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ANALYSIS AND DESIGN OF HIGHLY COMPACT BANDPASS WAVEGUIDE FILTER UTILIZING COMPLEMENTARY SPLIT RING RESONATORS (CSRR)

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Abstract—Split Ring Resonators (SRR) and Complementary Split Ring Resonators (CSRR) are widely used to design metamaterial structures. These structures when excited by suitable electromagnetic fields have resonance behavior and show unusual properties such as negative permeability and permittivity near the resonance frequency region. In this paper, CSRRs are used to design a bandpass waveguide filter in the X-band. The circuit model of these elements in the waveguide is similar to parallel L and C components that are placed in parallel form in a transmission line. Resonance frequency and bandwidth of LC resonance circuit are adjusted by proper choice of the CSRR geometrical dimensions. Then, to design the miniaturized filter these structures are combined with proper admittance inverter. The admittance inverter is designed such that its electric length is very smaller than the conventional $\lambda/4$ transmission line. As a result, a filter is compacted about 66% in comparison to the $\lambda/4$ transmission line as admittance inverter. Simulation results by Ansoft HFSS (Based on the Finite Element Method) confirm the results of filter circuit model.

1. INTRODUCTION

Split Ring Resonators (SRR) are resonance structures that are used widely in electromagnetics [1–8]. For example, these structures are used in periodic configurations to design metamaterial structures [9–12]. Also, because of their resonance behavior SRRs and CSRRs can be used to design slow wave transmission lines, phase shifters, various kinds of microstrip filters, etc. [5–21]. In particular, SRR and CSRR structures have been used in the design of waveguide filters because a waveguide in each propagation mode operates as a transmission line and it is possible to use such resonance structures as filter elements [22–24]. The circuit model of SRR elements in the waveguide is parallel capacitance and inductance, placed in series in the transmission line and, therefore, these elements can be used to design bandstop filters [22, 23]. On the other hand, using the duality theorem and Babinet's principle, the CSRRs can be used to design bandpass filters [21, 24, 25].

In this paper, at first the characteristics of CSRR in waveguide are studied and then a suitable resonance circuit model is proposed for it. The resonance frequency and its bandwidth are adjusted by properly choosing the geometrical parameters of CSRR which are used in realizing the filter in the next steps. The bandpass filter is primarily designed using a combination of CSRRs as resonance elements and a $\lambda/4$ transmission line as an inverter. Next, to miniaturize the filter, the $\lambda/4$ transmission line inverter is replaced by a very smaller one compared to $\lambda/4$. However, because of the closeness of CSRR and inverter elements, there is very high coupling between them and this leads to incorrect response of filter. To solve this problem, the designed CSRR and inverter must be effectively modified such as to compensate for the coupling effect and obtain the required response. The bandpass filter considered here is a 3-section Chebyshev with 500 MHz Bandwidth around operating frequency 11.95 GHz (4.18% bandwidth). The insertion loss (IL) is taken as 0.5 dB. Such a filter has application in satellite earth station receiver subsystems. We use the WR90 standard waveguide to design the waveguide component and the RT/Duroid 5880 microstrip board to design the CSRR and inverter elements. All the filter designs have been confirmed by simulation using the HFSS software which is based on the finite element method (FEM) [28].

2. DESIGN OF CSRR AND INVERTER

2.1. Design of CSRR

The structure of a CSRR in waveguide as proposed by [24] is shown in Figure 1.

