*Progress In Electromagnetics Research B, Vol. 2, 73–82, 2008*

# EFFECT OF MULTIPLICATION REGION MOLE FRACTION ON CHARACTERISTICS OF $\mathbf{AL}_X\mathbf{GA}_{1-X}$AS-APDS IN THE LINEAR AND GEIGER

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**Abstract**—In this paper we introduce general numerical analysis for investigation the performance of avalanche photodiodes (APD) while we change the multiplication region mole fraction. We have found that the gain, breakdown voltage, and performance factor, at a given bias voltage, increase while the excess noise factor decreases through the decreases in fraction of Al in $\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$-APDs. For calculation the characteristics of $\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$-APDs we use the dead space multiplication theory (DSMT) and width independent ionization coefficient.

## 1. INTRODUCTION

APDs have an advantage over p-i-n detectors because of the internal gain that they provide [1]. For high bit rate, long haul fiber optic communications, the avalanche photodiode (APD) is frequently the photodetector of choice owing to its internal gain, which provides a sensitivity margin relative to p-i-n photodiodes. This gain, which has its origin in the statistical characteristics of the multiplication process, is accompanied by excess noise that arises from randomness in the coupled avalanching process of electrons and holes [2]. It is important to achieve high sensitivity in order to maximize the separation between optical repeaters and, thus, reduce the overall system cost. APDs can achieve sensitivity of 5 to 10 dB better than PINs, provided that the multiplication noise is low and the gain–bandwidth product of the APD is sufficiently high.

The multiplication region of an APD plays a critical role in determining the gain, the multiplication noise, and the gain–bandwidth product. A great deal of research has been devoted to reducing the multiplication noise of APDs. For reduction the excess noise that arises from the avalanche process, the multiplication region width should be reduced [3, 4]. APDs with thin avalanche regions also have the further advantage of a high gain-bandwidth product [5]. It has been demonstrated that the use of thin (less than $1\,\mu$m) APD multiplication regions serves to reduce excess noise [2]. Impact ionization is an important process in semiconductor devices operating at high electric fields, where it leads to avalanche multiplication. After an ionization event, a carrier needs to travel a certain distance, which is called the "dead space," before it can gain sufficient energy from the electric field to have a nonnegligible ionization probability [6]. This dead length can be ignored if it is small compared to the thickness of the multiplication region. On the other hand, when the thickness of the multiplication region is reduced to the point that it becomes comparable to a "few" dead space, the assumption of continuous ionization process fails [7]. With increasing electric field strength the distance over which a carrier must travel before its ionization coefficient comes into equilibrium with the electric field, becomes an increasingly significant fraction of the mean ionization path length. This is because $\alpha$ and $\beta$, which represent the inverse of the mean distance between successive ionization events, depend exponentially on inverse fields [8] and have been used to describe the multiplication process.

It is well known that for avalanche photodiodes with thin multiplication regions, the conventional carrier-multiplication theory [9] does not correctly predict the reduction in the noise characteristics. There has been a widespread research effort in the modeling of avalanche photodiodes (APDs) with thin multiplication layers that include the effect of dead space, whose impact considerable in the thin APDs [2–4, 7–10, 13]. A convenient and simple way to model the dead space is to assume that the density of impact ionization before the dead space is zero, after which it abruptly assumes a constant rate [10]. With this assumption, Hayat et al. [2, 10–12] formulated a dead-space-multiplication theory (DSMT) that permitted the gain, excess noise factor, breakdown probability, breakdown voltage, and gain probability distribution to be determined in the presence of dead space. Because the DSMT developed in [11] incorporates a nonuniform electric field, it can accommodate arbitrary history-dependent ionization coefficients.

As mention above for reduction the excess noise factor multiplication region width should be reduced. Unfortunately, at the high electric fields encountered in these thin multiplication regions, the tunneling current can be significant, and increasing the background shot noise. One way to overcome this problem is to use a wider bandgap material such as $\mathrm{Al}_x\mathrm{GaAs}_{1-x}$, with consequently reduced tunneling current [5]. Change of multiplication region mole fraction exchange the ionization coefficient parameters. Plimmer et al. [14] introduced the mole fraction dependence of $\alpha$ and $\beta$ in $\mathrm{Al}_x\mathrm{GaAs}_{1-x}$ .In this paper we considered the change of multiplication region mole fraction and used the width independent ionization coefficient introduced by Plimmer et al. [14] for providing the general numerical analysis to calculate the gain, breakdown probability, breakdown voltage, single photon quantum efficiency, performance factor, and excess noise factor of $\mathrm{Al}_x\mathrm{Ga}_{1-x}\mathrm{As}$ APDs. On the other hand the effect of multiplication region width while we change the multiplication region mole fraction has been study.

