A COMBINATION OF TIME DOMAIN FINITE ELEMENT-BOUNDARY INTEGRAL WITH TIME DOMAIN PHYSICAL OPTICS FOR CALCULATION OF ELECTROMAGNETIC SCATTERING OF 3-D STRUCTURES

F. Faghihi and H. Heydari

Center of Excellence for Power System Automation and Operation
Electrical Engineering Department
Iran University of Science and Technology
Tehran, Iran

Abstract—This paper presents a hybrid numerical approach combining an improved Time Domain Finite Element-Boundary Integral (FE-BI) method with Time Domain Physical Optics (TDPO) for calculations of electromagnetic scattering of 3-D combinative-complex objects. For complex-combined objects containing a small size and large size parts, using TDPO is an appropriate approach for coupling between two regions. Therefore, our technique calculates the objects complexity with the help of FE-BI and the combinatory structures by using of the TDPO. The hybridization algorithm for restrictive object is implemented and the numerical results validate the superiority of the proposed algorithm via realistic electromagnetic applications.

1. INTRODUCTION

The accurate and efficient evaluation of the scattering from a large and complex object is of great interest in science and engineering [1–6]. The finite element hybridized with the boundary integral equation method is approved to be a powerful numerical method for such complex problem simulation [3–5]. The hybrid method remains the advantages both of the finite element method (FEM) and the boundary integral (BI) method [7, 8], and has been widely used and well developed in frequency domain [8–10]. In recent years, as more interests have been focused on time domain method [11, 12]; the FE-BI method in time domain also has received increasing attention

[13, 14]. Furthermore, the development of high power microwave antenna, aviation and space techniques has motivated the interest in electromagnetic scattering problems involving combinative objects [15, 16]. The combinative objects are composed by two separate parts: one is a Small-Size (SS) structure and the other is a Large-Size (LS) part with respect to the wavelength of interest, such as line surface structure objects, a reflector antenna and a satellite with large wingspan. Generally, neither rigorous numerical schemes nor asymptotic techniques is straightforwardly be implemented in evaluating the time domain scattering for combinative objects precisely and professionally. Therefore the hybrid algorithm combining numerical scheme with asymptotic technique is invoked in dealing with this class of problems. For complex-combined objects including a small size and large size parts, using TDPO [17–19] is suitable technique for coupling between two regions. This paper proposes a time-domain hybrid approach that combines FE-BI method with TDPO for the scattering problem by Complex and combinative structures. The FE-BI and TDPO are taken separately to treat the SS structure and LS part and then it is implemented by two numerical examples based on basic issues in electromagnetic applications and for verification a comparison is made with far field to near field transformation.

2. FE-BI OVERVIEW FOR ELECTROMAGNETIC SCATTERING FROM COMPLEX OBJECTS

2.1. Principles of FE-BI For Solving a Typical Problem of Electromagnetic Scattering

Let us consider a typical problem of electromagnetic scattering by an arbitrary formed and inhomogeneous object. We initiate an artificial surface S to terminate the calculation domain for finite elements, and suppose that the area outside S is free space. Inside S, the field E satisfies [7, 20, 21]

$$\nabla \times \left[\mu_r^{-1} \nabla \times E(r,t)\right] + \mu_0 \varepsilon \partial_t^2 E(r,t) + \sigma \mu_0 \partial_t E(r,t) = 0 \quad r \in V \quad (1)$$

where V denotes the volume enclosed by S. For a exclusive solution, it is essential to impose a boundary condition on S. To be free of interior resonance, we select a boundary condition like the combined field integral equation (CFIE) [3, 14, 21] used in frequency domain

$$n \times \left[\mu_r^{-1} \nabla \times E(r,t)\right] + c^{-1} n \times \partial_t [n \times E(r,t)]\big|_{S^-}$$
$$= -n \times \mu_0 \partial_t H(r,t) + c^{-1} n \times \partial_t [n \times E(r,t)]\big|_{S^+} \quad (2)$$

where n refer to the outward unit vector normal to S, S^- refer to the surface just interior to S, and S^+ refer to the surface just exterior to S.

