Progress In Electromagnetics Research B, Vol. 1, 253–268, 2008

STUDY ON THE BLOCKAGE OF ELECTROMAGNETIC RAYS ANALYTICALLY

C. Liang, Z. Liu, and H. Di

School of Electronic Engineering
Xidian University
Xi'an 710071, Shaanxi, China

Abstract—The electromagnetic rays might be shaded when an obstacle occurs in its way. In this paper, the close analytic expressions determining whether a ray is shaded by boards, elliptic cylinders, elliptic spheres and elliptic cones are presented based on general principle of Geometrical Optics. In optical methods like GTD or UTD in computational electromagnetics which are based on various rays, what studied in this paper with the advantages of analytical measures can be useful to keep the rays valid. Several examples are given as further proof.

1. INTRODUCTION

When analyzing EMC problems of electrically large platforms, the Uniform Geometrical Theory of Diffraction (UTD) method is widely used and reasonable effective. The introduce of Geometrical Theory of Diffraction (GTD) by Keller was a very important development, because it made it possible to calculate the high frequency radiation from antennas and scatterers of a quite general shape and to understand the various radiation mechanisms involved. Unfortunately the GTD diffracted fields failed at and near the shadow boundaries of the incident and reflected fields, so the uniform GTD was developed to overcome this limitation. In the UTD the canonical problems are solved by uniform asymptotic methods, and the resulting diffracted field not only describes the field in the shadow region, it also compensates the discontinuities in the geometrical optics field at the shadow boundaries.

As a ray-based method, fields in UTD are supposed to be carried along various kinds of rays, thus the tracing of rays, which can be seen as the basis of this method, is first to consider, many works focus on this aspect [1–3]. Also, with the development of problems met in practice, lots of work on UTD try to extend the use of the method to wider areas [4–6]. Among works mentioned above, there is a tiny assumption that the ray is valid in the first place and the models like wedges exist alone. Apparently, if the model concerned is complex, the rays traced around one surface may be shaded by other part of the structure and further if a traced ray is blocked by any part of the model but not considered, result of pattern is possibly invalid. This problem of occlusion tends to be complex due to the changing kind of objects forming the whole model and the changing location of rays carrying the fields. Complex models in optical methods are always constructed by typical geometric objects like boards, cylinders and cones, thus it is needed to study the relationship between electromagnetic rays and obstacles.

Also, numerical measures are widely taken in studies mentioned above which tend to be complex and time consuming in calculation when models managed become large and complex, therefore the analytic analysis of UTD method is worthy considering due to the stability and rapidity of analytical measures. Studies have already been done on the tracing of rays analytically [7–9], thus to solve the whole problem analytically, it is necessary to find the conditions when a given ray is shaded by certain objects analytically.

The authors try to form a generalized problem in this paper and obstacles in space are assumed to be quadric surfaces satisfying the equation that follows

$$F\left(x_0, y_0, z_0\right) = 0 \tag{1}$$

where x_0, y_0, z_0 are the coordinates of a point on surface and the highest power of which is 2, (x_1, y_1, z_1) and (x_2, y_2, z_2) represent the start point and end point of a given electromagnetic ray. According to differential geometry, the line equation passing those two points mentioned above is easy to form as:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} \tag{2}$$

Then, the problem can be divided into four sub-problems:

(a). Determine whether the start point and the end point are outside of the obstacle;

(b). Determine whether the infinite line passing both (x_1, y_1, z_1) and (x_2, y_2, z_2) is across by the obstacle;

(c). Determine whether (x_1, y_1, z_1) and (x_2, y_2, z_2) are located at the same side of the obstacle;

(d). If the obstacle is the finite cylinder or cone and its z coordinate is within (z_{01}, z_{02}), determine whether the z coordinate of the intersecting point between the straight line and the finite cylinder or cone is within this interval.

Of the four aspects above, the most important is item (b), that is, to study whether the straight line (infinite) has any intersections with the obstacle. Shadow of a straight line can be seen in Fig. 1.

