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PATTERN SYNTHESIS OF CYLINDRICAL CONFORMAL ARRAY BY THE MODIFIED PARTICLE SWARM OPTIMIZATION ALGORITHM

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Abstract—In order to overcome drawbacks of standard particle swarm optimization (PSO) algorithm, such as prematurity and easily trapping in local optimum, a modified PSO algorithm which adopts a global best perturbation, is used to optimize the pattern of cylindrical conformal antenna array for sidelobe level (SLL) suppression and null control in certain directions. The convergence speed and accuracy of the algorithm are improved. Compared with genetic algorithm and simulated annealing, The PSO algorithm is much easier to understand and implement. Firstlypattern formula of conformal array is provided, then, the standard and modified PSO algorithm are introduced, at last, application examples and simulation results are presented. The results show that the Modified PSO algorithm is an effective and efficient method to solve multi-dimension and nonlinear problem.

1. INTRODUCTION

Conformal antenna arrays with low profile have widest application prospect for low RCS and small effects on aerodynamic performance of the carrier. Because of its curved structure and conformal to the carrier, the synthesis problem of the type of antenna array is multidimensional and nonlinear problem, which is more difficult than the synthesis of linear and planar array. There are a wide variety of techniques that have been developed for the synthesis of linear and planar arrays [1–15]. The more complicated problem of synthesizing radiation patterns of conformal antennas has also been considered. The least-squares method is used to synthesize low sidelobe beams from conformal array consisting of few elements and large radius of curvature [16]. An iterative full polarimetric least-square synthesis technique was presented for the optimization of the element excitations of an active conformal array antenna [17]. [18] uses an aperture projection method with mutual coupling compensation to synthesis low sidelobe pattern of the conformal array. [19] uses the alternating projection method for synthesizing a low sidelobe pattern for a conformal array of 116 microstrip patch elements wrapped around a representative aircraft wing profile. An efficient algorithm based on adaptive array method is used to synthesis arbitrary array pattern [20, 21]. In addition, a simulated annealing technique previously developed for circular arc arrays is presented for the synthesis for three-dimensional conformal arrays in [22]. Based on the computationally efficient domain decomposition /reciprocity procedure, Allard et al. apply genetic algorithm to pattern synthesis of conformal array [23].

In the paper, a modified PSO algorithm is used to optimize the pattern of cylindrical conformal antenna array. That is to say, in the case of the known geometry distribution of array element, excitation amplitude of every array element is optimized to realize desired average SLL in certain regions and null control in the prescribed direction.

2. PATTERN FORMULA OF CONFORMAL ARRAY

For conformal antenna array with arbitrary geometry distribution, its far field pattern formula may be expressed as

$$F(\theta,\phi)=\sum_{n=1}^{N}A_n e^{j\beta_n}\boldsymbol{E}_n(\theta_n,\phi_n)e^{ik\boldsymbol{R_n}\cdot\boldsymbol{e_r}} \tag{1}$$

where $\boldsymbol{E}_n(\theta_n,\ \phi_n)$, A_n, β_n and $\boldsymbol{R}_n$ are the element pattern, the excitation amplitude, the excitation phase and position vector of nth element respectively.

$\boldsymbol{R}_n$ and $\boldsymbol{e}_r$ are expressed as follow

$$\boldsymbol{R}_n = \vec{\boldsymbol{e}}_x x(n)+\vec{\boldsymbol{e}}_y y(n)+\vec{\boldsymbol{e}}_z z(n) \tag{2}$$

$$\boldsymbol{e}_r = \vec{\boldsymbol{e}}_x \sin\theta\cos\phi+\vec{\boldsymbol{e}}_y \sin\theta\sin\phi+\vec{\boldsymbol{e}}_z\cos\theta \tag{3}$$

It is assumed that $(\theta_0,\ \phi_0)$ is the desired steering angle, So then,

$$\beta_n=-k\boldsymbol{R}_n\cdot\boldsymbol{e}_{r0} \tag{4}$$

where

$$\boldsymbol{e}_{r0}=\vec{\boldsymbol{e}}_x\sin\theta_0\cos\phi_0+\vec{\boldsymbol{e}}_y\sin\theta_0\sin\phi_0+\vec{\boldsymbol{e}}_z\cos\theta_0 \tag{5}$$

According to above mentioned formula, formula (1) may be written in the form

$$F(\theta, \phi; \theta_0, \phi_0) = \sum_{n=1}^{N} A_n \boldsymbol{E}_n(\theta_n, \phi_n) e^{ik\boldsymbol{R_n}\cdot(\boldsymbol{e_r}-\boldsymbol{e_{r0}})} \tag{6}$$

The element pattern is expressed as in the spherical coordinate

$$\boldsymbol{E}(\theta, \phi) = \vec{\boldsymbol{e}}_\theta E_\theta(\theta, \phi) + \vec{\boldsymbol{e}}_\phi E_\phi(\theta, \phi) \tag{7}$$

where E_θ and $E\phi$ are θ and ϕ components respectively.