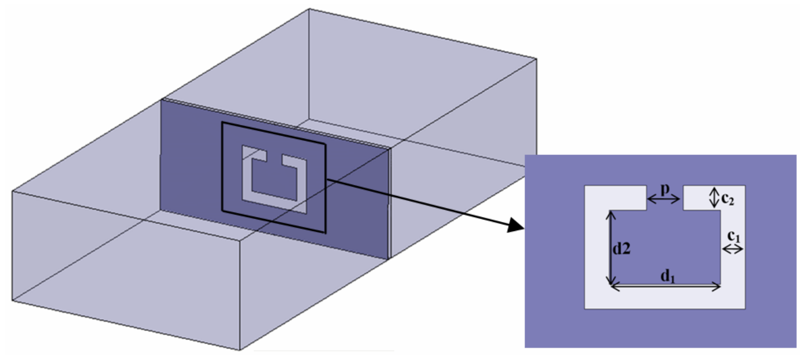
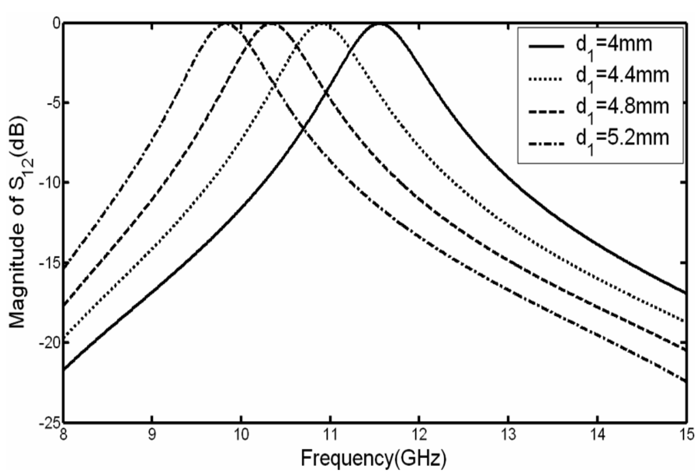


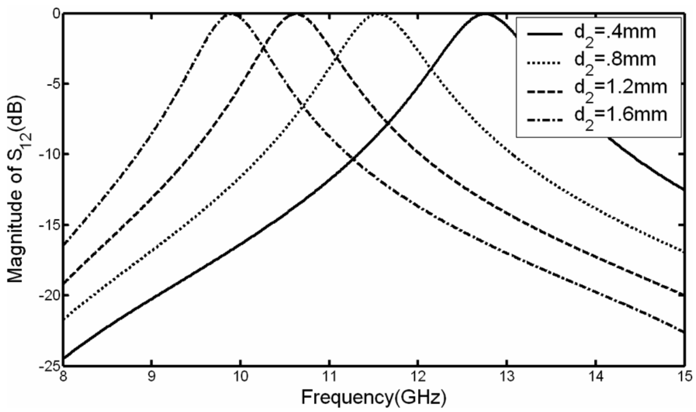
Figure 1. CSRR configuration in waveguide.

The effect of the various geometrical parameters on S_{12} including the longitudinal and horizontal slot widths (d_1, d_2) and also the length of its strip (p) are displayed in Figure 2. It must be noticed that the variations of the c_1 and c_2 are similar to d_1 and d_2, respectively.

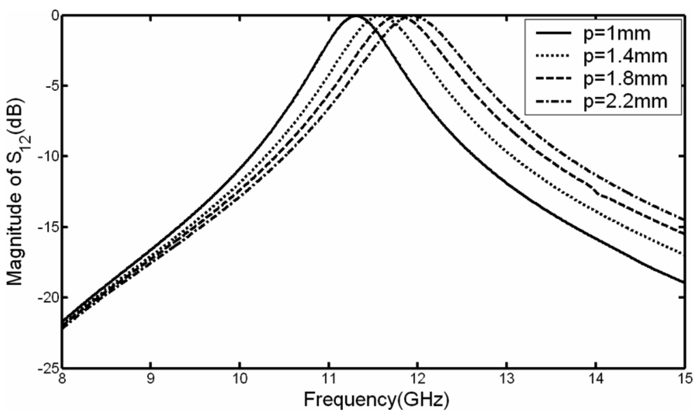
The manner of variation of the resonance frequency and



(a)



(b)



(c)