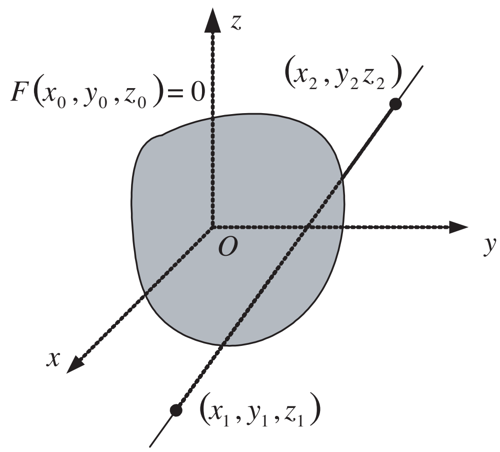


Figure 1. Correlation between a straight line and an obstacle.

Substituted (x_1, y_1, z_1) and (x_2, y_2, z_2) into the left side of formula (1), if the values of function F satisfy the following inequations:

$$\begin{cases} F(x_1, y_1, z_1) > 0 \\ F(x_2, y_2, z_2) > 0 \end{cases} \tag{3}$$

the start point and the end point are outside the obstacle. So inequations (3) are the criterions of problem (a).

From the straight line equation of (2), we obtain

$$\begin{cases} x = g(z) = x_1 + (x_2 - x_1)\left(\dfrac{z - z_1}{z_2 - z_1}\right) \\ y = h(z) = y_1 + (y_2 - y_1)\left(\dfrac{z - z_1}{z_2 - z_1}\right) \end{cases} \tag{4}$$

Let $z = z_0$, and substitute (4) into (1), we yield

$$F(g(z_0), h(z_0), z_0) = 0 \tag{5}$$

Notice that F is conicoid, then (5) can be written into quadratic equation with one variable like:

$$Az_0^2 + 2Bz_0 + C = 0 \tag{6}$$

where A, B and C are known coefficients and z_0 is the unknown to be calculated.

The formula above represents the intersection between line and surface and the discriminant of quadratic form (6) is

$$\Delta = B^2 - AC \tag{7}$$

Then the condition determining whether a ray is shaded is

$$\Delta = \begin{cases} \leq 0, & \text{not shaded} \\ > 0, & \text{shaded} \end{cases} \tag{8}$$

The intersecting points are

$$z_0 = \frac{1}{A}\left\{-B \pm \sqrt{B^2 - AC}\right\} \tag{9}$$

Then

$$z_0 = \begin{cases} \in [z_1, z_2], & \text{shaded} \\ \notin [z_1, z_2], & \text{not shaded} \end{cases} \tag{10}$$

Formulas (1) to (10) give the generalized theory of shadow analysis analytically

2. RAY AND FLAT PLATE

Flat plate is the simplest and most common geometry. Without loss of generality, suppose that the plate is on x-o-y plane, as illustrated in Fig. 2, and the function of the plate is

$$F(x_0, y_0, 0) = 0 \tag{11}$$

To ensure that (x_1, y_1, z_1) and (x_2, y_2, z_2) locate on the different side of the plate, the z-coordinates of the two points must satisfy

$$z_1 z_2 < 0 \tag{12}$$

That is, z_1 and z_2 takes contrary sign.

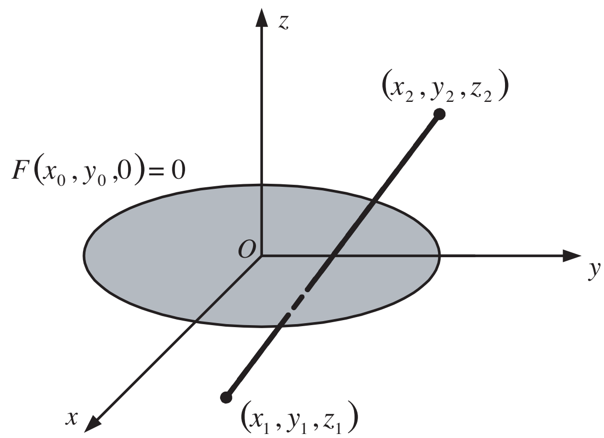


Figure 2. Correlation between a straight line and an plate.