The solution to the boundary-value problem defined by (1) and (2) can be obtained by seeking the stationary point of the functional [4, 5, 13, 14]

$$
\begin{aligned}
F\left[E(r,t)\right] = \ & \frac{1}{2}\iiint_V \{\mu_r^{-1}\left[\nabla \times E(r,t)\right]\cdot\left[\nabla \times E(r,t)\right] \\
& +\mu_0\varepsilon\partial_t^2 E(r,t)\cdot E(r,t) + \sigma\mu_0\partial_t E(r,t)\cdot E(r,t)\}dV \\
& +\frac{1}{2}\iint_S \{c^{-1}\partial_t\left[n \times E(r,t)\right]\cdot\left[n \times E(r,t)\right] \\
& +2E(r,t)\cdot V_S(r,t)\}dS
\end{aligned}
\tag{3}
$$

where $V_S(r,t)$ represents the right side of (2). That is,

$$
V_s(r,t) = -n \times \mu_0\partial_t H(r,t) + c^{-1}n \times \partial_t\left[n \times E(r,t)\right]
\tag{4}
$$

The electric and magnetic field in (4) are given as

$$
E(r,t)=E^i(r,t)+\int_{-\infty}^t c^2\nabla\nabla\cdot A(r,\tau)d\tau-\partial_t A(r,t)-\varepsilon^{-1}\nabla\times F(r,t)
\tag{5}
$$

$$
H(r,t)=H^i(r,t)+\int_{-\infty}^t c^2\nabla\nabla\cdot F(r,\tau)d\tau-\partial_t F(r,t)+\mu^{-1}\nabla\times A(r,t)
\tag{6}
$$

where

$$
A(r,t) = \mu_0\iint_S J(r',t) * g(|r-r'|,t)dS'
\tag{7}
$$

$$
F(r,t) = \varepsilon_0\iint_S M(r',t) * g(|r-r'|,t)dS'
\tag{8}
$$

In the above equations, "$*$" stands for the convolution, $g(|r-r'|,t)$ denotes the 3D free-space Green's function, and J and M are the equivalent electric and magnetic currents, defined on surface S, respectively as

$$
M(r',t) = E(r',t) \times n \qquad\qquad r' \in S
\tag{9}
$$

$$
J(r',t) = -n \times \left\{\mu^{-1}\partial_t^{-1}\left[\nabla' \times E(r',t)\right]\right\} \quad r' \in S
\tag{10}
$$

The notation ∂_t^{-1} in (10) denotes temporal integration.

2.2. Singular Integrals Behavior Using Whitney Form for Complex Objects Computation

We discretize volume V by tetrahedral, and use Whitney 1-form basis functions [22] to approximate the electric field in (3)

$$E(r,t) = \sum_{i=1}^{N} \tilde{e}_i(t) W_i(r) \tag{11}$$

where N denotes the total number of edges, and W_i denotes Whitney 1-form basis function and $\tilde{e}_i(t)$ refers to its coefficient.

According to the finite elements procedure, the space discretization of (3) results in a second-order differential system [3, 4, 13]

$$T\frac{d^2\tilde{e}(t)}{dt^2} + (R+Q)\frac{d\tilde{e}(t)}{dt} + S\tilde{e}(t) + v(t) = 0 \tag{12}$$

The calculation of matrices T, R, Q, and S is straightforward [23], and is not given here.