Although all of the elements have identical radiation patterns, they are in fact not identical in the same coordinate for the influence of the shape of the platform. A more general superposition computation must be performed in terms of formula (6). Therefore, to compute the whole array pattern, it is first necessary to transform individual element pattern in respective local coordinate into element pattern in a uniform coordinate. The concerning knowledge are seen in [24].

In this design, the dipole element is used for simplifying the analysis, the normalized element pattern function of which is expressed as follow

$$\boldsymbol{E}(\theta, \phi) = \vec{\boldsymbol{e}}_\phi \sqrt{\cos\theta} \tag{8}$$

3. PSO ALGORITHM

3.1. Standard PSO Algorithm

The PSO algorithm is an evolutionary algorithm capable of solving difficult multidimensional optimization problems in various fields. Since its introduction in 1995 by Kennedy and Eberhart [25], the PSO has gained an increasing popularity as an efficient alternative to genetic algorithm and simulated annealing in solving optimization design problems in antenna arrays [26]. As an evolutionary algorithm, the PSO algorithm depends on the social interaction between independent agents, here called particles, during their search for the optimum solution using the concept of fitness. After defining the solution space and the fitness function, the PSO algorithm starts by randomly initializing the position and velocity of each particle in the swarm, That is $\boldsymbol{X}_m = (x_{m1}, x_{m2}, \ldots, x_{mN})$, $\boldsymbol{V}_m = (v_{m1}, v_{m2}, \ldots, v_{mN})$, $1 \leq m \leq M$ (M is the number of particles in the swarm).

For an N-dimensional problem, the iterative formula of standard PSO algorithm as follows:

$$v_{mn}^t = \omega \cdot v_{mn}^{t-1} + c_1 \cdot U_{n1}^t \cdot (p_{mn}^t - x_{mn}^{t-1}) + c_2 \cdot U_{n2}^t \cdot (g_n^t - x_{mn}^{t-1}) \tag{9}$$

$$x_{mn}^t = x_{mn}^{t-1} + k \cdot v_{mn}^t \tag{10}$$

where formula (9) is particle's velocity updating equation, formula (10) is particle's position updating equation, the superscripts t and $t-1$ refer to the time index of the current and the previous iterations. p_{mn}^t is the personal best position which defines the position at which each particle attained its best fitness value up to the present iteration. g_n^t is the global best position which defines the position in the solution space at which the best fitness value was achieved by all particles. U_{n1} and U_{n2} are two uniformly distributed random numbers in the interval $[0, 1]$. The parameters c_1 and c_2 specify the relative weight of the personal best position versus the global best position. Previous work has shown that a value of 2.0 is a good choice for both parameters [27]. The parameter ω is a number, called the "inertial weight," in the range $[0, 1]$, and it Specifies the weight by which the particle's current velocity depends on its previous velocity and how far the particle is from its personal best and global best positions. Previous work [28] has shown that the PSO algorithm converges faster if ω is linearly damped with iterations starting at 0.9 and decreasing linearly to 0.4 at the last iteration. In addition, k is constrained factor for the purpose of controlling convergence speed, which value depends on specific problem. For current design, a value of 1.0 is chosen.

3.2. Modified PSO Algorithm

To avoid the problem of prematurity and easily trapping in local optimum, a modified PSO algorithm is proposed, that the only global best particle is perturbed in every iteration of the algorithm and other particles are updated according to original updating method. However, a modified PSO algorithm in [29] is based on fitness distance ratio, which is different from the updating formula (9) of this design.

During the iteration, the current global best particle is updated as follow

$$v_{mn}^t = 0 \tag{11}$$

$$x_{mn}^t = x_{mn}^{t-1} + x_{mn}^{t-1} \cdot p_p \cdot (U - 0.5) \tag{12}$$

where P_p is perturbation parameter, the value of which determines the speed of convergence and the ability of deviating from local optimum. If its value is too small, the solution will easily trap into the local optimum again; if it is too big, the original best particle will enter into the position at which the fitness value is worse. Many different optimization experiments using MPSO algorithm have shown that the value of 1.3 is a good choice for the parameter. In addition, U is random number in the interval $[0, 1]$.

By using formula (11) and (12), stagnant particle may be activated again so that it is more probability to find the global best.

