

## **ENHANCEMENT OF OMNIDIRECTIONAL REFLECTION IN PHOTONIC CRYSTAL HETEROSTRUCTURES**

**R. Srivastava and ShyamPati**

Department of Physics  
Udai Pratap Autonomous College  
Varanasi 221007, India

**S. P. Ojha**

Chaudhary Charan Singh University  
Meerut 200005, India

**Abstract**—In this paper we have theoretically studied the omnidirectional total reflection frequency range of a multilayered dielectric heterostructures. Three structures of  $\text{Na}_3\text{AlF}_6/\text{Ge}$  multilayer have been studied. The thickness of the two layers of the first and second structure is differing from each other and the third photonic structure is the combination of first and second structures. Using the Transfer Matrix Method (TMM) and the Bloch theorem, the reflectivity of one dimensional periodic structure for TE- and TM-modes at different angles of incidence is calculated. From the analysis it is found that the proposed structure has very wide range of omnidirectional total frequency bands for both polarizations.

### **1. INTRODUCTION**

During the last 15 years, photonic crystals (PCs) have drawn much attention as a new kind of optical materials. These materials are based on the interaction between an optical field and materials exhibiting periodicity on the scale of wave length [1–5]. Photonic crystals are composite structures with a periodic arrangement of refractive index in one-dimension (1D), two-dimensions (2D), or three-dimensions (3D) [6, 7]. The main feature of photonic crystals is that they can prohibit the propagation of electromagnetic waves within a certain frequency range called photonic band gap (PBG), which is analogous

to the electronic band gap in ordinary materials. The materials containing PBG have many potential applications in optoelectronics and optical communication [8–12]. Ojha et al. [13] observed filtering properties in PBG materials and extended the idea for constructing monochromators [14].

Reflectors are one of the most widely used optical devices. There are two types of reflectors, the metallic reflectors and multilayer dielectric reflectors. Metallic reflector can reflect light over a wide range of frequencies for arbitrary incident angles. At infrared, optical or higher frequencies, there is a considerable power loss owing to absorption. However, Multilayer dielectric reflectors can have an extremely low loss, means they have a high reflectivity over a broad range of frequencies at all incident angles, i.e., an omnidirectional total reflection, [15–29] if the refractive index and thickness of the constituent dielectric layers are properly chosen. This kind of omnidirectional dielectric reflectors can have potential applications in many ways, for instance, as micro-cavities, antenna substrates, or coaxial waveguides, etc. [30].

For the optical range within which the main application is expected, most of experimental effort has been concentrated on 2D and 3D photonic crystals. However, due to technological problem and high cost, the application of 2D and 3D photonic crystals is limited. The 1D photonic crystals are attractive since their production is more feasible at any wavelength scale and their analytical and numerical calculations are simpler. These crystals can exhibit the property of omnidirectional reflection [16, 17, 31]. A one-dimensional photonic crystal is a periodic multilayer structure consisting two type of layer which differs in the dielectric constant. The index of refraction is periodic in the  $y$  coordinate and consists of an endlessly repeating stack of dielectric slabs, which alternate in thickness from  $d_1$  to  $d_2$  and in index of refraction from  $n_1$  to  $n_2$ . Incident light can be either s-polarized or p-polarized [32, 33].

In 1998 Fink et al. [16] first reported that one dimensional dielectric lattice displays total omnidirection reflection for incident light under certain conditions. Chigrin et al. [19] described the effect at optical frequencies (604.3–638.4 nm) using 19 layers of  $\text{Na}_3\text{AlF}_6/\text{ZnSe}$ .

Large band gap is one aim in the study of photonic crystals. Large refractive index contrast and specific structures are required to obtain a wide band gap. Disordered 1D photonic crystal has been studied for band gap extension [34, 35]. Combinations of two or more photonic crystals have been used to enlarge the frequency range of reflection [32, 33].