Figure 2. HFSS simulation results of S_{12} of waveguide filter (a) $d_2 = 0.8\,\mathrm{mm}$, $c_1 = 1.5\,\mathrm{mm}$, $c_2 = 0.25\,\mathrm{mm}$, $p = 1.4\,\mathrm{mm}$. (b) $d_1 = 4\,\mathrm{mm}$, $c_1 = 1.5\,\mathrm{mm}$, $c_2 = 0.25\,\mathrm{mm}$, $p = 1.4\,\mathrm{mm}$. (c) $d_1 = 4\,\mathrm{mm}$, $d_2 = 0.8\,\mathrm{mm}$, $c_1 = 1.5\,\mathrm{mm}$, $c_2 = 0.25\,\mathrm{mm}$.

bandwidth of the CSRR structure versus the geometrical parameters are summarized in Table 1.

Thus, the CSRR element can be modeled by a parallel LC circuit placed in parallel form across a transmission line (Figure 3). The values of the L and C components are related to the geometrical parameters of the CSRR.

Table 1. Variation of resonance frequency (f_0) and relative bandwidth (BW) of CSRR structure versus different parameters.

Parameter	f_0	BW
d_1 ↑	↓	↑
d_2 ↑	↓	↓
c_1 ↑	↓	↑
c_2 ↑	↓	↑
p ↑	↑	↑

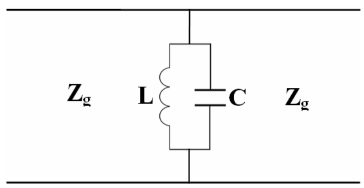


Figure 3. Circuit model of CSRR in waveguide.

2.2. Design of Inverter

In general, the inverter is a 2-port network which inverts the impedance or admittance of its load. In the special case where there is only a reactive load it is sufficient to produce $\pi/2$ phase shift in S_{12} (scattering parameter) to have inverter. A common structure used as an inverter in microstrip and waveguide filters is the $\lambda/4$ transmission line. To reduce the filter length it is necessary to design the inverter with length smaller than $\lambda/4$. This can be done by inserting an iris structure in the waveguide. By adjusting the iris hole (k), the effective electrical length θ of the inverter is reduced considerably. Figure 4 shows the inverter configuration.

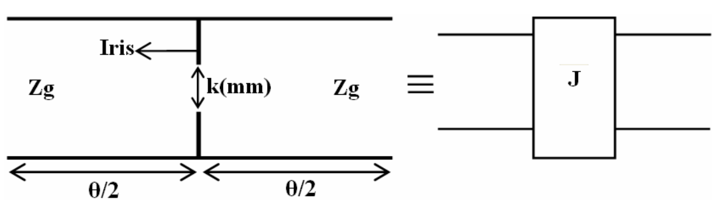


Figure 4. Iris configuration as inverter.

A model used to design an inverter is described in [27] and we call it symmetrical Π circuit model. However, numerical simulations of the iris show that this model is not sufficiently accurate to represent the iris as an inverter. Therefore, in this section a more general circuit model which we call as asymmetric Π model is proposed. The circuit model of the iris combined with the waveguide as transmission line is depicted in Figure 5.

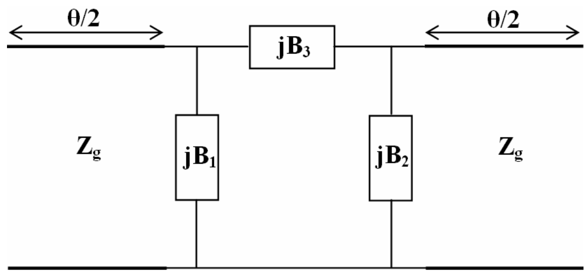


Figure 5. Circuit model of iris combined with the waveguide as transmission line.