The intersecting point of the straight line and x-o-y plane is easy to get as

$$\begin{cases} x = \dfrac{z_2 x_1 - z_1 x_2}{z_2 - z_1} \\ y = \dfrac{z_2 y_1 - z_1 y_2}{z_2 - z_1} \\ z = 0 \end{cases} \tag{13}$$

Then the correlation between the ray and the flat plate can be known through

$$\begin{cases} F(x, y, 0) < 0, & \text{shaded} \\ F(x, y, 0) \geq 0, & \text{not shaded} \end{cases} \tag{14}$$

3. RAY AND ELLIPTIC CYLINDER

In this section, the problem between straight line across (x_1, y_1, z_1), (x_2, y_2, z_2) and the two-dimensional elliptic cylinder is investigated (see Fig. 3). The elliptic cylinder is infinite along its axis which is the z-axis, so the function of the infinite elliptic cylinder can be written as:

$$\frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} = 1 \tag{15}$$

The projections of the cylinder and the line passing (x_1, y_1, z_1), (x_2, y_2, z_2) are studied, as in Fig. 3.

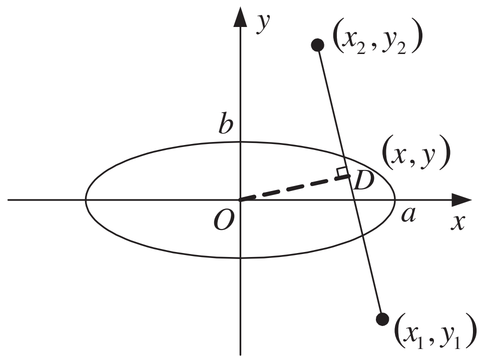


Figure 3. Correlation between a straight line and elliptic cylinder.

If the following inequality is satisfied, the two points are outside the elliptic cylinder

$$\begin{cases} \dfrac{x_1^2}{a^2} + \dfrac{y_1^2}{b^2} > 1 \\ \dfrac{x_2^2}{a^2} + \dfrac{y_2^2}{b^2} > 1 \end{cases} \tag{16}$$

Substitute $y_0 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right) x_0 + \left(\frac{x_2 y_1 - x_1 y_2}{x_2 - x_1}\right)$ into (15) which can be rewritten as

$$b^2 x_0^2 + a^2 y_0^2 = a^2 b^2 \tag{17}$$

Then the quadratic equation of x can be achieved as

$$\begin{aligned} &\left[b^2 (x_2 - x_1)^2 + a^2 (y_2 - y_1)^2\right] x_0^2 + 2a^2 (y_2 - y_1)(x_2 y_1 - x_1 y_2) x_0 \\ &+a^2 \left[(x_2 y_1 - x_1 y_2)^2 - b^2 (x_2 - x_1)^2\right] = 0 \end{aligned} \tag{18}$$

(18) is the quadratic equation of x_0. According to the discriminant of the quadratic equation, the criterion of shadow is

$$\frac{(x_2 y_1 - x_1 y_2)^2}{b^2 (x_2 - x_1)^2 + a^2 (y_2 - y_1)^2} = \begin{cases} \geq 1, & \text{not shaded} \\ < 1, & \text{shaded} \end{cases} \tag{19}$$

Specially, when $a = b = R$ which represents circular cylinder, one can get:

$$\frac{(x_2 y_1 - x_1 y_2)^2}{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \begin{cases} \geq R^2, & \text{not shaded} \\ < R^2, & \text{shaded} \end{cases} \tag{20}$$

It can be seen from (19) and (20) that whether the ray is shaded only relates to (x_1, y_1), (x_2, y_2) and is independent to z_1, z_2.

