

**RIGOROUS ANALYSIS OF UNIAXIAL
DISCONTINUITIES MICROWAVE COMPONENTS
USING A NEW MULTIMODAL VARIATIONAL
FORMULATION**

D. Lilonga-Boyenga and C. Nziengui-Mabika

ENSP
Université Marien Ngouabi
BP 69 Brazzaville, Congo

G. Okoumou-Moko

Faculté des Sciences
Université Marien Ngouabi
BP 69 Brazzaville, Congo

Abstract—A new multimodal variational formulation (NVMF) analysis is used for a rigorous analysis of four microwave subsystems with multiple discontinuities: one double-step and one quadruple-step empty-ridged waveguide discontinuity, one iris-coupled cavities filter with four resonators and one impedance transformer. The S-parameters of each structure are deduced from its total impedance matrix, without cascading the S-parameters of individual discontinuities as with the most methods based on mode-matching technique; the convergence study versus the accessible modes is no long necessary, which makes this passive microwave circuit's analysis and design tool very efficient.

1. INTRODUCTION

The analysis of discontinuities in the passive microwave structures has a particular interest because of their importance in the realization of many devices used in the modern communication systems. The electromagnetic modelling of these discontinuities is a significant stake in the process of their analysis or design. During these last years, many numerical methods were developed to characterize the discontinuities in the guiding structures:

- the differential methods (finite element method [1], finite difference method [2]), which are methods based on the discretization of the Helmholtz's equation in space;
- the integral methods (mode-matching [3], generalized scattering parameters [4], Green functions [5], multimodal variational method (MVM) [6], ...), which allow the determination of the distribution of the fields and the currents on surfaces of discontinuity starting from the boundary conditions on these surfaces.

The MVM apply to uniaxial discontinuities. Assimilating each discontinuity to a multipôle, a complete structure comprising several discontinuities is divided into several constituent blocks formed by one individual discontinuity following by one section of finite length waveguide. The scattering parameters S of the total structure are calculated by the chaining process of individual scattering matrix of these different constituent blocks [7–9]. Furthermore, the introduction of the concept of accessible modes (coupled with the discontinuity) and non accessible (localized) modes into this method, reduced considerably the size of the matrix to be treated [9] and also returns the MVM particularly efficient in numerical point of view compared to the method of the generalized scattering parameters, based on the same mode-matching technique. This approach allowed the analysis and the design with a good precision of a large variety of microwave components: dielectric and ferrite loaded waveguide filters and phase-shifts, multimode cavity filters, antennas [6, 7, 9]. However in the above mentioned studies, the coupling between discontinuities is empirically managed by a choice of the number of modes accessible according to the studied structure, which requires a systematic study of convergence if one wants to analyze a new structure. The other problem of convergence occurs when the number of accessible modes in the intermediate waveguides exceeds a certain threshold (approximately 1/3 of the total number of modes). In order to avoid this study of convergence, a new multimodal variational formulation was proposed [10]. This one integrates the whole of discontinuities in the calculation of the total matrix impedance of the structure and takes into account all higher modes of the intermediate guides, thus decreasing the computing time while improving the precision of the results. The NVMF has been applied successfully to the design of a dielectric E-plane loaded rectangular waveguide evanescent-mode bandpass filter in K-band [10]. We present here the implementation of this formulation in the analysis of another kind of discontinuities and particularly the analysis of a double and a quadruple empty-ridged waveguides discontinuity, a fourth order filter with inductive irises and

an impedance transformer Ku- K bands.

2. NEW MULTIMODAL VARIATIONAL FORMULATION APPROACH

The N -uniaxial discontinuities structure studied is schematically represented in Figure 1. z is the common propagation direction of all waveguides and (z_1, z_2, \dots, z_N) denote the positions of discontinuities plane or positions of interfaces between the different waveguides.