## 2. THEORETICAL BACKGROUND

Consider an APD with a multiplication region of width $w$. A parent photo-electron is injected into the multiplication region at $x = 0$. After traveling a random distance (in the $x$-direction), called the electron free-path distance, the electron impact ionizes resulting in a secondary electron and a hole. Upon ionization, the regenerated parent electron and the offspring electron continue to travel and may initiate further impact ionizations independently of each other. The offspring hole, on the other hand, travels in the $-x$-direction and impact ionizes after traveling a hole random free-path distance. The hole ionization results in a secondary hole and an electron. These newly created carriers proceed to generate their own offspring's, and so on.

This avalanche of ionization events continues until all carriers exit the multiplication region. Avalanche-multiplication process can take place only after an electron or hole has acquired sufficient kinetic energy to collide with the lattice and ionize another electron-hole pair. The smallest value of the ionizing particle kinetic energy that can accommodate this process is termed the ionization threshold energy, denoted $E_{th,e}$ and $E_{th,h}$ for the electron and hole, respectively. The minimum distance that a newly generated carrier must travel in order to acquire this threshold energy is termed the carrier dead space, and is denoted $d_e$ and $d_h$ for electrons and holes, respectively.

As pointed out by Okuto et al. [15], a carrier starting with near zero energy, relative to the band edge, will have an almost zero chance of having an ionizing collision until it has gained sufficient energy from the electric field to attain the necessary energy to permit impact ionization. Assuming the absence of phonon scattering, the presence of a uniform electric field in the multiplication region gives rise to a constant force so that

$$d_{e,h} = \frac{E_{th}}{q\varepsilon} \tag{1}$$

where $q$ is the electronic charge. A model for the electron and hole impact ionization coefficients of *enabled* carriers has been developed by Saleh et al. [2]. For the electrons, the model is given by:

$$\alpha(\varepsilon) = A \exp\left[-\left(\frac{\varepsilon_c}{\varepsilon}\right)^m\right] \tag{2}$$

A similar formula exists for the holes. This model has been shown to correctly predict the excess noise factors independently of the width of the multiplication layer. The gain statistics for double-carrier multiplication APDs, in the presence of dead space and a uniform or a nonuniform electric field, have been developed and reported in [2, 10–12]. The theory involves recurrence equations of certain intermediate random variables. In the case of electron injection at the edge of the multiplication region, the random gain of the APD is given by $G = (Z(0)+Y(0))/2$, which can be further reduced to $G = (Z(0)+1)/2$ since $Y(0) = 1$. The averages of $Z(x)$ and $Y(x)$, denoted by $z(x)$ and $y(x)$, respectively, obey the following set of coupled integral equations:

$$< Z(x) >= z(x) = [1 - H_e(W - x)] + \int_x^W [2z(\xi) + y(\xi)]h_e(\xi - x)d\xi \tag{3}$$

and

$$< Y(x) >= y(x) = [1 - H_h(x)] + \int_0^x [2y(\xi) + z(\xi)]h_h(x - \xi)d\xi \tag{4}$$

where $z(x) =< Z(x) >$ and $y(x) =< Y(x) >$ are the means of $Z(x)$ and $Y(x)$, respectively. Where

$$H_e(x) = \int_{-\infty}^x h_e(\xi)d\xi \tag{5}$$

and

$$H_h(x) = \int_{-\infty}^x h_h(\xi)d\xi \tag{6}$$

Here, $h_e(x)$ and $h_h(x)$ are the probability density functions (pdfs) of the random free-path lengths $X_e$ and $X_h$ of the electron and hole, respectively. In the dead space model, for example

$$h_e(x_e) = \begin{cases} 0, & x_e < d_e \\ \alpha e^{-\alpha(x_e-d_e)}, & x_e \geq d_e \end{cases} \tag{7}$$

and

$$h_h(x_h) = \begin{cases} 0, & x_h < d_h \\ \beta e^{-\beta(x_h-d_h)}, & x_h \geq d_h \end{cases} \tag{8}$$

where $\alpha$ and $\beta$ are the ionization rates for electrons and holes that have traveled beyond the dead space, respectively.