Photonic crystals are used in manufacturing optical devices and to

increase the capability of optoelectronic circuits. Many optical devices are designed using the property of high reflectance, attributed to high dielectric contrast at the  $\text{Na}_3\text{AlF}_6/\text{Ge}$  interface ( $\Delta n \approx 2$ ) [34, 36]. These structures are suitable for antireflection coating and design the interference filter and thin films near infrared-visible region [37]. Such a structure can be used as an useful optical devices in the optical industry.

In this article, we study theoretically the reflection properties of  $\text{Na}_3\text{AlF}_6/\text{Ge}$  1D photonic crystals. Two structures of same period and different layer thicknesses are studied and then these two structures are combined to form a single PC. We show that it is possible to enhance the total reflection frequency range of omnidirectional reflector in this combined PC. The condition for obtaining this large omnidirectional PBG is that the directional PBGs of constituent 1D PCs should overlap each other.

## 2. THEORY

The propagation of electromagnetic radiation in a simple periodic layered medium, consist of alternating layer of transparent materials with different refractive indices, we take  $x$  axis along the direction normal to the layer and assume that the materials are nonmagnetic. The index of refraction is

$$\eta(x) = \begin{cases} \eta_1 & 0 < x < b \\ \eta_2 & b < x < \Lambda \end{cases} \quad (1)$$

with

$$\eta(x) = \eta(x + \Lambda) \quad (2)$$

To solve propagation of electromagnetic radiation in this medium we use transfer matrix formulation [38, 39]. The electric field is given as

$$E = E(x) \cdot e^{i(\omega t - \beta \cdot z)} \quad (3)$$

where  $\beta$  is the  $z$  component of the wave vector. The electric field distribution  $E(x)$  within each homogenous layer can be expressed as the sum of an incident plane wave and a reflected wave. The complex amplitudes of these two waves constitute the component of a column vector. The electric field of  $n$ th cell can be written as

$$E(x) = \begin{cases} a_n \cdot e^{-i \cdot k_{1\pi}(x-n\Lambda)} + b_n \cdot e^{i \cdot k_{1\pi}(x-n\Lambda)} & n\Lambda - a < x < n\Lambda \\ c_n \cdot e^{-i \cdot k_{2\pi}(x-n\Lambda+a)} + d_n \cdot e^{i \cdot k_{1\pi}(x-n\Lambda+a)} & (n-1)\Lambda < x < n\Lambda - a \end{cases} \quad (4)$$

with

$$\begin{aligned} k_{1x} &= \left[ \left( \eta_1 \cdot \frac{\omega}{c} \right)^2 - \beta^2 \right]^{\frac{1}{2}} = \frac{\eta_1 \cdot \omega}{c} \cdot \cos \theta_1 \\ k_{2x} &= \left[ \left( \eta_2 \cdot \frac{\omega}{c} \right)^2 - \beta^2 \right]^{\frac{1}{2}} = \frac{\eta_2 \cdot \omega}{c} \cdot \cos \theta_2 \end{aligned} \quad (5)$$

where  $\theta_1$  and  $\theta_2$  are the ray angles in the layer.

Using matrix method

$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = \frac{1}{2} \begin{bmatrix} e^{(i \cdot k_{2x} \cdot b)} \cdot \left( 1 + \frac{k_{2x}}{k_{1k}} \right) & e^{-(i \cdot k_{2x} \cdot b)} \cdot \left( 1 - \frac{k_{2x}}{k_{1k}} \right) \\ e^{(i \cdot k_{2x} \cdot b)} \cdot \left( 1 - \frac{k_{2x}}{k_{1k}} \right) & e^{-(i \cdot k_{2x} \cdot b)} \cdot \left( 1 + \frac{k_{2x}}{k_{1k}} \right) \end{bmatrix} \cdot \begin{pmatrix} c_n \\ d_n \end{pmatrix} \quad (6)$$

and similarly

$$\begin{pmatrix} c_n \\ d_n \end{pmatrix} = \frac{1}{2} \begin{bmatrix} e^{(i \cdot k_{1x} \cdot a)} \cdot \left( 1 + \frac{k_{1x}}{k_{2k}} \right) & e^{-(i \cdot k_{1x} \cdot a)} \cdot \left( 1 - \frac{k_{1x}}{k_{2k}} \right) \\ e^{(i \cdot k_{1x} \cdot a)} \cdot \left( 1 - \frac{k_{1x}}{k_{2k}} \right) & e^{-(i \cdot k_{1x} \cdot a)} \cdot \left( 1 + \frac{k_{1x}}{k_{2k}} \right) \end{bmatrix} \cdot \begin{pmatrix} a_n \\ b_n \end{pmatrix} \quad (7)$$