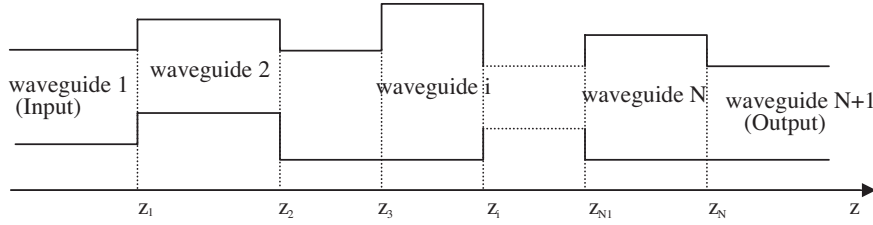


Figure 1. N -uniaxial discontinuities structure.

Analyzing the problems of uniaxial discontinuities [10], it was shown that the vector density of current \hat{J}_d and the tangential electric field vector E_t on the interfaces are related by:

$$\hat{J}_d = \hat{Y} \hat{E}_t \quad (1)$$

\hat{Y} represents the operator admittance of the entire structure and

$$\vec{J}_d = [\vec{J}_d(z_1) \vec{J}_d(z_2) \vec{J}_d(z_3) \dots \vec{J}_d(z_1)]^T \quad (2a)$$

$$\vec{E}_t = [\vec{E}_t(z_1) \vec{E}_t(z_2) \vec{E}_t(z_3) \dots \vec{E}_t(z_1)]^T \quad (2b)$$

where T represents the transpose symbol.

The boundary conditions on the interfaces of discontinuities are written as follow:

$$\begin{cases} \vec{J}_d = 0 & \text{on dielectric interfaces} \\ \vec{E}_t = 0 & \text{on metallic interfaces} \end{cases} \quad (3)$$

The variational form associated to a system of Equations (3) is:

$$f(\vec{E}_t) = \langle \vec{E}_t | \hat{Y} \vec{E}_t \rangle \quad (4)$$

The solution of the system of Equations (3) is obtained by minimizing this above variational form. On one basis of appropriate trial functions, the minimization of the variational form (4), for m_1 accessible modes to the input and m_2 accessible modes to the output, leads to the following total impedance matrix of the entire structure [10]

$$Z = -jN^{-1}|N|^{+\frac{1}{2}}U^{T*}Q^{-1}U|N|^{-\frac{1}{2}} \quad (5)$$

From which one can deduce the scattering matrix of the structure:

$$S = (Z + I)^{-1}(Z - I) \quad (6)$$

I denotes the unity matrix N , U and Q are the matrix depending on the scalar products between waveguides modes which describe the interaction between them [10].

3. NUMERICAL RESULTS

A computer matlab code based on the NVMF was developed and we had applied it to the analysis of four following structures:

As the first analysis, we consider the case of a double step-discontinuity: Standard waveguide WR75 — ridged waveguide — standard waveguide WR75 shown in Figure 2.

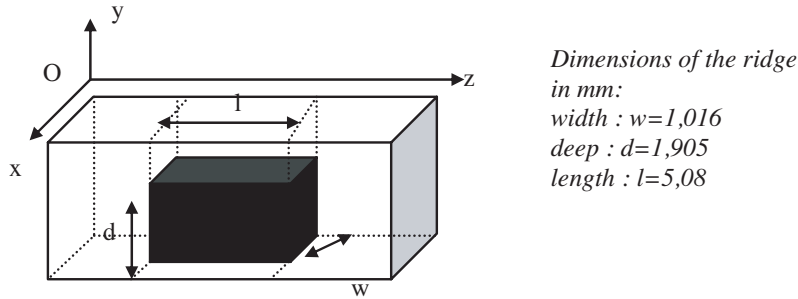
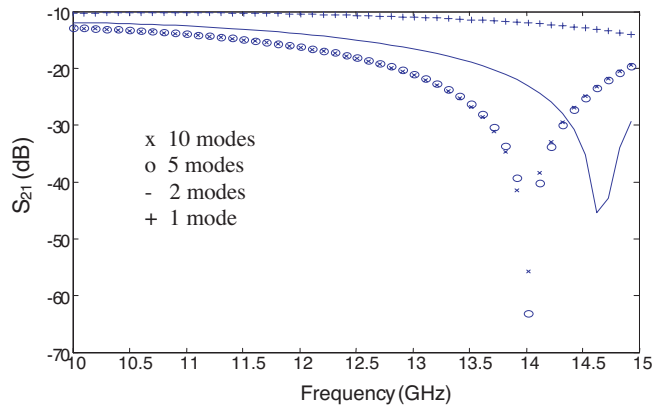