Finally, the calculation of column vector $v(t)$ must be performed. Its elements are given by

$$
\begin{aligned}
v_i^m(t) &= \iint_{S^m} dS W_i^m(r) \cdot \{c^{-1} n \times n \times [\partial_t E^i(r,t) + c^2 \nabla\nabla \cdot A(r,t) \\
&\quad - \partial_t^2 A(r,t) - \varepsilon^{-1}\partial_t \nabla \times F(r,t)] - \mu_0 n \times [\partial_t H^i(r,t) \\
&\quad + c^2 \nabla\nabla \cdot F(r,t) - \partial_t^2 F(r,t) + \mu^{-1}\partial_t \nabla \times A(r,t)]\}
\end{aligned}
\tag{13}
$$

$v_i^m(t)$ can be calculated by numerical method e.g., by Gaussian quadrature. Applying a traditional central difference scheme to (12) yields

$$A\tilde{e}^{n+1} = (2T - \Delta t^2 S)\tilde{e}^n + [0.5\Delta t(R+Q) - T]\tilde{e}^{n-1} - \Delta t^2 v^n \tag{14}$$

where $\tilde{e}^n = \tilde{e}(n\Delta t)$, $v^n = v(n\Delta t)$, and $A = T + 0.5\Delta t(R+Q)$.

It is worth mentioning out here that by adopting the central difference scheme, the required electric and magnetic currents on the surface S for the computation of column vector v^n at each time step are known quantities, and so the sparseness and symmetry of finite element matrices are well preserved. Specifically, while it reduces the calculation domain, the proposed method using one auxiliary boundary will not contaminate the finite element matrices [3, 13, 14].

Apparently, to update electric field with (14), we have to solve a matrix equation at each time step. However, matrix A is time invariant, and is highly sparse. Hence, the matrix equation can be

solved efficiently. Generally, a direct solver is suitable for small- and medium-sized problems, and matrix A has to be factorized only once. Whereas an iterative solver is more attractive for large-scale problems, and the preconditioner also has to be constructed only once.

3. TDPO AS A USEFUL METHOD FOR COMBINATIVE STRUCTURES

Physical optics is a high frequency approximation technique. The Physical Optics (PO) is then used to study the scattering on the ground plane. It is based on an exact formulation of the diffraction problem well known as the Chu-Stratton equation [17]. Three simplifying assumptions are carried out to reduce the vector integral equation to a simple definite integral over the scatterer surface. It is clearly assumed that:

(a): the surface field over the shadowed portion of the body is *zero,*

(b): the observation point is removed far from the object in term of wavelength and scattering object dimensions,

(c): the dimensions of curvature of the scatterer are large compared to the wavelength.

For a perfectly conducting body, the far scattering field in time-domain is given [17, 18]:

$$E(r,t) = (Z_0/4\pi rc) \times \iint_S' \hat{r} \times [\hat{r} \times \frac{\partial}{\partial t} J_S(r', t - \tau)]dS' \quad (15)$$

$$H(r,t) = -(1/4\pi rc) \times \iint_S' \hat{r} \times \frac{\partial}{\partial t} J_S(r', t - \tau)dS' \quad (16)$$

where, τ is the time retardation.

Surface-current density distribution, J_S is written as

$$J_S(r',t) = \begin{cases} 2\widehat{n'} \times h^{inc}(r',t) & \text{in the lit region} \\ \\ 0 & \text{otherwise} \end{cases} \quad (17)$$

where h^{inc} is the incident magnetic field. The scattered field can be determined by Equations (15) and (16) if the magnetic field incident on the scatterer is known.

The FE-BI method has been demonstrated to be a precise and professional method to simulate the interaction of electromagnetic waves with all kinds of obstacles, including the target of complex configuration [3, 9, 13]. Considering the configuration of combinative

objects, the computation domain is firstly split into FE-BI region
and TDPO region, enclosing the SS and LS part, respectively. The
proposed method uses the two kinds of approach in considering that
the total radiated field $e(r, t)$ can be divided into two terms:

$$e(r, t) = e_i(r, t) + e_d(r, t) \tag{18}$$

The first term $e_i(r, t)$, represents the far field directly radiated
by the complex element. It can be simply calculated from the FE-BI
algorithm. The second term $e_d(r, t)$ represents the far field scattered
by the metallic plane. It can be determined thanks to the TDPO
terminology if the magnetic field incident on the scatterer h^{inc}, is
known.