If the line is shaded, then two others factors are taken into account:

(i). Determine whether (x_1, y_1, z_1), (x_2, y_2, z_2) locate at the same side of the elliptic cylinder. Suppose the line treated is shaded by the elliptic cylinder, a line OD is made normal to the line as in Fig. 3, then the following equation can be achieved:

$$x(x_2 - x_1) + y(y_2 - y_1) = 0 \tag{21}$$

where (x, y) denotes the coordinates of point D. Combining (4) with (21), we get

$$\begin{cases} x = \dfrac{(y_2 - y_1)(x_1 y_2 - x_2 y_1)}{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ y = \dfrac{-(x_2 - x_1)(x_1 y_2 - x_2 y_1)}{(x_2 - x_1)^2 + (y_2 - y_1)^2} \end{cases} \tag{22}$$

Parameter k is introduced here as

$$k = \frac{x_1 - x}{x_2 - x} \tag{23}$$

Then

$$k = \begin{cases} > 0 & \text{at the same side and not shaded} \\ < 0 & \text{shaded} \end{cases} \tag{24}$$

(ii). Consider the case when the length of the cylinder is practically limited, that is, $z \in [z_1, z_2]$. From the coordinates x_0 of the intersecting point derived, the following result can be achieved easily

$$z_0 = z_1 + (z_2 - z_1)\frac{x_0 - x_1}{x_2 - x_1} \tag{25}$$

Therefore if cylinder is limited, then

$$z_0 = \begin{cases} \in [z_1, z_2], & \text{shaded} \\ \notin [z_1, z_2], & \text{not shaded} \end{cases} \tag{26}$$

4. RAY AND ELLIPSOID

In this Section, the problem concentrates on three-dimensional ellipsoid, and its expression is

$$\frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} + \frac{z_0^2}{c^2} = 1 \tag{27}$$

Remove the denominators of the left side of Equation (27), we have

$$b^2c^2x_0^2 + a^2c^2y_0^2 + a^2b^2z_0^2 = a^2b^2c^2 \tag{28}$$

Meanwhile, a line can be found through (x_1, y_1, z_1) and (x_2, y_2, z_2), as shown in Fig. 4

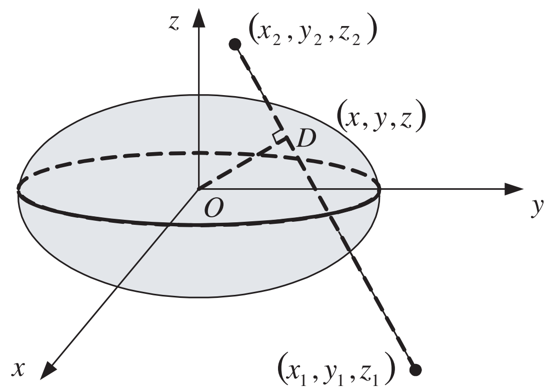


Figure 4. Correlation between a straight line and ellipsoid.

First, the two points are outside the ellipsoid if

$$\begin{cases} \dfrac{x_1^2}{a^2} + \dfrac{y_1^2}{b^2} + \dfrac{z_1^2}{c^2} > 1 \\ \dfrac{x_2^2}{a^2} + \dfrac{y_2^2}{b^2} + \dfrac{z_2^2}{c^2} > 1 \end{cases} \tag{29}$$

The discriminant functions of the two points (x_1, y_1, z_1) and (x_2, y_2, z_2) outside the ellipsoid are

$$\begin{cases} x_0 = \dfrac{x_2 - x_1}{z_2 - z_1} z_0 + \dfrac{z_2x_1 - z_1x_2}{z_2 - z_1} \\ y_0 = \dfrac{y_2 - y_1}{z_2 - z_1} z_0 + \dfrac{z_2y_1 - z_1y_2}{z_2 - z_1} \end{cases} \tag{30}$$