The mean gain and excess noise factor given by

$$< G >= 0.5(z(0) + 1) \tag{9}$$

and

$$F = \frac{< G^2 >}{< G >^2} = \frac{z_2(0) + 2z(0) + 1}{[z(0) + 1]^2} \tag{10}$$

Here, $z_2(0) =< z(0)^2 >$ and $y_2(0) =< y(0)^2 >$ are the second moments of $Z(x)$ and $Y(x)$, respectively. These quantities are given by

$$\begin{aligned} z_2(x) &= \left[1 - \int_0^{W-x} h_e(\xi)d\xi\right] \\ &\quad + \int_x^W \Big[2z_2(\xi)+y_2(\xi)+4z(\xi)y(\xi)+2z^2(\xi)\Big]\times h_e(\xi-x)d\xi, \end{aligned} \tag{11}$$

and

$$\begin{aligned} y_2(x) &= \left[1 - \int_0^{x} h_h(\xi)d\xi\right] \\ &\quad + \int_0^x \Big[2y_2(\xi)+z_2(\xi)+4z(\xi)y(\xi)+2y^2(\xi)\Big]\times h_h(x-\xi)d\xi, \end{aligned} \tag{12}$$

Equations (10) and (11) are valid only for $0 < x < W$. The Breakdown voltage is defined as the reverse-bias voltage across the multiplication region at which the mean gain becomes infinite. Let $P_Z(x)$ be defined as the probability that $Z(x)$ is finite, and similarly, let $P_Y(x) \equiv P\{Y(x) < \infty\}$. McIntyre [7] invoked the recurrence argument and characterized $P_Z$ and $P_Y$ through the following two nonlinear integral equations:

$$P_Z(x) = \int_{W-x}^{\infty} h_e(\xi)d\xi + \int_0^{W-x} P_Z^2(x+\xi)P_Y(x+\xi)h_e(\xi)d\xi \tag{13}$$

and

$$P_Y(x) = \int_x^\infty h_h(\xi)d\xi + \int_0^x P_Y^2(x-\xi)P_Z(x-\xi)h_h(\xi)d\xi \qquad (14)$$

The first of these equations states that if a cold electron is injected (created) at $x$, no breakdown occurs if it manages to escape with no ionizations or if it does have a first ionization at $\xi$ with a probability $P_Y(x+\xi)d\xi$, all three resulting cold carriers (two electrons and one hole), escape without initiating a breakdown. The second equation is an equivalent expression for holes. The recurrence equations presented in this section can be solved by means of a simple iterative numerical technique. We use these coupled equations for calculation the characteristics of $Al_xGa_{1-x}As$-APDs when we change the multiplication region mole fraction.

## 3. SIMULATION AND RESULTS

To see the effects of change of multiplication region mole fraction on the characteristics of avalanche photodiodes we use the nonlocalized ionization coefficient model (width independent ionization coefficient) derived by Plimmer et al. [14] and DSMT introduce in the previous section to characterize the behavior of the $Al_xGa_{1-x}As$-APDs. In our calculations, we assumed a constant electric field profile within the multiplication region and used the simple approximation $V = \varepsilon W$ for the reverse bias voltage. In Fig. 1 we introduce the mean gain

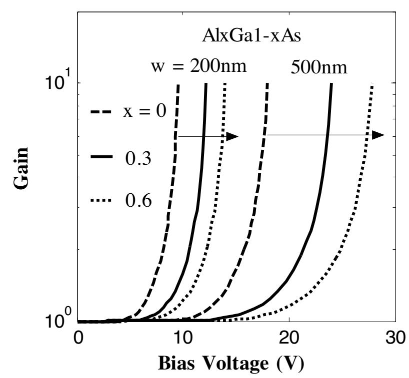


**Figure 1.** Logarithmic plots of $Al_xGa_{1-x}As$-APDs gain versus bias voltage as a function of multiplication region mole fraction for different multiplication region width. The dashed lines show the calculated gain for GaAs-APDs, the solid lines were calculated for $Al_{0.3}Ga_{0.7}As$-APDs and the dotted lines are the values calculated for $Al_{0.6}Ga_{0.4}As$-APDs.

versus bias voltage for different multiplication region mole fraction as a function of multiplication region width, as shown in Fig. 1 by increasing the fraction of Al in the multiplication region decreases the mean gain for the same applied voltage on the other hand while decreases the thickness of multiplication region width impact of change of multiplication region mole fraction reduces. We continue by demonstrating the predicted excess noise factor for the thin homojunction $Al_xGa_{1-x}As$-APDs.