The far field can be divided into several terms as follows [18, 19]:
The dominant technique of the FE-BI/TDPO hybrid approach then
consists in the interaction between the two regions. First, we consider
the influence of FE-BI region onto TDPO region, as shown in Fig. 1,
providing the primary scattered field by SS complex configuration has
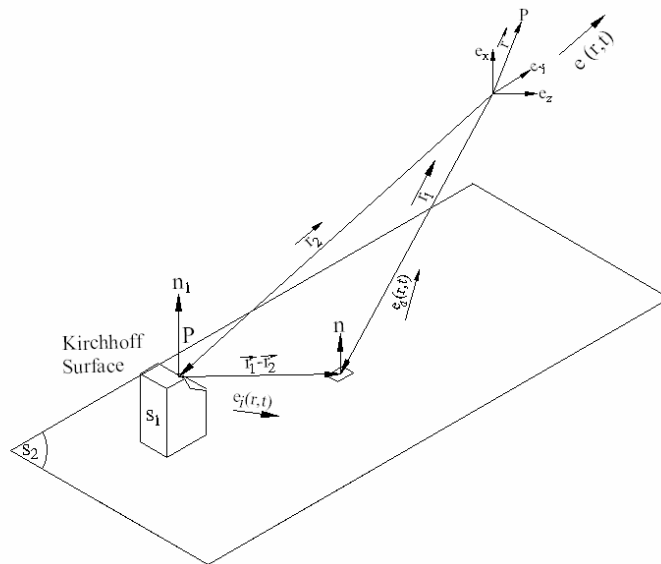been obtained by using FE-BI method.

Figure 1. FE-BI/TDPO hybrid method for field calculation from
complex objects.

In order to find the illuminating field onto LS part in TDPO
region from SS configuration in FE-BI region, the near-to-near field

extrapolation procedure in FE-BI is invoked, because FE-BI region is close to TDPO region.

Now, we present an algorithm of the proposed method as bellow:

1- Applying initial value for the incident field

2- "A" calculation: computing the electric and magnetic field from complex objects with FE-BI

3- "B" calculation: computing the electric and magnetic field for LS part with TDPO

4- "C" calculation: computing the electric and magnetic field in TDPO region using results of primary scattering field from "A" calculation

5- "D" calculation: computing the electric and magnetic field in FE-BI region using results of primary scattering field from "B" calculation

6- Final result: scattered field in observation point for far area is obtained from the primary scattering results ("A" and "B" calculation) and secondary scattering results ("C" and "D" calculation).

This algorithm is performed for each time step.

It is evident that the LS part in TDOP region is only a transfer stage, and no additional computer memory is required.

4. COMPUTATIONAL RESULTS

In this section two numerical examples which refer to fundamental problems in electromagnetic applications are given to demonstrate the validity of the proposed method and a comparison is made with the results of far field to near field transformation which is presented in [24].

Example 1- To determine the radiation characteristics of a monopole antenna mounted on a large but finite size ground plane has been considered (Fig. 2). The $\lambda/4$ antenna is placed on the center of a plane of dimension $5\lambda \times 5\lambda$ and is excited on their base. The FE-BI volume and the ground plane surface are discretized in $\lambda/30$. Furthermore, the radiation characteristics of the same $\lambda/4$ antenna mounted on an infinite plane is performed. It illustrates the effects of the finite scatterer dimensions that are well taken into account by the proposed method for $a = 5\lambda$, $f = 1\,\text{GHz}$.