3.3. Fitness Function

In antenna problems, there are many factors that can be used to evaluate the fitness such as directivity, gain, SLL, size, and weight, depending on the application. For the current problem, the geometry layout of array element is first assumed to be known, and then optimize excitation amplitude distribution of array element with minimum average SLL and null control in specific directions. To achieve this goal, the following function is used to evaluate the fitness

$$Fitness = w1 \cdot \sum_i \frac{1}{\Delta\phi_i} \int_{\phi_{li}}^{\phi_{ui}} |F(90°, \phi)/F(90°, 0°)|^2 d\phi + w2 \cdot \sum_k |F(90°, \phi_k)/F(90°, 0°)|^2 \tag{13}$$

where $[\phi_{li}, \phi_{ui}]$ are the spatial regions in which the SLL is suppressed, $\Delta\phi = \phi_{ui} - \phi_{li}$, and ϕ_k are the directions of the nulls. $w1$ and $w2$ stand for the weight coefficients which are 0.3 and 0.7 respectively. For the problem at hand, the particle's position vector that resulted in the minimum value of the fitness function given in formula (13) is chosen as the global best position vector up to the current iteration.

4. DESIGN EXAMPLES

The modified PSO algorithm(MPSO) presented in the previous section is applied to the synthesis of antenna arrays uniformly placed on a circular cylinder which has a radius of 15λ (λ is operating wavelength). The spacing between the rows is assumed to 0.5λ; the interelement spacing along the arc is also assumed to 0.5λ. The geometry layout of the array is seen in Fig. 1.

The first example illustrates the synthesis of a 4-by-24 element array for SLL suppression in the regions [$-90°$, $-6°$] and [$6°$, $90°$] and prescribed nulls at $-50°$, $50°$. For the design, only one dimension scan in azimuth direction is considered. The excitation amplitude of four array elements at the same column along axial direction is the same. The excitation amplitude of 24 elements in azimuth direction is optimized only. The PSO algorithm which uses 20 particles starts by randomly initializing the position matrix X and the velocity matrix V. By iterations of about 2000 steps, the amplitude distribution results

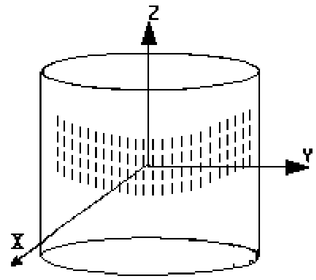

Figure 1. Uniformly spaced cylindrical array.

of array elements are obtained, which are seen from the Table 1. The corresponding pattern is shown in the Fig. 2. In addition, the simulation results using chebyshev amplitude distribution based on linear and planar array synthesis are also provide in the Fig. 2. Its corresponding convergence curve is shown in Fig. 4.

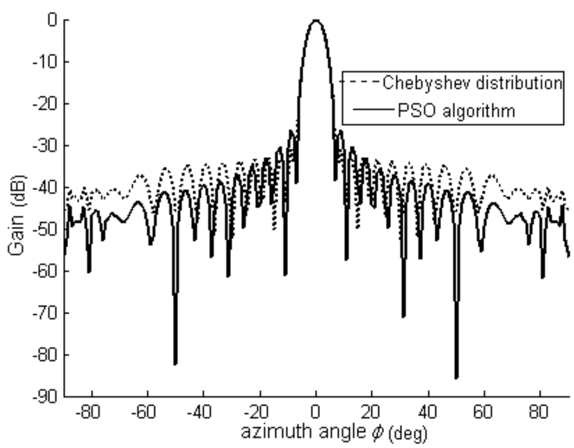

Figure 2. Normalized pattern of 4-by-4 element array.

The second example illustrates the synthesis of a 3-by-30 element array for SLL suppression in the regions $[-90°, -5°]$ and $[5°, 90°]$ and prescribed nulls at $-60°$, $-20°$, $-20°$, $46°$. Only one dimension scan in azimuth direction is also considered. By iterations of about 2000 steps, the amplitude distribution results of array elements are obtained, which are seen from the Table 2. The corresponding pattern is shown in the Fig. 3. Its corresponding convergence curve is shown in Fig. 5.

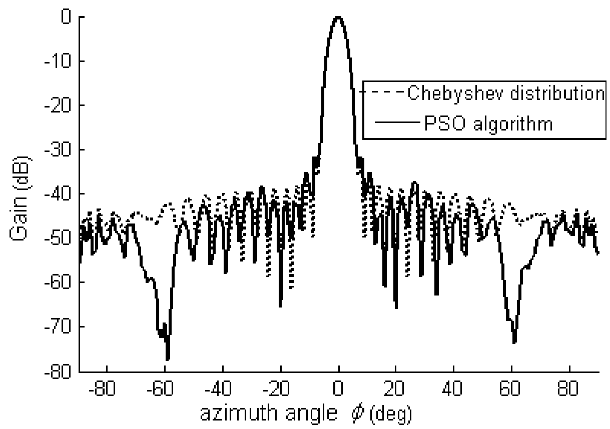

Figure 3. Normalized pattern of 3-by-30 element array.