Combining (30) with (28) achieves the quadratic equation of coordinate z_0 of the intersecting points

$$\begin{aligned} &\left[b^2c^2(x_2 - x_1)^2 + a^2c^2(y_2 - y_1)^2 + a^2b^2(z_2 - z_1)^2\right] z_0^2 \\ &+2\left[b^2c^2(x_2 - x_1)(z_2x_1 - z_1x_2) + a^2c^2(y_2 - y_1)(z_2y_1 - z_1y_2)\right] z_0 \\ &+\left[b^2c^2(z_2x_1 - z_1x_2)^2 + a^2c^2(z_2y_1 - z_1y_2)^2 - a^2b^2c^2(z_2 - z_1)^2\right] \end{aligned} \tag{31}$$

If there is the line passes the object without intersection, then

$$\begin{aligned}&\left[b^2c^2\left(x_2-x_1\right)\left(z_2x_1-z_1x_2\right)+a^2c^2\left(y_2-y_1\right)\left(z_2y_1-z_1y_2\right)\right]^2\\&\leq\left[b^2c^2\left(x_2-x_1\right)^2+a^2c^2\left(y_2-y_1\right)^2+a^2b^2\left(z_2-z_1\right)^2\right]\\&\cdot\left[b^2c^2\left(z_2x_1-z_1x_2\right)^2+a^2c^2\left(z_2y_1-z_1y_2\right)^2-a^2b^2c^2\left(z_2-z_1\right)^2\right]\end{aligned}\tag{32}$$

The correlation between the ray and the ellipsoid can be achieved through:

$$\frac{c^2(x_2y_1-x_1y_2)^2+a^2(y_2z_1-y_1z_2)^2+b^2(z_2x_1-z_1x_2)^2}{b^2c^2\left(x_2-x_1\right)^2+a^2c^2\left(y_2-y_1\right)^2+a^2b^2\left(z_2-z_1\right)^2}=\begin{cases}\geq 1, & \text{not shaded}\\ < 1, & \text{shaded}\end{cases}\tag{33}$$

Specially, when $a = b = c = R$ which represents sphere, the condition is

$$\frac{(x_2y_1-x_1y_2)^2+(y_2z_1-y_1z_2)^2+(z_2x_1-z_1x_2)^2}{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2+\left(z_2-z_1\right)^2}=\begin{cases}\geq R^2, & \text{not shaded}\\ < R^2, & \text{shaded}\end{cases}\tag{34}$$

If the line is found to be shaded, the next to determine is whether (x_1, y_1, z_1), (x_2, y_2, z_2) are located on the same side of the ellipsoid. Also, a line OD is drawn perpendicular to the straight line which expressions are (30). Point D is the perpendicular foot and its coordinate is (x, y, z). According to the orthogonality condition, yields

$$x\left(x_2-x_1\right)+y\left(y_2-y_1\right)+z\left(z_2-z_1\right)=0\tag{35}$$

It can be achieved easily that

$$z=\frac{-\left(x_2-x_1\right)\left(z_2x_1-z_1x_2\right)-\left(y_2-y_1\right)\left(z_2y_1-z_1y_2\right)}{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2+\left(z_2-z_1\right)^2}\tag{36}$$

Parameter k is introduced as

$$k=\frac{z_1-z}{z_2-z}\tag{37}$$

Then

$$k=\begin{cases}>0, & \text{at the same side and not shaded}\\ <0, & \text{shaded}\end{cases}\tag{38}$$

5. RAY AND ELLIPTIC CONE

Consider a half elliptic cone with $z_0 > 0$, as illustrated in Fig. 5. And its expression is

$$\frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} - z_0^2 = 0, \quad (z_0 > 0) \tag{39}$$

Simplify formular (39), we have

$$b^2 x_0^2 + a^2 y_0^2 - a^2 b^2 z_0^2 = 0, \quad (z_0 > 0) \tag{40}$$

A line can be found through (x_1, y_1, z_1) and (x_2, y_2, z_2), as shown in Fig. 5