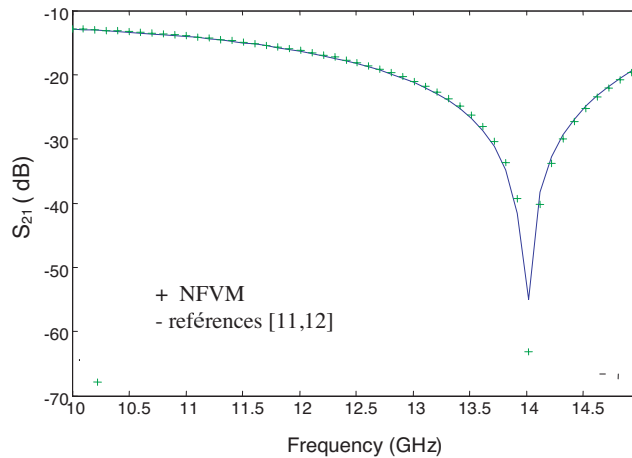
Figure 2. Double step-discontinuity WR75-Ridged waveguide-WR75.

The Figure 3(a) represents the convergence curve of the transmission coefficient of the double step-discontinuity versus the number of accessible modes in the ridged waveguide using the classical MVM. It is seen that the results converge starting from 5 accessible modes in the ridged waveguide.

On the Figure 3(b), we compared the results of our model which excludes the problems involved the convergence study and those



(a)

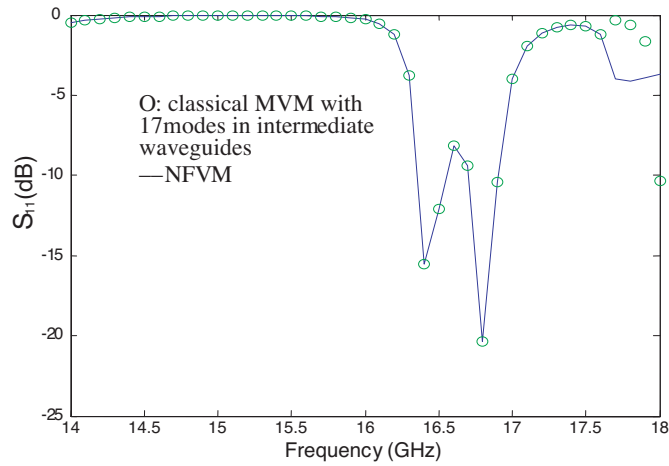


(b)

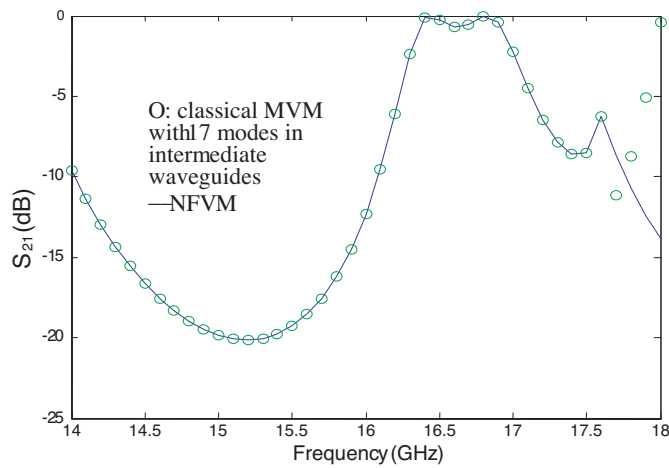
Figure 3. (a) Convergence curve of transmission coefficient versus ridged waveguide number of accessible modes. (b) Transmission coefficient of double step-discontinuity.

given by the references and obtained using the traditional MVM [11] and the higher order large domain FEM and also experimental measurements [12]. For this calculation, we used 100 modes in the empty access waveguides and 40 modes in the ridged waveguide. One notes a good agreement between our results and those of the references. However, a more marked resonance with the NVMF at the frequency of 14 GHz is observed.