The results given by the proposed method are compared with the results based on near-field to far-field transformation method [24] and indicate a good agreement which is shown in Fig. 3.
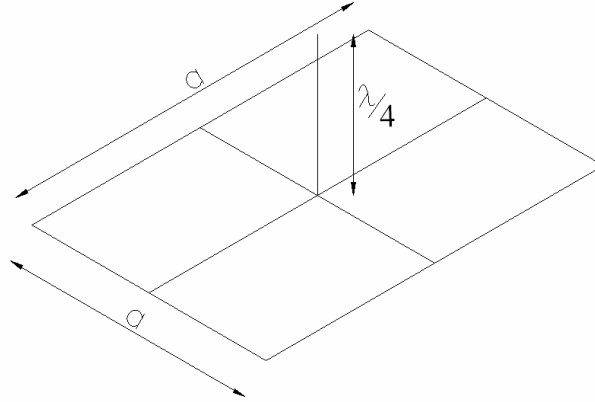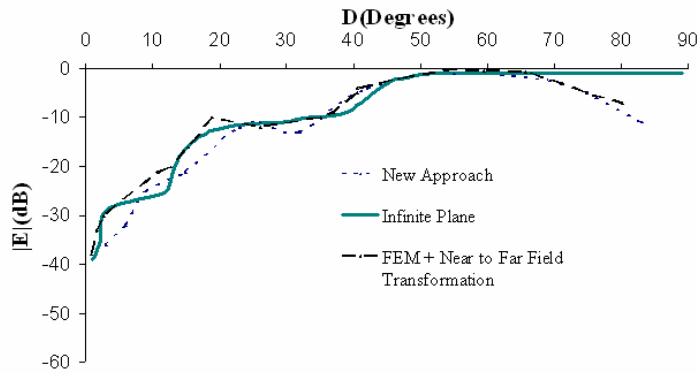
Figure 2. Monopole above finite square ground plane.

Figure 3. Amplitude pattern for a monopole antenna.

Example 2- We consider another example of the backscattering by combinative objects composed by a perfect electric conducting (PEC) cube and plate [19, 25–28], as shown in Fig. 4. A modulated Gaussian pulse with frequency ranging from 200 MHz to 300 MHz, and pulse width 30 ns excites the combinative complex object. The incident wave travels in the xoz plane with $\theta = 45°$ with its electric field parallel to the y-axis. The backscattering for co-polarization is to be established. First, we consider the primary scattering coming from the PEC cube set as an SS structure, and plate as the LS part, respectively. The backscattered waveform shown in Fig. 5, is based on the calculation of the proposed method through the identification between FE-BI and TDPO regions for the combinative object. The results obtained by

the proposed method are compared with the results based on near-field to far-field transformation method [24] and indicates an excellent conformity as shown in Fig. 5.
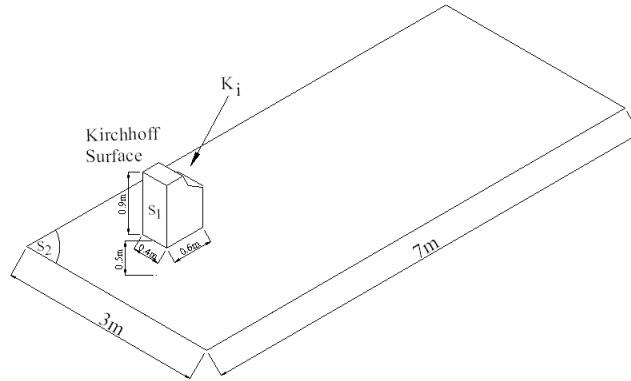

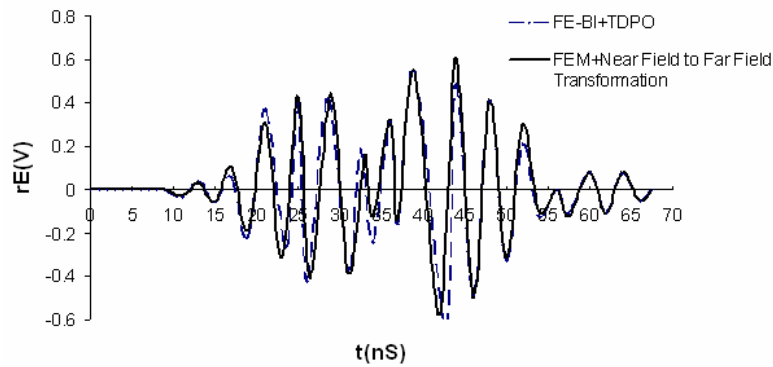**Figure 4.** Combinative object composed by cube and plate.