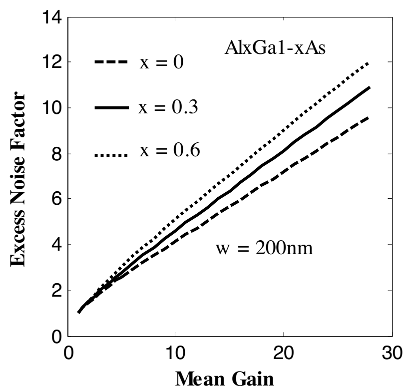


**Figure 2.** Predicted excess noise factor versus mean gain as a function of multiplication region mole fraction for $Al_xGa_{1-x}As$-APDs.

In Fig. 2 we introduce excess noise factor versus mean gain for different multiplication region mole fraction as shown in Fig. 2 by increasing the fraction of Al in multiplication region increases the excess noise factor for the same applied voltage. Note that the breakdown voltage is the voltage corresponding to the point when the breakdown probability begins to exceed zero. In Fig. 3, with this assumption we introduce the breakdown probability versus bias voltage for different multiplication region mole fraction. We also note that the calculated values of the breakdown probability near breakdown are sensitive to precision error (resulting from discretizing the recurrence equations) [12].

In Fig. 4 we introduce the breakdown voltage versus multiplication region width in the wide range of multiplication region width as shown in this figure by increasing the multiplication region width increases the breakdown voltage. Change of multiplication region mole fraction also affects the performance factor of APDs. For this reason in Fig. 5 we compare the performance factor of $Al_xGa_{1-x}As$-APDs as a function of mean gain. For calculation of this characteristic we assumed the fix value of circuit to noise parameter.

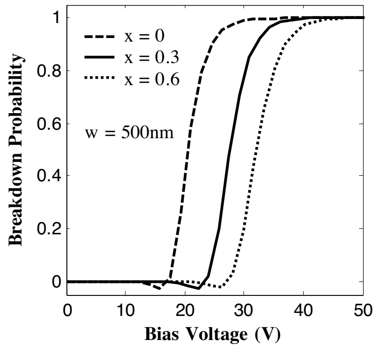


**Figure 3.** Predicted breakdown probability versus bias voltage for different multiplication region mole fraction in $Al_xGa_{1-x}As$-APDs.

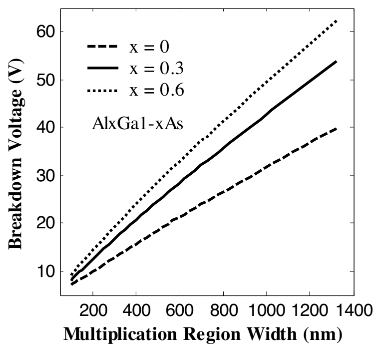


**Figure 4.** Predicted breakdown voltages as a function of multiplication region mole fraction in the wide range of multiplication region width for $Al_xGa_{1-x}As$-APDs. The thicknesses of multiplication region of APDs are marked on their curves.

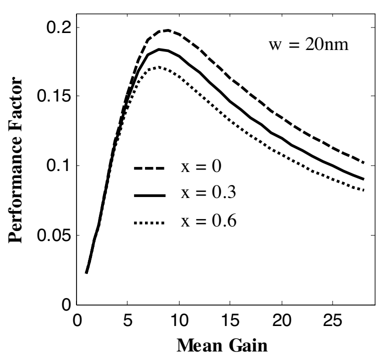


**Figure 5.** Predicted performance factor versus mean gain as a function of multiplication region mole fraction for $Al_xGa_{1-x}As$-APDs.

## 4. CONCLUSIONS

We have examined the mean gain, excess noise factor, breakdown probability, breakdown voltage, and performance factor of homojunction APD when we considered the change of multiplication region mole fraction for different multiplication region width. We have found that the change of multiplication region mole fraction strongly affected by change the multiplication region width and its effect vanished through the reduction of multiplication region width. Our calculations also showed that when we decrease the fraction of Al in $\mathrm{Al}_x\mathrm{Ga}_{1-x}$As-APDs, the performance of device increases but the most important trade off in $\mathrm{Al}_x\mathrm{Ga}_{1-x}$As-APDs is between detection wavelength and performance of APD.

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