The second analysis is devoted to a quadruple step-discontinuity:



(a)



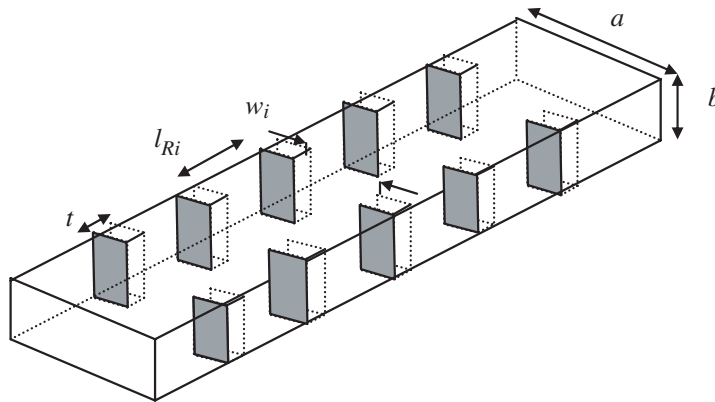
(b)

Figure 4. (a) Reflection coefficient of quadruple step-discontinuity. (b) Transmission coefficient of quadruple step-discontinuity.

Standard WR75-ridged waveguide-standard WR75-ridged waveguide-standard WR75.

The ridged waveguides are identical to that of the preceding section. The empty central standard WR75 has 15 mm of length. We initially calculated the scattering parameters of the structure using the traditional MVM and then using the new formulation in the frequency range of 14–18 GHz. As show it the Figures 4(a) and 4(b), to ensure

the convergence of the results with the traditional MVM, 17 accessible modes at least are necessary in the intermediate waveguides, whereas, if the matrix of the scalar products between modes are available, this same result is obtained in less 10s with the NVMF in only one analysis without preliminary study of convergence, which once more shows the effectiveness of this new formulation. The variation observed for frequencies higher than 17.6 GHz is explained by the fact that starting from this frequency, the 2nd mode of the standard WR75 is excited and it should be taken into account by increasing the number of accessible modes in the intermediate guides in the traditional MVM approach to ensure the convergence of the results.



Dimensions of filter:

Housing empty waveguide : $a=15.799$ mm ; $b=7.899$ mm

Resonators lengths: $l_{R1}=l_{R4}= 8.191$ mm ; $l_{R2}=l_{R3}= 8.884$ mm

Irises widths: $w_1=w_5= 6.080$ mm ; $w_2=w_4= 4.278$ mm ; $w_3=4.010$ mm;

Irises thickness : $t=190$ μ m

Figure 5. Fourth orders iris coupled-cavities filter.

The third structure analyzed is a fourth iris coupled-cavities filter made up of 4 resonators (Figure 5) and designed for a telecommunications satellite [13]. The results of our simulations are given in Figure 6. They show a good agreement with those obtained by the classical MVM with 2 accessible modes in the resonators (all modes of the irises being regarded as accessible) and those obtained by Arndt [13] using the generalized scattering parameters method by considering 45 modes in each resonator and each iris. Our results were

obtained with 100 modes in the empty waveguides, 60, 50 and 40 modes in the irises. Calculations are carried out during 15 s approximately on a standard PC of a processor of 1.5 GHz when the matrix of the scalar products between modes are available.