Table 1. Normalized amplitude distribution of array element in azimuth direction.

Array Element No.	Amplitude Excitation	Array Element No.	Amplitude Excitation
1	0.1934	13	0.9983
2	0.2661	14	0.9718
3	0.3505	15	0.9381
4	0.4446	16	0.8747
5	0.5384	17	0.8012
6	0.6312	18	0.7214
7	0.7277	19	0.6238
8	0.8063	20	0.5309
9	0.8800	21	0.4388
10	0.9425	22	0.3439
11	0.9736	23	0.2604
12	1.0000	24	0.1891

It can be seen from Fig. 2 and Fig. 3 that chebyshev amplitude distribution and MPSO algorithm offer very similar 3-dB beamwidth, but chebyshev amplitude distribution can not realize prescribed nulls and the corresponding average SLL will become much larger.

In addition, optimization designs using standard PSO algorithm and genetic algorithm are also made. These optimization algorithms can attain similar results, and but standard PSO algorithm and genetic algorithm need more larger number of the iterations than MPSO algorithm, namely, slower convergence speed than MPSO algorithm. In the design, the simulation results using standard PSO algorithm and genetic algorithm are not provided. However, the convergence curves using standard PSO algorithm and genetic algorithm are provided in Fig. 4 and Fig. 5 for comparison.

Table 2. Normalized amplitude distribution of array element in azimuth direction.

Array Element No.	Amplitude Excitation	Array Element No.	Amplitude Excitation
1	0.1052	16	1.0000
2	0.1520	17	0.9843
3	0.1753	18	0.9438
4	0.2523	19	0.8937
5	0.3145	20	0.8473
6	0.4278	21	0.7745
7	0.4901	22	0.6929
8	0.5783	23	0.6155
9	0.6572	24	0.5322
10	0.7394	25	0.4807
11	0.8213	26	0.3610
12	0.8728	27	0.2863
13	0.9198	28	0.1967
14	0.9680	29	0.1707
15	0.9947	30	0.1275

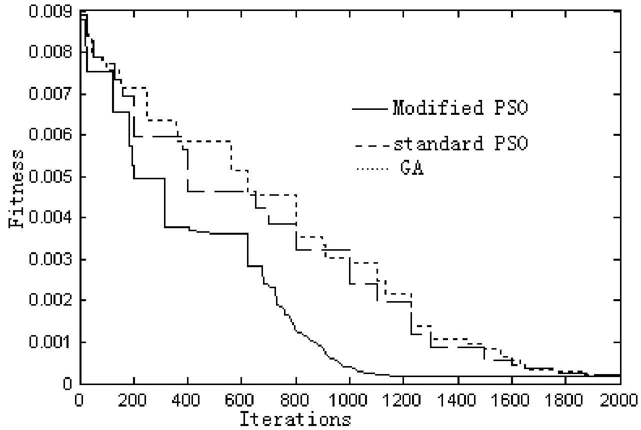

Figure 4. Convergence curve of the fitness value of 4-by-24 element array.

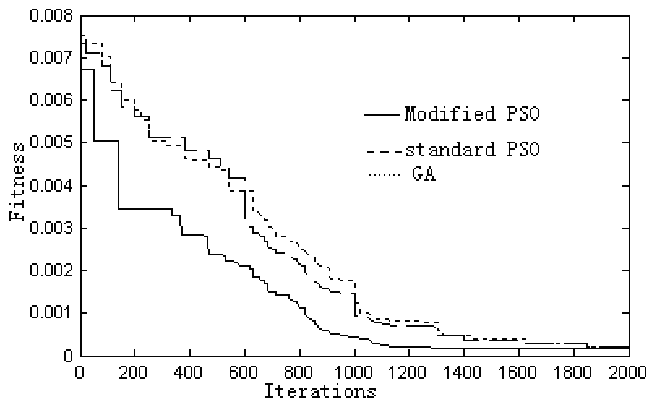

Figure 5. Convergence curve of the fitness value of 3-by-30 element array.

5. CONCLUSION

This paper illustrated the use of the modified particle swarm optimization algorithm in the pattern synthesis of cylindrical conformal array for the purpose of suppressed sidelobe in certain regions and null placement in prescribed directions. The modified PSO algorithm was successfully used to optimize the excitation amplitude of array elements to realize the array pattern with desired sidelobe level and null placement in prescribed directions. More flexible control of the array pattern can be achieved by using the modified or multi-objective PSO algorithm to optimize, not only the excitation amplitude, but also the excitation phase and location of each element in the array.

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