The total transmission matrix of the structure is:

$$[ABCD]_{total} = [ABCD]_{TL} \times [ABCD]_{\pi} \times [ABCD]_{TL} \tag{1}$$

where,

$[ABCD]_{TL}$: Transmission Matrix of Waveguide Half-Section and

$[ABCD]_{\pi}$: Transmission Matrix of π Network

Knowing the total transmission matrix, the transmission matrix of Π section and consequently the parameters of circuit model of iris ($B_{1,2,3}$) can be obtained. The necessary condition to obtain an inverter is that the A and D components of the total transmission matrix be zero [27]. Therefore, after some computation the electric length of the inverter and also its admittance coefficient are:

$$\theta = \arctan[(2B_3+B_1+B_2)/(-Y_0+B_1B_2/Y_0+B_1B_3/Y_0+B_2B_3/Y_0)] \tag{2}$$

$$J = Y_0\sqrt{[(\alpha\sin\theta + \beta\cos\theta - \delta)\,/\,(\alpha\sin\theta + \beta\cos\theta + \delta)]} \tag{3}$$

where,

$$\alpha = Y_0 + Y_0B_2/(2B_3) + Y_0B_1/(2B_3)$$
$$\beta = B_1/2 + B_2/2 + B_1B_2/(2B_3) - Y_0^2/(2B_3)$$
$$\delta = B_1/2 + B_2/2 + B_1B_2/(2B_3) + Y_0^2/(2B_3)$$

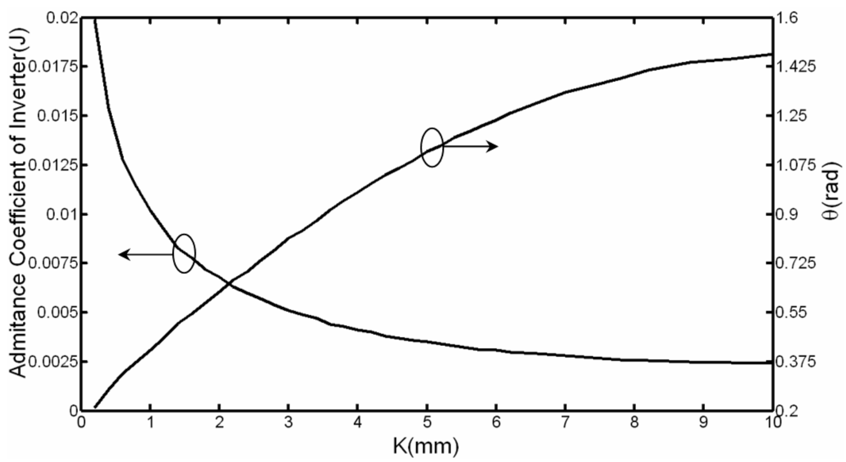


Figure 6. Admittance coefficient (J) and electrical length (θ_{rad}) versus k (mm).

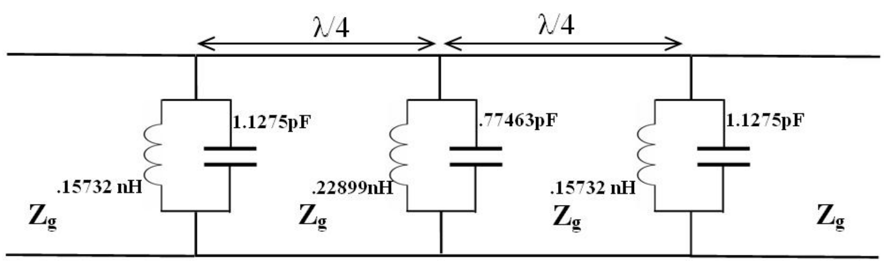
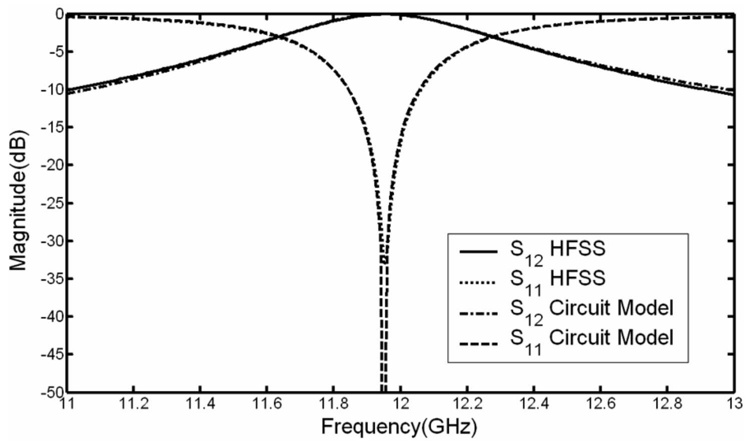