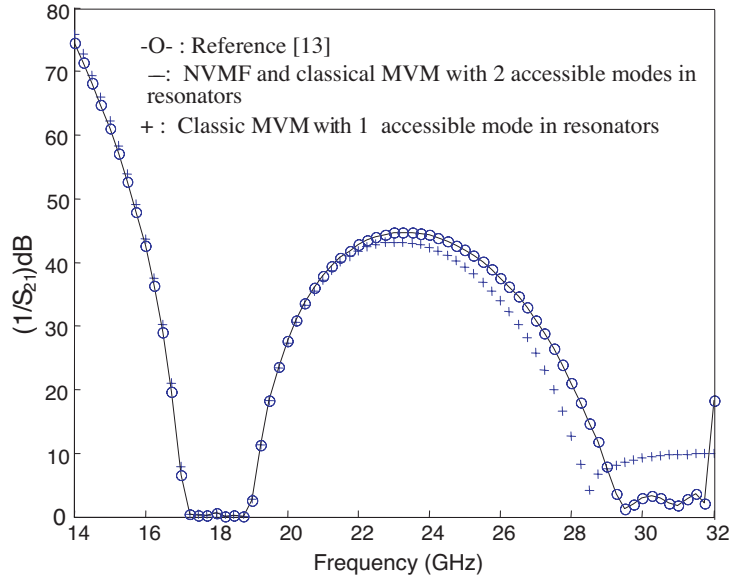
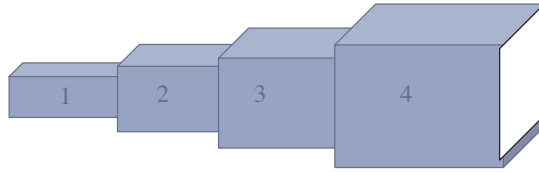


Figure 6. Insertion loss of fourth orders iris coupled-cavities filter.

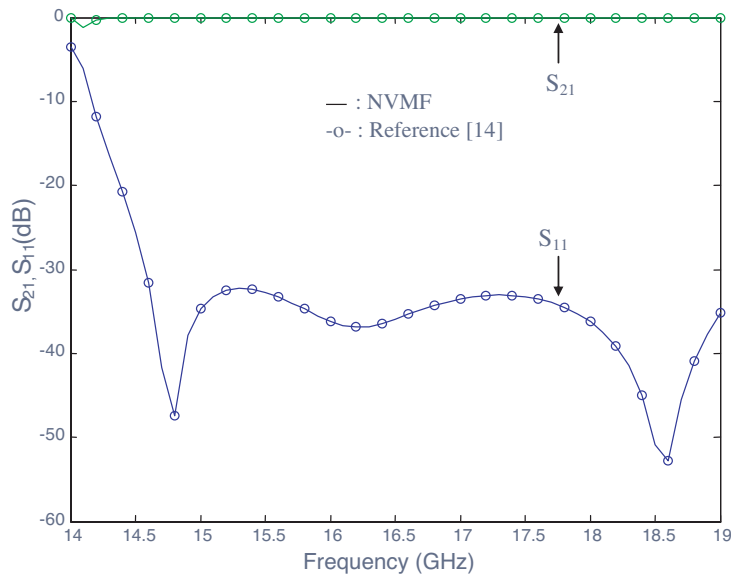
The fourth analysis finally is devoted to one impedance transformer (Figure 7(a)) allowing to pass from the K-band to Ku-band, made up of a triple discontinuity between uniaxial concentric rectangular empty waveguides. The access waveguides are constituted by the standard WR42 (10.668×4.318 mm) at the input and by the standard WR62 (15.8×7.9 mm) at the output. Dimensions of the two intermediate waveguides ($a \times b \times L$) are respectively: $10.85 \times 4.88 \times 7.15$ mm and $12.19 \times 6.53 \times 7.23$ mm.

The Figure 7(b) illustrates frequency response of this impedance transformer. These results were obtained with 36 TE modes and 30 TM modes in the input waveguide, 35 TE modes and 30 TM modes in the second waveguide, 64 TE modes and 40 TM modes in the third waveguide, and 104 TE modes and 80 TM modes in the output waveguide. As one can note it, these results are in agreement with those given by the reference [14]. The computation is carried out during less 20s on a standard PC of a processor of 1.5 GHz when the matrix of the scalar products between modes are available.



Dimensions of waveguides in mm:
 waveguide 1 10.668 x 4.318 (1)
 waveguide 2 10.850 x 4.880 (2)
 waveguide 3 12.190 x 6.530 (3)
 waveguide 4 15.900 x 7.900 (4)

(a)



(b)

Figure 7. (a) Impedance transformer K-band–Ku-band. (b) Response curves of impedance transformer K-band–Ku-band.