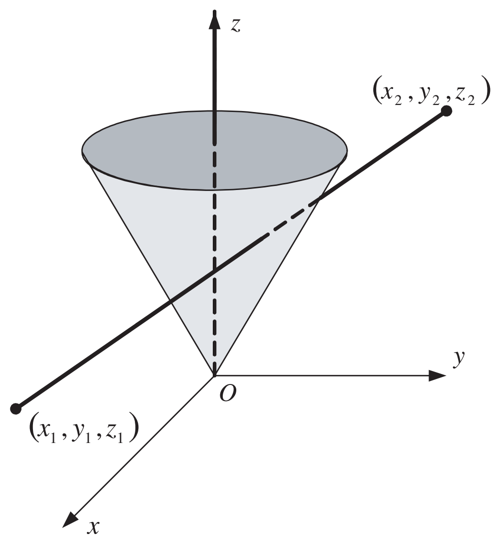


Figure 5. Correlation between a straight line and elliptic cone.

What needs doing first is to determine whether the two points are outside the elliptic cone:

(i). if $z_i < 0 (i = 1, 2)$, then point i is outside the elliptic cone;

(ii).

$$\begin{cases} \dfrac{x_1^2}{a^2} + \dfrac{y_1^2}{b^2} - z_1^2 > 0 \\ \dfrac{x_2^2}{a^2} + \dfrac{y_2^2}{b^2} - z_2^2 > 0 \end{cases} \tag{41}$$

if the inequality is satisfied, it also represents that the point is outside the elliptic cone.

Consider the line equation (30) and combine it with (40), again the quadratic equation of coordinate z_0 of the intersecting point is obtained

$$\begin{aligned}&\left[b^2\left(x_2-x_1\right)^2+a^2\left(y_2-y_1\right)^2-a^2b^2\left(z_2-z_1\right)^2\right]z_0^2\\&+2\left[b^2\left(x_2-x_1\right)\left(z_2x_1-z_1x_2\right)+a^2\left(y_2-y_1\right)\left(z_2y_1-z_1y_2\right)\right]z_0\\&+\left[b^2\left(z_2x_1-z_1x_2\right)^2+a^2\left(z_2y_1-z_1y_2\right)^2\right]=0\end{aligned}\tag{42}$$

If there is no occlusion, then

$$\begin{aligned}&\left[b^2\left(x_2-x_1\right)\left(z_2x_1-z_1x_2\right)+a^2\left(y_2-y_1\right)\left(z_2y_1-z_1y_2\right)\right]^2\\&\le\left[b^2\left(x_2-x_1\right)^2+a^2\left(y_2-y_1\right)^2-a^2b^2\left(z_2-z_1\right)^2\right]\\&\left[b^2\left(z_2x_1-z_1x_2\right)^2+a^2\left(z_2y_1-z_1y_2\right)^2\right]\end{aligned}\tag{43}$$

Therefore the analytical criterion of shadow is

$$\left(x_2y_1-x_1y_2\right)^2-a^2\left(y_2z_1-y_1z_2\right)^2-b^2\left(z_2x_1-z_1x_2\right)^2=\begin{cases}\ge 0, & \text{not shaded}\\ <0, & \text{shaded}\end{cases}\tag{44}$$

Specially when $a = b = k$, then the half elliptic cone degenerates to half circular cone as

$$x_0^2+y_0^2-k^2z_0^2=0\tag{45}$$

The correlation between the ray and the elliptic cone can be achieved through

$$\left(x_2y_1-x_1y_2\right)^2-k^2\left(y_2z_1-y_1z_2\right)^2-k^2\left(z_2x_1-z_1x_2\right)^2=\begin{cases}\ge 0, & \text{not shaded}\\ <0, & \text{shaded}\end{cases}\tag{46}$$

If the line is found to be shaded, two aspects needs considering:

(i). Whether (x_1, y_1, z_1), (x_2, y_2, z_2) locate at the same side of the ellipsoid. The same as that in the case of elliptic cylinder, this is dealt with by two-dimensional means as shown in Fig. 5, according to the orthogonality condition, yields the coordinate as

$$x=\frac{\left(x_1y_2-x_2y_1\right)\left(y_2-y_1\right)}{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}\tag{47}$$