By eliminating

$$\begin{pmatrix} c_n \\ d_n \end{pmatrix}$$

the matrix equation

$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} a_n \\ b_n \end{pmatrix} \quad (8)$$

is obtained

$$\begin{aligned} A_{\text{TE}} &= e^{i \cdot k_{1x} \cdot a} \left[ \cos(k_{2x} \cdot b) + \frac{1}{2} i \left( \frac{k_{2x}}{k_{1x}} - \frac{k_{1x}}{k_{2x}} \right) \sin(k_{2x} \cdot b) \right] \\ B_{\text{TE}} &= e^{-i \cdot k_{1x} \cdot a} \left[ \frac{1}{2} i \left( \frac{k_{2x}}{k_{1x}} - \frac{k_{1x}}{k_{2x}} \right) \sin(k_{2x} \cdot b) \right] \\ C_{\text{TE}} &= e^{i \cdot k_{1x} \cdot a} \left[ -\frac{1}{2} i \left( \frac{k_{2x}}{k_{1x}} - \frac{k_{1x}}{k_{2x}} \right) \sin(k_{2x} \cdot b) \right] \end{aligned} \quad (9)$$

$$D_{\text{TE}} = e^{-i \cdot k_{1 \cdot x} \cdot a} \left[ \cos(k_{2 \cdot x} \cdot b) - \frac{1}{2} i \left( \frac{k_{2 \cdot x}}{k_{1 \cdot x}} + \frac{k_{1 \cdot x}}{k_{2 \cdot x}} \right) \sin(k_{2 \cdot x} \cdot b) \right]$$

This is for s polarization of electromagnetic wave. Similarly for p polarization

$$\begin{aligned} A_{\text{TM}} &= e^{i \cdot k_{1 \cdot x} \cdot a} \left[ \cos(k_{2 \cdot x} \cdot b) + \frac{1}{2} i \left( \frac{k_{1 \cdot x} \eta_2^2}{k_{2 \cdot x} \eta_1^2} + \frac{k_{2 \cdot x} \eta_1^2}{k_{1 \cdot x} \eta_2^2} \right) \sin(k_{2 \cdot x} \cdot b) \right] \\ B_{\text{TM}} &= e^{-i \cdot k_{1 \cdot x} \cdot a} \left[ \frac{1}{2} i \left( \frac{k_{1 \cdot x} \eta_2^2}{k_{2 \cdot x} \eta_1^2} - \frac{k_{2 \cdot x} \eta_1^2}{k_{1 \cdot x} \eta_2^2} \right) \sin(k_{2 \cdot x} \cdot b) \right] \\ C_{\text{TM}} &= e^{i \cdot k_{1 \cdot x} \cdot a} \left[ -\frac{1}{2} i \left( \frac{k_{1 \cdot x} \eta_2^2}{k_{2 \cdot x} \eta_1^2} - \frac{k_{2 \cdot x} \eta_1^2}{k_{1 \cdot x} \eta_2^2} \right) \sin(k_{2 \cdot x} \cdot b) \right] \\ D_{\text{TM}} &= e^{-i \cdot k_{1 \cdot x} \cdot a} \left[ \cos(k_{2 \cdot x} \cdot b) - \frac{1}{2} i \left( \frac{k_{1 \cdot x} \eta_2^2}{k_{2 \cdot x} \eta_1^2} - \frac{k_{2 \cdot x} \eta_1^2}{k_{1 \cdot x} \eta_2^2} \right) \sin(k_{2 \cdot x} \cdot b) \right] \end{aligned} \quad (10)$$