4. CONCLUSION

In this work, we used the new variational multimodal formulation (NVMF) to analyze four filter structures with several uniaxial discontinuities. The results obtained were compared with results available in the literature. We noted a good agreement between our results and those of the references. This study made it possible to

show that the problems of uniaxial discontinuities can be tackled with a higher degree of accuracy and a greater flexibility with a gain of the significant computing time using the NVMF. This analysis tool will have to be able to apply to more complex and more varied structures such as open structures and the structures using the non radiative dielectric loaded waveguides.

REFERENCES

1. Ise, K., K. Inoue, and M. Koshiba, "Three dimensional finite element method with edge element of electromagnetic waveguide discontinuities," *IEEE Trans.-MTT*, Vol. 39, 1289–1295, August 1991.
2. Lotz, R., J. Ritter, and F. Arndt, "3D subgrid technique for the finite difference method in frequency domain," *IEEE-MTT-S Int. Microwave Symp. Digest*, 1275–1278, Baltimore, MD, June 1998.
3. Schmidt, R. and P. Russer, "Modeling of cascaded coplanar waveguide discontinuities by mode-matching approach," *IEEE Trans.-MTT*, Vol. 43, 2910–2917, 1995.
4. Arndt, F., J. Bornemann, et al., "Modal S-matrix method for optimum design of inductively direct-coupled cavity filters," *IEEE Proceeding*, Vol. 133, No. 5, Oct. 1986.
5. Soler, F. J. P., F. D. Q. Pereira, J. P. Garcia, D. C. Rebenaque and A. Á. Melcon, "Analysis of inductive waveguide microwave components using alternative port treatment and efficient fast multipole," *Progress In Electromagnetics Research*, PIER 68, 71–90, 2007.
6. Tao, J. W. and H. Baudrand, "Multimodal variational analysis of uniaxial waveguide discontinuities," *IEEE Trans.-MTT*, Vol. 39, 506–516, March 1991.
7. Nanan, J. C., J. W. Tao, H. Baudrand, B. Théron, and S. Vigneron, "A two-step synthesis of broadband rigged bandpass filters with improved performances," *IEEE Trans.-MTT*, Vol. 39, 2192–2197, December 1991.
8. Lilonga-Boyenga, D., J. Tao, and T. H. Vuong, "A new multimodal variational analysis of uniaxial discontinuities in microwaves devices," *Proceeding Mediterranean Microwave Symposium MMS'2004*, 112–114, Marseille, 2004.
9. Couffignal, P., H. Baudrand, and B. Theron, "A new rigorous method for determination of iris dimensions in dual-mode cavity filters," *IEEE Trans.-MTT*, Vol. 42, 1314–1320, July 1994.
10. Lilonga-Boyenga, D., J. Tao, and T. H. Vuong, "Uniaxial

- discontinuities by a new multimodal variational method — Application to filter design,” *International Journal of RF and Microwave Computer-Aided Engineering*, Vol. 17, No. 1, 77–82, January 2007.
11. Vuong, T. P., “Contribution à l’étude des discontinuités dans les guides métalliques creux — Applications aux antennes et filtres,” Thèse de doctorat INP Toulouse, December 1999.
 12. Ilić, M. M., A. Z. Ilić, and B. M. Notaros, “Higher order large-domain FEM modeling of 3-D multiport waveguide structures with arbitrary discontinuities,” *IEEE Trans.-MTT*, Vol. 52, 1608–1614, June 2004.
 13. Arndt, F., J. Bornemann, D. Heckmann, C. Pionek, H. Sommerow, and H. Schueler, “Modal S-matrix method for optimum design of inductively direct-coupled cavity filters,” *IEEE Proceedings*, Vol. 133, Pt. H, No. 5, 341–350, October 1986.
 14. Arndt, F., Th. Sieverding, et al., “Optimized-oriented design of rectangular and circular waveguide components with the use of efficient mode-matching simulators in commercial circuit CAD tools (invited article),” 37–50, John Wiley & Sons Inc., 1997.