idea can be used for arbitrary practical on-board antenna arrays. The Shelkunoff circle concept can be generalized to allow the synthesis of zeros at complex angles for arbitrary antenna arrays, by defining a new circle based on the true angles (not on the $kd\cos(\phi)$ angle as in equispaced antenna arrays). In contraposition to the Shelkunoff formulation, we define the z transform as

$$z = e^{j\phi} \tag{19}$$

The z-plane can be defined by representing z in terms of real and imaginary parts as in Figure 2. The unit circle in the z-plane represents the real angles ϕ. Complex angles ϕ are represented by points in the z-plane that do not lie on the unit circle. The complex angle represented by z is given by

$$\phi = -j\ln z \tag{20}$$

As in the Shelkunoff based methods, an antenna array can be synthesized by an arrangement of $N_a - 1$ zeros in the z-plane. Zeros on the unit circle simply represent zeros in the radiation diagram. While zeros inside or outside the unit circle do not correspond to zeros in the radiation pattern, those that are located near the unit circle lead to minima, termed 'filled nulls', in the radiation pattern at the real part of the angle that represents this zero.

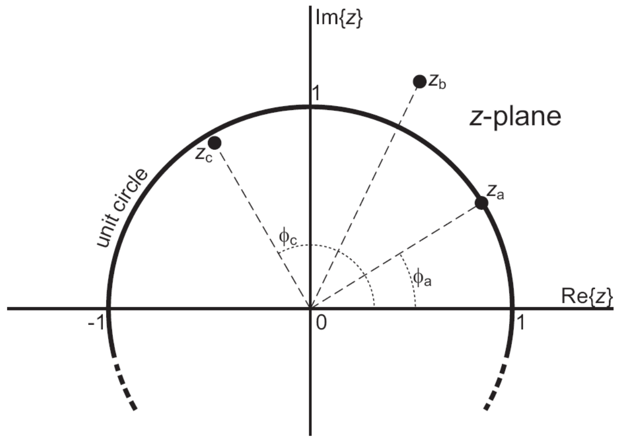

Figure 2. Zeros and unit circle in the z-plane. The zero z_a corresponds to a zero at the angle ϕ_a of the radiation pattern. The zero z_c near the unit circle corresponds to a 'filled null' at the angle ϕ_c of the radiation pattern. The zero z_b is distant from the unit circle.

6. SYNTHESIS OF ZEROS

In Shelkunoff based methods, there is a polynomial relationship between excitations and zeros on the Shelkunoff circle. While the proposed generalization does not permit such a straightforward relationship, a matrix relationship between zeros and excitations can be established.

Using the analytical continuation to complex-angle radiation in (11) or (12) and the matrix relation between radiation and coefficients of the antenna array in (16), one can define a matrix S_o that relates the excitations to the $N_a - 1$ complex angles that define the zeros in the z-plane:

$$E_o = R_o V_a \tag{21}$$

with

$$R_o = S_{ao} Y_{aa} + S_{so} Y_{sa} \tag{22}$$

where the dimension of the vector E_o is $(N_a - 1) \times 1$ and of the matrix R_o is $(N_a - 1) \times N_a$.

In a zero synthesizing procedure, one wants to impose zero radiation at these $N_a - 1$ complex angles. One can solve (21) by imposing $E_o = 0$:

$$0 = R_o V_a \tag{23}$$

With this procedure, there are $N_a - 1$ equations and N_a unknowns. An additional equation is therefore needed. Analogously with the Shelkunoff based methods, this equation must be a normalization of the coefficients (for example, the central excitation equal to unity). One thereby obtains the following equation for the excitation coefficients for a set of complex-angle zeros:

$$e = R_{o'} V_a \tag{24}$$

with

$$e = (0, 0, \ldots, 0, 1)^T \tag{25}$$

and

$$R_{o'} = \begin{pmatrix} R_o \\ R_{norm} \end{pmatrix} \tag{26}$$

where R_{norm} represents a normalization equation (for example, $R_{norm} = (0, 0, 0, \ldots, 0, 1, 0, \ldots, 0)$ or a vector giving an imposed value of the radiation in some direction).