According to the Bloch equation

$$E_k(x + \Lambda) = E_k(x) \quad (11)$$

and Bloch wave function

$$K(\beta \cdot \omega) = \frac{1}{\Lambda} \cos^{-1} \left[ \frac{1}{2} (A + D) \right] \quad (12)$$

Regimes  $1/2[A + D] < 1$  corresponds to real and thus to propagating Bloch waves; when  $1/2[A + D] > 1$  however  $= m\pi/\Lambda + i \cdot K_i$  which has an imaginary parts  $K_i$  so that the Bloch wave is evanescent. These are so called forbidden bands of the periodic medium the band edges are the regimes where  $1/2[A + D] = 1$  the band structure for a typical periodic layered is obtained for TE and TM waves. The TM is forbidden band shrink to zero when  $\beta = \eta_2 \omega / c \cdot \sin \theta_B$  with  $\theta_B$  the Brewster angle.  $\beta = 0$  for normal incident (TE). The dispersion relation  $\omega$  verse  $K$  for normal incident can be written as

$$\cos(K\Lambda) = \cos(k_1 \cdot a) \cos(k_2 \cdot b) - \frac{1}{2} \left( \frac{\eta_2}{\eta_1} + \frac{\eta_1}{\eta_2} \right) \sin(k_1 \cdot a) \sin(k_2 \cdot b) \quad (13)$$

where

$$k_1 = \frac{\eta_1 \cdot \omega}{c} \quad \text{and} \quad k_2 = \frac{\eta_2 \cdot \omega}{c}$$

### 3. RESULT AND DISCUSSION

We have studied two 1D photonic crystals PC1 and PC2 consisting  $\text{Na}_3\text{AlF}_6/\text{Ge}$  periodic multilayer structure of the same period, but with different layer thicknesses. PC1 has 10 pairs of  $\text{Na}_3\text{AlF}_6/\text{Ge}$  as two dielectric layers with and layer thicknesses  $d_1 = 0.75d$  and  $d_2 = 0.25d$  respectively and PC2 has 10 pairs of two dielectric layers of  $\text{Na}_3\text{AlF}_6/\text{Ge}$  with layer thicknesses  $d_1 = 0.50d$  and  $d_2 = 0.50d$  respectively, where  $d$  is the stack thickness. The refractive indices of the crystals are  $n_1 = 1.34$  and  $n_2 = 4.2$ . Both crystals are then combined side by side to constitute the third structure. Hence the third structure (PC1/PC2) is a combination of first (PC1) and second (PC2) structures. The reflectance spectra of two 1D photonic crystals and their combined structure are given in the Table 1, 2 and 3 respectively and then plotted and compared in Fig. 1, Fig. 2 and Fig. 3.

**Table 1.** For PC1.

Angle(deg)	TE	TM
0	0.161-0.323	0.161-0.323
45	0.164-0.360	0.186-0.337
85	0.167-0.407	0.232-0.352

ODR = 0.232-0.323

**Table 2.** For PC2.

Angle(deg)	TE	TM
0	0.131-0.208	0.131-0.208
	0.301-0.415	0.301-0.415
45	0.132-0.216	0.148-0.208
	0.307-0.438	0.323-0.425
85	0.134-0.226	0.178-0.204
	0.314-0.460	0.358-0.433