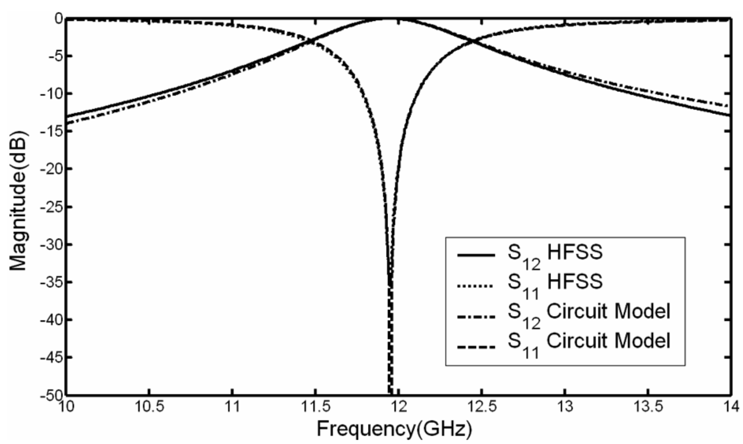
Figure 7. Circuit model of 3-section Chebyshev filter.

Consequently, by varying the iris slot width (k), the parameters of its circuit model (B_1, B_2, B_3) vary such that we obtain the inverter with required electrical length. Figure 6 shows the variations of electrical length of inverter (θ_{rad}) and its admittance coefficient (J) versus k (mm).

It must be noted that in the $\lambda/4$ transmission line inverter the admittance coefficient is the characteristic admittance of the transmission line, but in the modified one this parameter is determined for each electrical length using the formula (3) or results of Figure 6. Also, the whole designed inverter can be replaced by a T-circuit model as discussed in the next sections.



(a)



(b)

Figure 8. 3-section Chebyshev filter (a)- 1st and 3rd elements with $d_1 = 4.9\,\text{mm}$, $d_2 = 1\,\text{mm}$, $c_1 = 0.5\,\text{mm}$, $c_2 = 0.5\,\text{mm}$, $p = 1.4\,\text{mm}$ (b)- 2nd element with $d_1 = 4.3\,\text{mm}$, $d_2 = 0.95\,\text{mm}$, $c_1 = 0.84\,\text{mm}$, $c_2 = 0.84\,\text{mm}$, $p = 1.95\,\text{mm}$.

3. DESIGN AND REALIZATION OF FILTER

3.1. Design of Filter Using the $\lambda/4$ Transmission Line as Inverter

A bandpass filter with specifications defined in this paper, designed according to the method of [15], is shown in Figure 7.

The resonance circuits in Figure 7 are realized using CSRR structures and the inverters are realized by $\lambda/4$ waveguide sections. The S-parameters of the resonance circuits of theoretical model in Figure 7 and the S-parameters of their equivalent CSRR are shown in Figure 8.