Figure 5. Backscattered waveform in time domain for co-polarization.

5. CONCLUSIONS

A time-domain hybrid approach that combines the FE-BI with TDPO has been developed. The approach can be applied to an analysis of electromagnetic scattering by complex and combinative objects having both SS and LS. The first-order scattering field by LS part in TDPO region is considered as the illuminating field on the LS structure in FE-BI region, when analyzing the coupling of TDPO to FE-BI region.

The illuminating wave from LS part to SS structure can be introduced through the connection boundary in FE-BI region. Computational results for both elementary structure and complex configuration validate the superiority of the proposed algorithm via realistic electromagnetic applications. Also, computational comparisons are made with the method based on near-field to far-field transformation and recently proposed method.

It is worth mentioning that there is another approach for validating the proposed method of this work that can be performed by field evaluation in a closer area.

ACKNOWLEDGMENT

REFERENCES

1. Rokhlin, V., "Rapid solution of integral equations of scattering theory in two dimensions," *J. Comput. Phys.*, Vol. 86, 414–439, 1990.

2. Wei, S., Z. Huang, X. Wu, and M. Chen, "Application of the symplectic finite-difference time-domain scheme to electromagnetic simulation," *J. Comput. Phys.*, Vol. 225, 33–50, 2007.

3. Jian, L. and J. M. Jin, "A highly effective preconditioner for solving the finite element-boundary integral matrix equation of 3-D scattering," *IEEE Trans. Antennas Propagat.*, Vol. 50, 1212–1221, 2002.

4. Liu, J. and J. M. Jin, "A novel hybridization of higher order finite element and boundary integral methods for electromagnetic scattering and radiation problems," *IEEE Trans. Antennas Propagat.*, Vol. 49, 1794–1806, 2001.

5. Qiu, Z. J., J. D. Xu, G. Wei, and X. Y. Hou, "An improved time domain finite element-boundary integral scheme for electromagnetic scattering from 3-D objects," *Progress In Electromagnetics Research*, PIER 75, 119–135, 2007.

6. Kashdan, E. and E. Turkel, "High-order accurate modeling of electromagnetic wave propagation across media — Grid conforming bodies," *J. Comput. Phys.*, Vol. 218, 816–835, 2006.

7. Jin, J. M., *The Finite Element Method in Electromagnetic*, 2nd edition, Wiley, New York, 2002.

8. Qiu, Z. J., X. Y. Hou, X. Li, and J. D. Xu, "On the condition number of matrices from various hybrid vector FEMBEM formulations for 3-D scattering," *J. Electromagn. Waves Appl.*, Vol. 20, 1797–1806, 2006.

9. Botha, M. M. and J. M. Jin, "Adaptive finite element-boundary integral analysis for electromagnetic fields in 3-D," *IEEE Trans. Antennas Propagat.*, Vol. 53, 1710–1720, 2005.

10. Paulsen, K. D., D. R. Lynch, and J. W. Strobehn, "Three dimensional finite, boundary, and hybrid element solutions of Maxwell equations for lossy dielectric media," *IEEE Trans. Microwave Theory Tech.*, Vol. 36, 682–693, 1988.

11. Gong, Z. and G. Q. Zhu, "FDTD analysis of an anisotropically coated missile," *Progress In Electromagnetics Research*, PIER 64, 69–80, 2006.