Finally, the excitations can be calculated as

$$V_a = R_{o'}^{-1} e \tag{27}$$

From V_a, one can calculate the radiation pattern using (16) to obtain the fit to a target pattern in the synthesis procedure.

7. APPLICATION TO PATTERN SYNTHESIS

For the synthesis procedure, one wants to determine the positions of the $N_a - 1$ zeros in the z-plane that minimize a goodness-of-fit function. In our procedure, we use the following fitness function:

$$f(Z) = \alpha_m f_m + \alpha_d f_d + \alpha_b f_b \tag{28}$$

where the subscript m refers to the fulfilment of a mask pattern, d to the dynamic range of the excitation vector, and b to the maximum brightness of the excitation vector.

In the present work, we calculated f_m as

$$f_m = \int_\phi M(\phi) g(|E(\phi)|^2) d\phi \tag{29}$$

where $M(\cdot)$ represents a mask, and $g(\cdot)$ is a function reflecting the degree of fulfilment of the mask.

The functions f_d and f_b delimit the dynamic range between array excitations:

$$f_d = \frac{\max\limits_i(|\{V_a\}_i|)}{\min\limits_i(|\{V_a\}_i|)}, \qquad f_b = \max_i \left[\frac{|\{V_a\}_i|}{\left|\{V_a\}_{i\pm1}\right|} \right]. \tag{30}$$

These two values must be low in order to obtain excitations that can be implemented in practice. The weights α_m, α_d and α_b are to match the dimensions of (28) and to control the optimization process.

In the synthesis procedure, the perturbation of the complex angle zeros is done in a process of optimization. For a new set of the distribution of complex angle zeros, one obtains the excitations using (27). Then with these excitations, one calculates the radiation pattern to obtain f_d using (16). Note that the matrix R_a in (15) is calculated only once at the beginning of the process.

8. RESULTS

The results to be described for on-board antenna arrays are obtained for TM polarization. The radiating elements are electric currents on an infinite z-axis electric conductor. A Genetic Algorithm procedure is used for the perturbation and optimization of the complex angles zeros.

We considered the on-board antenna array of Figure 3 with 41 radiating elements. The separation between these elements and the conducting platform is 0.25λ, between the horizontal elements is 0.5λ, and between the diagonal elements is 0.71λ.

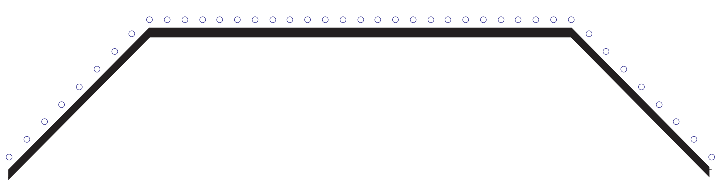

Figure 3. Antenna array on board a perfect electrically conducting platform.

For this radiating structure, we synthesize a pencil beam in the direction of the axis with low sidelobe radiation of -40 dB. The result of the synthesis is shown in Figure 4. The values of f_d and f_b of (30) characterizing the dynamic range of the amplitude excitations are $f_d = 14.52$ and $f_b = 1.62$.

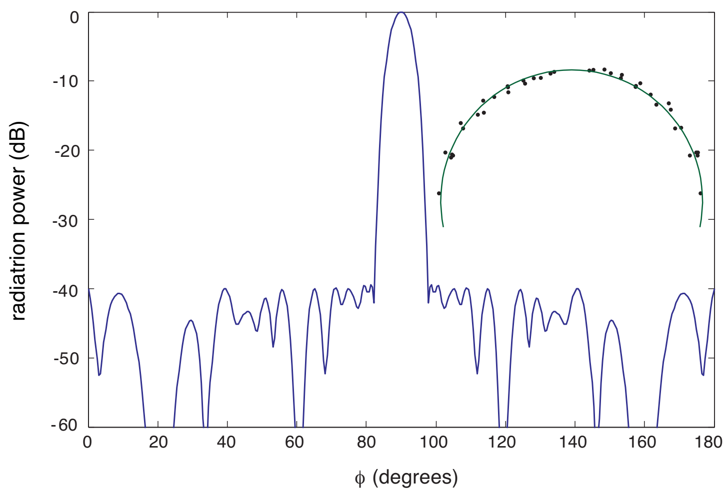

Figure 4. Example 1. Radiation pattern for a pencil beam. The distribution of zeros in the z-plane is also shown.