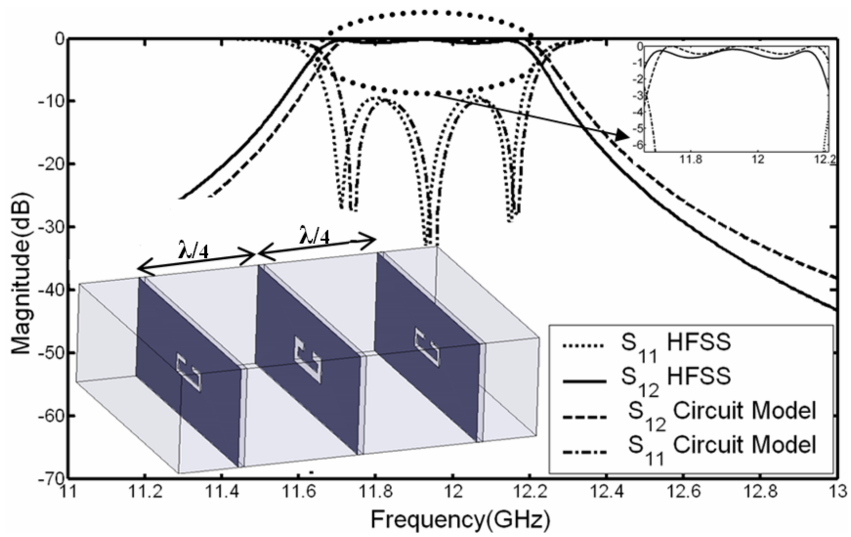


Figure 9. 3-sections Chebyshev filter.

Figure 9 shows the fina1 designed filter realized with CSRR structures and $\lambda/4$ transmission lines as inverters.

3.2. Miniaturization of the Filter

The first step in the miniaturization of the filter is using suitable resonance components that occupy small region in the waveguide, as discussed in Section 2.2. In the next step, the $\lambda/4$ transmission line inverter must be replaced by the new compact inverter introduced in the same section. In this situation because of using new inverter with new admittance coefficient (J), the resonance circuit associated with inverter must be modified suitably. In the designed filter of Figure 7, the second resonance frequency must be modified as follows.

Design of new inverter with $J = 0.0081$ leads to an inverter of $\lambda/12$ electrical length. This yields about 66% miniaturization compared to the filter with $\lambda/4$ transmission line inverters. Accordingly, the value of k must be about 1.48 mm (from Figure 6). In this case, the

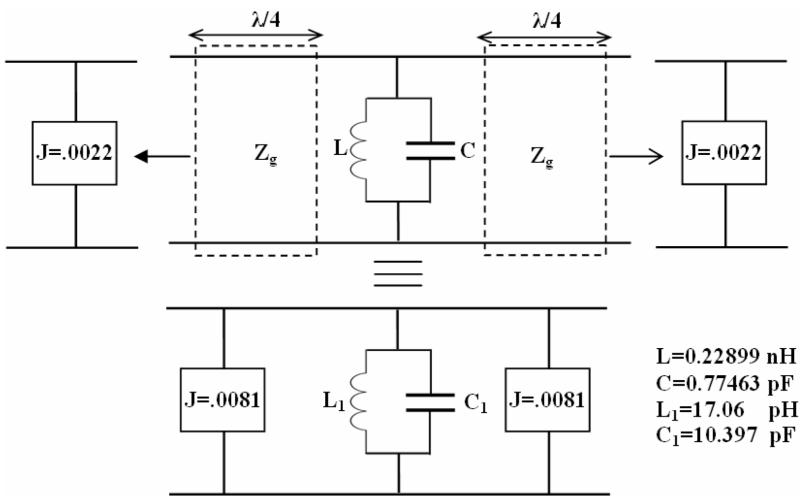


Figure 10. 2nd element of filter with $\lambda/4$ inverter equal to 2nd element of miniaturized filter with new inverter.

transmission matrix $[ABCD]$ of the iris will be:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.9739 & 4.3425j \\ 0.0075j & 0.9932 \end{bmatrix}. \tag{4}$$

This means that the parameters of circuit model of iris are as follows:

$$B_1 = 0.0016, \; B_2 = 0.006, \; B_3 = -0.2003.$$

Now, the circuit model of the inverter can be represented by a T-circuit model in the filter as depicted in Figure 11.