12. Remis, R. F., "On the stability of the finite-difference time-domain method," *J. of Computational Physics*, Vol. 163, 249–261, 2000.

13. Jiao, D., A. A. Ergin, B. Shanker, E. Michielssen, and J. M. Jin, "A fast higher-order time-domain finite element-boundary integral method for 3-D electromagnetic scattering analysis," *IEEE Trans. Antennas Propagat.*, Vol. 50, 1192–1202, 2002.

14. Cowen Mc, A., A. J. Radcliffe, and M. S. Towers, "Time-domain modeling of scattering from arbitrary cylinders in two dimensions using a hybrid finite-element and integral equation method," *IEEE Trans. Magn.*, Vol. 39, 1227–1229, 2003.

15. Wang, Y., K. C. Sujeet, and S. N. Safieddin, "FDTD/raytracing analysis method for wave penetration through inhomogeneous walls," *IEEE Trans. on Antennas and Propagat.*, Vol. 50, 1598–1604, 2002.

16. Nie, X. C., Y. B. Gan, N. Yuan, C. F. Wang, and L. W. Li, "An efficient hybrid method for analysis of slot arrays enclosed by a large radome," *J. of Electromagn. Wave and Appl.*, Vol. 20, 249–264, 2006.

17. Sun, E. Y. and W. V. T. Rusch, "Time-domain physical-optics," *IEEE Trans. on Antennas and Propagat.*, Vol. 42, 9–15, 1994.

18. Beillard, B., L. Andrieu, Y. Chevalier, and B. Jecko, "Technique combining the finite difference time domain and the uniform

theory of diffraction," *Microwave and Optical Technology Letters*, Vol. 16, 10–16, 1997.

19. Yang, L. X., D. B. Ge, and B. Wei, "FDTD/TDPO hybrid approach for analysis of the EM scattering of combinative objects," *Progress In Electromagnetics Research*, PIER 76, 275–284, 2007.

20. Sadiku and Matthew, *Numerical Techniques in Electromagnetic*, 2nd edition, CRC Press, 2001.

21. Rao, S. M. and D. R. Wilton, "E-field, H-field, and combined field solution for arbitrarily shaped three-dimensional dielectric bodies," *Electromagn.*, Vol. 10, 407–421, 1990.

22. Bossavit, A., "Whitney forms: A class of finite elements for three-dimensional computation in electromagnetism," *IEE Proc.*, Vol. 135, 493–500, 1988.

23. Christie, I., D. F. Griffiths, A. R. Mitchell, and O. C. Zienkiewicz, "Finite element methods for second order differential equations with significant first derivatives," *International Journal for Numerical Methods in Engineering*, Vol. 10, 1389–1396, 1976.

24. Costanzo, S. and G. Di Massa, "Near-field to far-field transformation with planar spiral scanning," *Progress In Electromagnetics Research*, PIER 73, 49–59, 2007.

25. Pingenot, J., R. N. Rieben, D. A. White, and D. G. Dudley, "Full wave analysis of RF signal attenuation in a rough surface cave using a higher order time domain vector finite element method," *J. Electromagn. Waves Appl.*, Vol. 20, 1695–1705, 2006.

26. Zhang, Y. J. and E. P. Li, "Scattering of three-dimensional chiral objects above a perfect conducting plane by hybrid finite element method," *J. Electromagn. Waves Appl.*, Vol. 19, 1535–1546, 2005.

27. Chen, K.-S., A. K. Fung, J. C. Shi, and H.-W. Lee, "Interpretation of backscattering mechanisms from non-Gaussian correlated randomly rough surfaces," *J. Electromagn. Waves Appl.*, Vol. 20, No. 1, 105–118, 2006.

28. Fung, A. K. and N. C. Kuo, "Backscattering from multi-scale and exponentially correlated surfaces," *J. Electromagn. Waves Appl.*, Vol. 20, No. 1, 3–11, 2006.