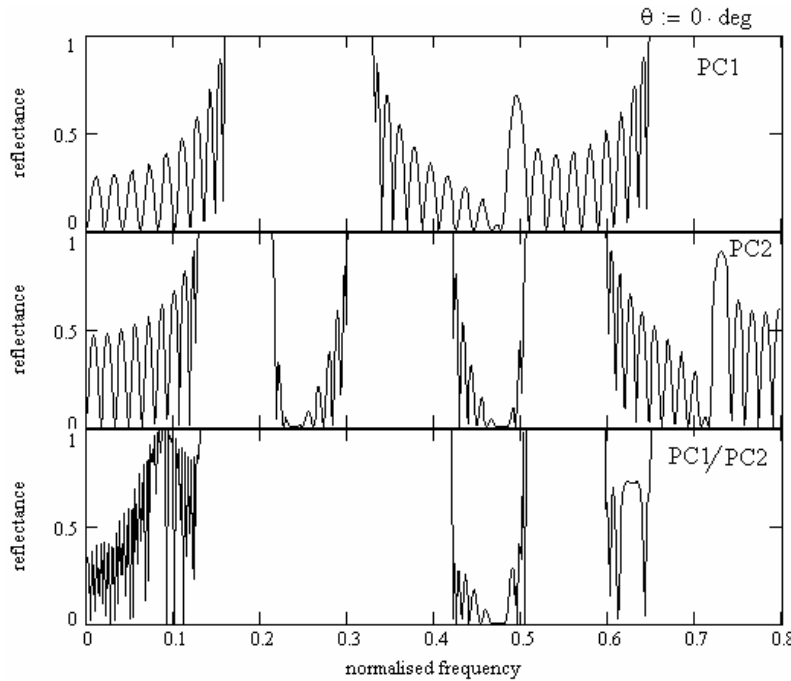
ODR = 0.204-0.131 and 0.358-0.415

It is clear from the Table 1 that the ODR range for PC1 is 0.232–0.323 and it is shown in Fig. 4(a). The ODR range for PC2 is 0.204–0.131 and 0.358–0.415 and it is shown in Fig. 4(b). The ODR range

**Table 3.** For PC1/PC2.

Angle(deg)	TE	TM
0	0.129-0.417	0.129-0.417
45	0.131-0.438	0.146-0.427
85	0.134-0.462	0.172-0.434

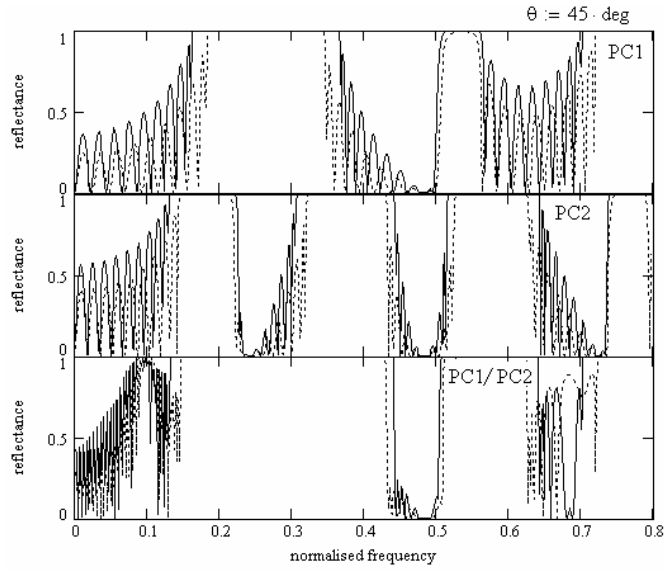
ODR = 0.172-0.417



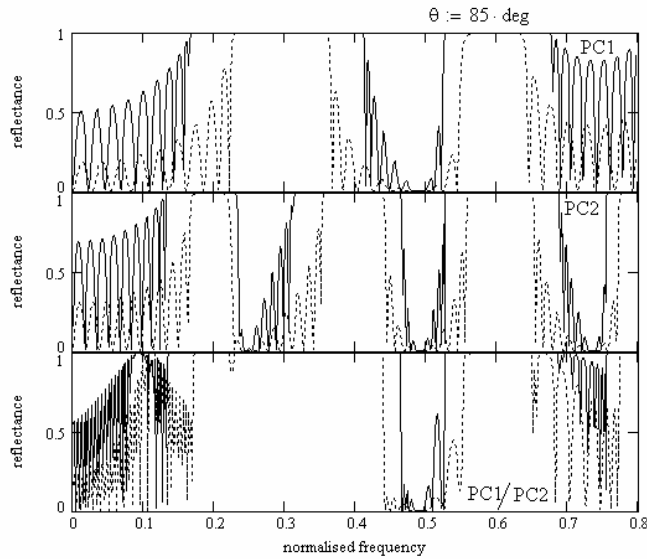
**Figure 1.** Calculated reflectance spectra of PC1 and PC2, and PC1/PC2 at the incident angle  $0^\circ$ . TM and TE polarizations are plotted as dotted and solid lines respectively.

for PC1/PC2 is 0.172–0.417 and it is shown in Fig. 5.