Parameter k is introduced again as

$$k=\frac{x_1-x}{x_2-x}\tag{48}$$

$$k = \begin{cases} > 0 & \text{on the same side and not shaded} \\ < 0 & \text{shaded} \end{cases} \tag{49}$$

(ii). If the elliptic cone is limited in $z \in [z_1, z_2]$, then

$$z_0 = z_1 + (z_2 - z_1)\,\frac{x_0 - x_1}{x_2 - x_1} \tag{50}$$

The correlation between the ray and the circular cone can be achieved through:

$$z_0 = \begin{cases} \in [z_1, z_2] & \text{shaded} \\ \notin [z_1, z_2] & \text{not shaded} \end{cases} \tag{51}$$

6. DISSCUSSION

After careful study of occlusions further, there is a case needs to be discussed when dealing with finite elliptic cylinder. This case occurs when at least one of the two points forming the line is inside the projection of the elliptic cylinder. Take it for example when the point (x_1, y_1, z_1) is in the projection, that is

$$\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} < 1 \tag{52}$$

What is given in Fig. 6 is a limited elliptic cylinder with a height of H and a centre of O.

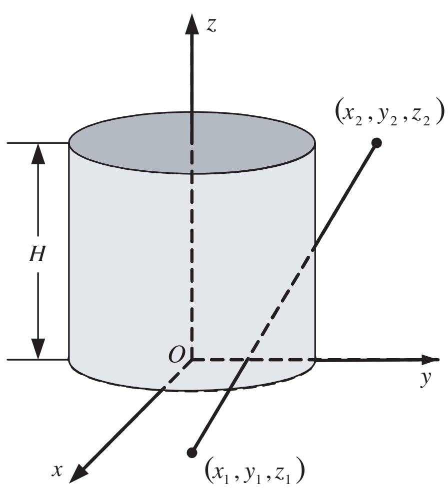


Figure 6. Special case of limited elliptic cylinder.

This special case can be seen as the line linking (x_1, y_1, z_1) and (x_2, y_2, z_2) intersects the elliptic cylinder at xoy plane ($z = 0$). Consider any point on the line, like:

$$\begin{cases} x = \dfrac{x_2 - x_1}{z_2 - z_1} z + \dfrac{z_2 x_1 - z_1 x_2}{z_2 - z_1} \\ y = \dfrac{y_2 - y_1}{z_2 - z_1} z + \dfrac{z_2 y_1 - z_1 y_2}{z_2 - z_1} \end{cases} \tag{53}$$

When $z = 0$, obtains

$$\begin{cases} x = \dfrac{z_2 x_1 - z_1 x_2}{z_2 - z_1} \\ y = \dfrac{z_2 y_1 - z_1 y_2}{z_2 - z_1} \end{cases} \tag{54}$$

Therefore the correlation of ray and object in this special case is determined by

$$\frac{(z_2 x_1 - z_1 x_2)^2}{a^2} + \frac{(z_2 y_1 - z_1 y_2)^2}{b^2} = \begin{cases} \geq (z_2 - z_1)^2 & \text{not shaded} \\ < (z_2 - z_1)^2 & \text{shaded} \end{cases} \tag{55}$$

7. NUMERICAL RESULTS

In UTD method, cylinders and cones are the most widely used and two examples of occlusions considering infinite elliptic cylinder and cone are given below first.

Case1. Infinite elliptic cylinder is placed along z axis, the long and the short radii are $a = 4$, $b = 3$, source point is $(x_1, y_1, z_1) = (5, 0, 0)$, when observers change, the shadow is shown in Table 1.