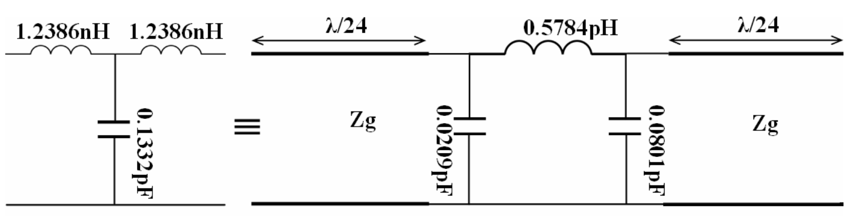


Figure 11. Circuit model admittance inverter with transmission line and final T-circuit model.

The final circuit model of the filter after using the new inverters and modification of resonance circuits is shown in Figure 12.

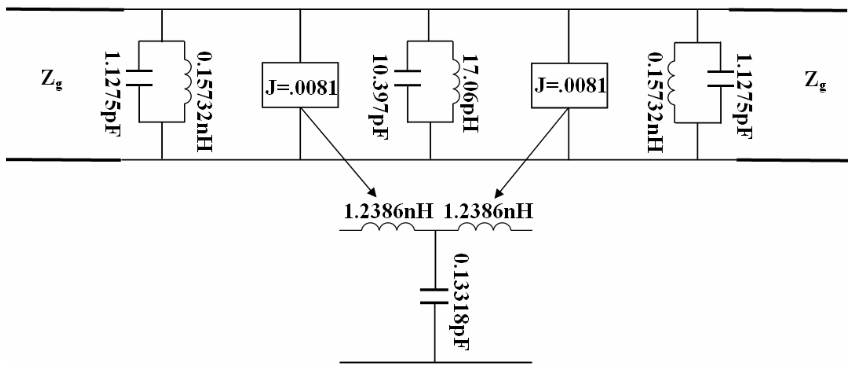


Figure 12. Circuit model of miniaturized 3-section Chebyshev filter without coupling.

An important point that must be noted is that, because of the large amount of coupling between the resonance elements and inverters, the result of circuit model is not compatible with the numerical simulation results. This is due to the compactness of the inverters and their proximity to the resonance elements. To solve this problem, a modification in the design is presented in the next section.

3.3. Modification of Miniaturized Designed Filter to Compensate for Coupling

The coupling between elements is represented in the circuit model shown in Figure 13.

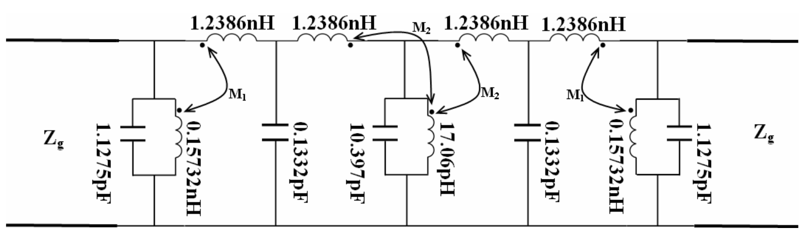


Figure 13. Miniaturized 3-sections Chebyshev filter with coupling.

This coupling affects the admittance coefficient of the inverters and also the parameters of the resonance circuits (resonance frequency and bandwidth). To compensate for this effect, the resonance elements and inverters must be changed suitably. In realization of the filter, the second resonator and the two adjacent inverters constitute one block of

Table 2. Changes of resonance frequency, resonance bandwidth and k parameter.