From the Tables 1, 2 and 3, we see that, although omnidirectional photonic band gaps for the two photonic crystals PC1 and PC2 do not overlap, the directional photonic band gaps for PC1 and PC2 overlap each other at different angles and for both the polarizations, TE and TM, except at  $85^\circ$ , where there is some non-overlapping region, which appears as transmission peaks in the reflectance spectra of PC1/PC2.

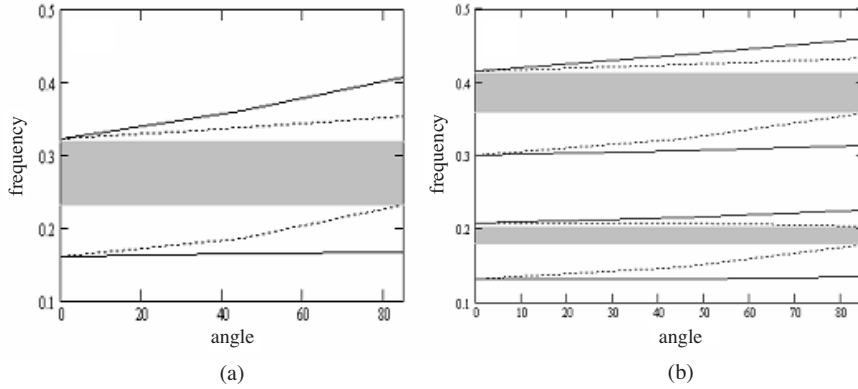


**Figure 2.** Calculated reflectance spectra of PC1 and PC2, and PC1/PC2 at the incident angle  $45^\circ$ . TM and TE polarizations are plotted as dotted and solid lines respectively.

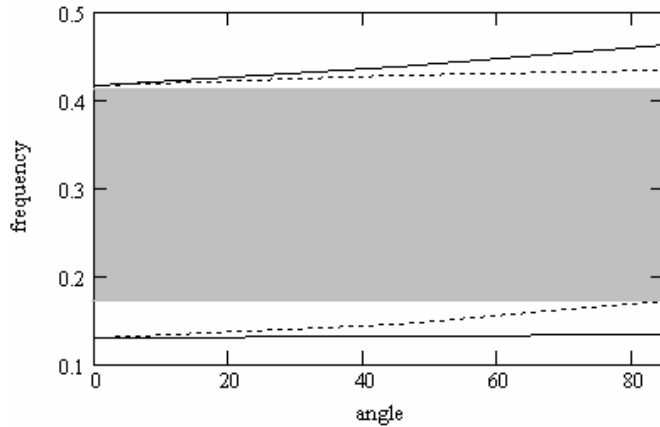


**Figure 3.** Calculated reflectance spectra of PC1 and PC2 and PC1/PC2 at the incident angle  $85^\circ$ . TM and TE polarizations are plotted as dotted and solid lines respectively.





**Figure 4.** Omnidirection reflection band for (a) PC1 and (b) PC2; TM and TE polarizations are plotted as dotted and solid lines respectively.



**Figure 5.** Omnidirection reflection band for PC1/PC2; TM and TE polarizations are plotted as dotted and solid lines respectively.

**4. CONCLUSION**

From the above study, we see that the 100% (approx.) reflection range becomes larger as compared to PC1 and PC2, for both TE and TM polarization and for all incident angles. Thus, the omnidirectional reflection ranges are enhanced. Therefore, large omnidirectional reflection range can be obtained for  $\text{Na}_3\text{AlF}_6/\text{Ge}$  structure by combining the two photonic crystals as considered here. For obtaining this large frequency range, the directional photonic band gaps of two constituent PCs should overlap each other.

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