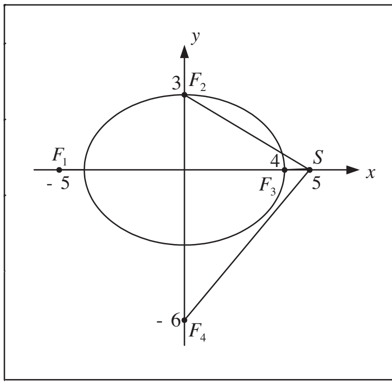
Case2. Elliptic cone with its vertex at the original point, the long and the short radii are $a = 4$, $b = 3$, $c = 2$, source point is $(x_1, y_1, z_1) = (5, 0, 1)$, when observers change, the shadow is shown in Table 2.

Results above are achieved by analytical measures thus convenient to use and it can also be seen that if complex models constructed by lots of objects mentioned above are managed, great resources will be saved.

Case3. What is shown in Fig. 7 is a plane modeled by several typical objects including cones, cylinders and boards, the result of pattern is illustrated in Fig. 8.

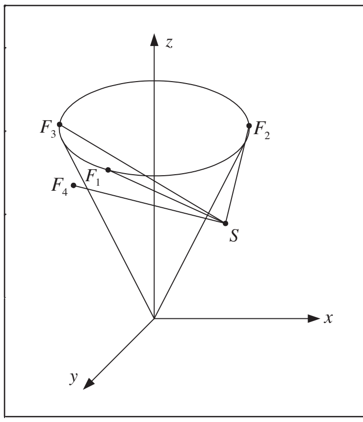
It is clear that using analytical measures [7–9] including methods presented in this paper as an important part are applied as in this example great resources can be saved and time spent is shortened with in minutes.

Table 1. Observers change around elliptic cylinder.



Observers	Occlusions
$F_1(-5,0,0)$	Shaded
$F_2(0,3,0)$	Shaded
$F_3(4,0,0)$	Not shaded
$F_4(0,-6,0)$	Not shaded

Table 2. Observers change around elliptic cone.



Observers	Occlusions
$F_1(-5,0,0)$	Shaded
$F_2(0,3,0)$	Shaded
$F_3(4,0,0)$	Not shaded
$F_4(0,-6,0)$	Not shaded

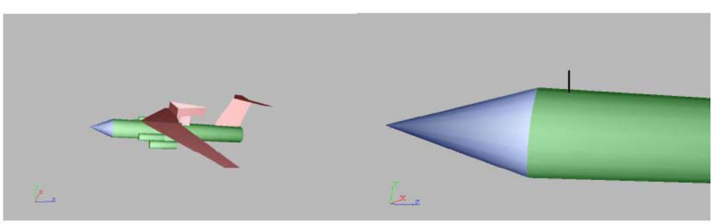


Figure 7. Place an monopole on an plane model.

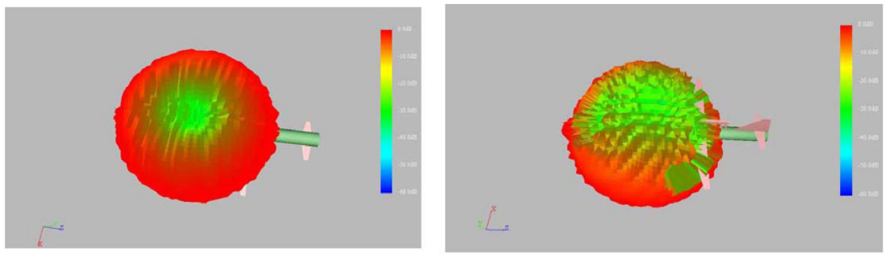


Figure 8. Result of pattern with the method presented in this paper taken into account.

8. CONCLUSION

In the ray-based method of Geometrical Theory of Diffraction (GTD and UTD), the traced rays like reflected rays and diffracted rays may be shaded by other part of the model thus leads to invalidity. To determining whether a ray is shaded by any part of the whole model is of the most important job which guarantees the accuracy of ray tracing when applying UTD method. The close analytic expressions to determining the conditions of occlusions between given ray and boards, elliptic cylinders, elliptic spheres and elliptic cones are given in this paper based on general principle of Geometrical Optics, which can obviously extend analytical UTD method into higher level.

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