Element	f_0	BW	k
1st and 3rd elements	↑	↑	–
2nd element	↑	↓	–
Iris	–	–	↑

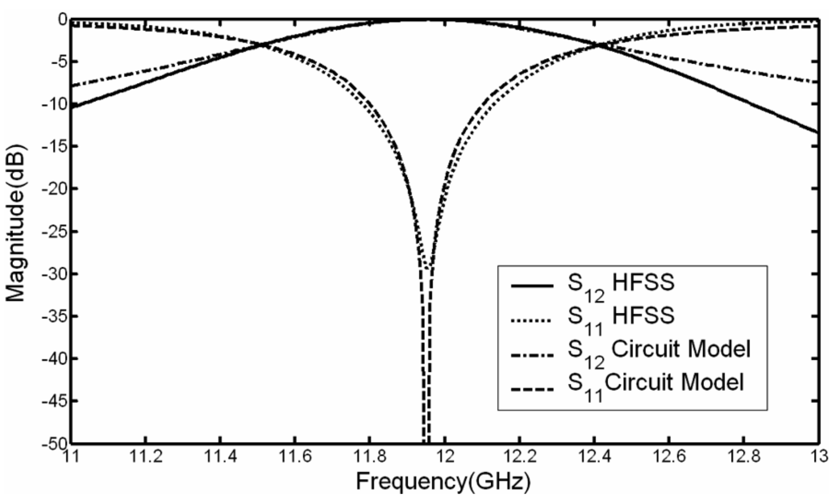


Figure 14. 2nd element of miniaturized filter.

the theoretical designed filter. Therefore, the combination is analyzed together. Table 2 shows the type of effects on different parameters of resonators and inverters in compensating for the coupling.

After exerting these variations, the theoretical and the simulated S-parameters of combination of the modified second resonator and two adjacent inverters are depicted in Figure 14.

Table 3 shows the amount of different parameters of different sections of compensated filter.

Finally, the simulation results of the compensated and uncompensated designed filters are depicted in Figure 15 which shows good agreement with the theoretical design.

Table 3. Amounts of parameters of miniaturized filter elements.

Element	D_1	d_2	c_1	c_2	P	k
1st and 3rd elements	3.75mm	1.09mm	1mm	1mm	2.1mm	-
2nd element	0.8mm	3.91mm	0.2mm	0.2mm	0.3mm	-
Iris	-	-	-	-	-	1.7mm

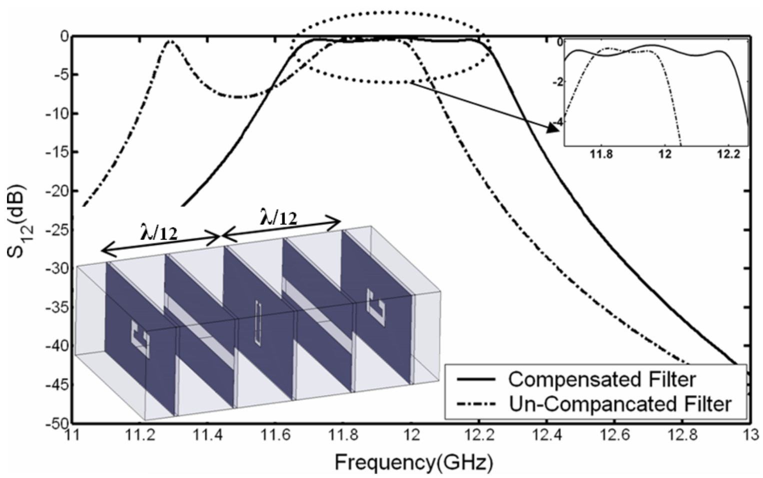


Figure 15. S_{12} of the miniaturized 3-section Chebyshev filter.

4. CONCLUSION

In this paper, the capability of rectangular CSRR elements to design waveguide bandpass filters and its miniaturization method have been demonstrated. It has been shown that a filter may be compacted by 66% using such elements. Also, the coupling between elements has been taken care of in the process of design.

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