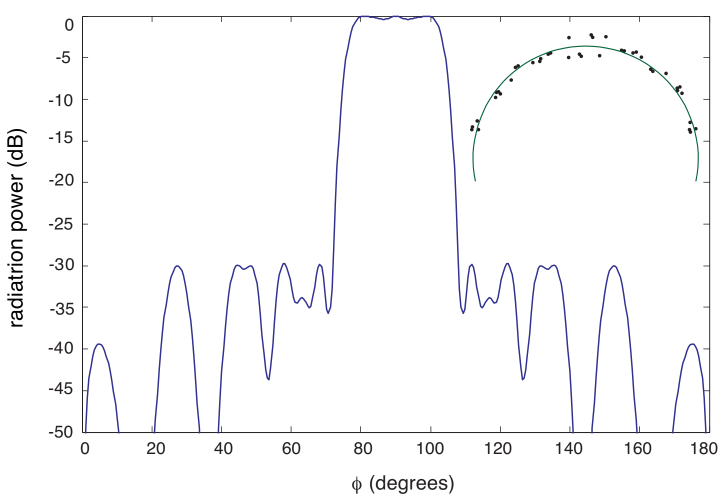

Figure 5. Example 2. Radiation pattern for a main broad lobe. The distribution of zeros in the z-plane is also shown.

In the second example we synthesize a main broad lobe with a ripple of ± 0.25 dB and a maximum sidelobe level of -30 dB. The results are shown in Figure 5, and the dynamic range of the excitations are characterized by $f_d = 8.84$ and $f_b = 1.71$.

9. CONCLUSIONS

We have presented a novel strategy for the synthesis of real patterns of on-board antenna arrays, considering both the mutual coupling and the electromagnetic coupling with the structure. The technique is based on the analytical continuation of the Green's functions of the radiation in the complex z-plane. The method allows global optimization searches for the locations of the zeros as in analogous techniques based on the Shelkunoff circle. Only a single inversion of a large matrix is needed to derive the complete electromagnetic behaviour of the problem, with the expressions used in the iteration process only involving low-dimensional matrices. Illustrative results for TM polarization showed the method to perform well for the synthesis of high performance patterns for on-board antenna arrays.

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REFERENCES

1. Shelkunoff, S. A., "A mathematical theory of linear arrays," *Bell Syst. Tech. J.*, Vol. 22, 80–107, 1943.
2. Woodward, P. M. and J. P. Lawson, "The theoretical precision with which an arbitrary radiation pattern may be obtained from a source of finite size," *Proc. IEEE*, Vol. 95, No. 1, 120–126, 1948.
3. Orchard, H. J., R. S. Elliott, and G. J. Stern, "Optimising the synthesis of shaped beam antenna patterns," *IEE Proc.*, Vol. 132, Pt. H, No. 1, 63–68, Feb. 1985.
4. Ares-Pena, F., "A note on the limitations of Orchard's method," *IEEE Antennas Propagat. Mag.*, Vol. 44, No. 1, 109, Feb. 2002.
5. Cid, J. M., J. A. Rodriguez, F. Ares, and F. Moreno, "Synthesis of satellite footprints by perturbation of Woodward-Lawson solutions for planar array antennas," *Journal of Electromagnetic Waves and Applications*, Vol. 14, 3–10, 2000.
6. Ares, F., S. R. Rengarajan, E. Villanueva, E. Skochinski, and E. Moreno, "Application of genetic algorithms and simulated annealing technique in optimising the aperture distributions of antenna array patterns," *Electron. Lett.*, Vol. 32, No. 3, 148–149, 1996.

7. Kedar, A., P. N. S. Kutiyal, M. Garg, and U. K. Revankar, "Wide band low profile linear microstrip antenna array with cosecant square-shaped beam pattern," *Microwave Opt. Technol. Lett.*, Vol. 49, No. 4, 963–965, 2007.
8. Vescovo, R., "Beam scanning with null and excitation constraints for linear arrays of antennas," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 2, 267–277, 2007.
9. Mouhamadou, M., P. Vaudon, and M. Rammal, "Smart antenna array patterns synthesis: Null steering and multi-user beamforming by phase control," *Progress In Electromagnetics Research*, PIER 60, 95–106, 2006.
10. Dessouky, M., H. Sharshar, and Y. Albagory, "Efficient sidelobe reduction technique for small-sized concentric circular arrays," *Progress In Electromagnetics Research*, PIER 65, 187–200, 2006.
11. Mahanti, G. K., A. Chakrabarty, and S. Das, "Phase-only and amplitude-phase synthesis of dual-pattern linear antenna arrays using floating-point genetic algorithms," *Progress In Electromagnetics Research*, PIER 68, 247–259, 2007.
12. Zhang, Y. F. and W. Cao, "Array pattern synthesis based on weighted biorthogonal modes," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 10, 1367–1376, 2006.
13. Rodriguez, J. A., L. Landesa, J. L. Rodriguez, F. Obelleiro, F. Ares, and A. G. Pino, "Pattern synthesis of array antennas with arbitrary elements by simulated annealing and adaptive array theory," *Microwave Opt. Technol. Lett.*, Vol. 20, No. 1, 48–50, 1999.
14. Obelleiro, F., L. Landesa, J. M. Taboada, and J. L. Rodriguez, "Synthesis of onboard array antennas including interaction with the mounting platform and mutual coupling effects," *IEEE Antennas Propagat. Mag.*, Vol. 43, No. 2, 76–82, 2001.
15. Landesa, L., J. L. Rodriguez, F. Obelleiro, and A. G. Pino, "Pattern synthesis of array antennas in presence of dielectric bodies," *IEEE Trans. Magnetics*, Vol. 35, No. 3, 1522–1525, 1999.
16. Landesa, L., J. M. Taboada, F. Obelleiro, J. L. Rodriguez, "Design of on-board array antennas by pattern optimization," *Microwave Opt. Technol. Lett.*, Vol. 21, No. 6, 446–448, 1999.
17. Lozano, M. V. and F. Ares, "Antenna array pattern synthesis in the presence of near-zone scatterers: Three-dimensional vector case," *Journal of Electromagnetic Waves and Applications*, Vol. 13, 1493–1507, 1999.
18. Su, T. and H. Ling, "Array beamforming in the presence of a

mounting tower using genetic algorithms," *IEEE Trans. Antennas Propagat.*, Vol. 53, No. 6, 2011–2019, 2005.

19. Capozzoli, A. and G. D'Elia, "Global optimization: Applications to advanced reflector antennas synthesis and diagnosis techniques," *Progress In Electromagnetics Research*, PIER 56, 195–232, 2006.
20. Capozzoli, A. and G. D'Elia, "Global optimization and antennas synthesis and diagnosis, Part two: Applications to advanced reflector antennas synthesis and diagnosis techniques," *Progress In Electromagnetics Research*, PIER 56, 233–261, 2006.
21. Ayestaran, R. G., F. Las-Heras, and J. A. Martínez, "Non-uniform-antenna array synthesis using neural networks," *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 8, 1001–1011, 2007.
22. Steyskal, H., "Synthesis of antenna patterns with imposed near-field nulls," *Electron. Lett.*, Vol. 30, No. 4, 2000–2001, 1994.
23. Allen, O. E. and W. Wasylkiwskyj, "Circularly polarized array pattern synthesis with electromagnetic compatibility constraints," *IEEE Trans. Electromagnetic Compatibility*, Vol. 48, No. 1, 251–254, 2006.
24. Landesa, L., F. Obelleiro, J. L. Rodriguez, J. A. Rodriguez, F. Ares, and A. G. Pino, "Pattern synthesis of array antennas with additional isolation of near field arbitrary objects," *Electron. Lett.*, Vol. 34, No. 16, 1540–1542, 1998.
25. Harrington, R. F., *Field Computations by Moment Method in Electromagnetics*, Krieger, Malabar, Fla., 1982.