

**PERMITTIVITY MEASUREMENT WITH A
NON-STANDARD WAVEGUIDE BY USING TRL
CALIBRATION AND FRACTIONAL LINEAR DATA
FITTING**

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Abstract—Modifications in the measurement of the complex permittivity are described, based on the transmission and reflection coefficients of a dielectric slab. The measurement uses TRL two-port calibration to bring the reference planes accurately to the sample surface. The complex permittivity as a function of frequency is computed by minimizing the difference between the measured and the ideal scattering parameters. An alternative procedure for determining the complex permittivity uses the fractional linear data fitting to a Q-circle of the virtual short-circuit and/or virtual open circuit data. In that case, the sample must be a multiple of one-quarter wavelength long within the measured range of frequencies. Comparison with results obtained by other traditional procedures is provided.

1. INTRODUCTION

Microwave material characterization is a classical topic which has an extensive work published in the past few decades. Transmission line approach to extract microwave properties is a powerful technique which provides the complex permittivity of materials under interest within the desired frequency range. Many modifications are suggested to the transmission/reflection methods and reflection methods to extract complex permittivity accurately [1–4]. The present work also falls

under this category and will try to explore different possible ways to extract dielectric constant with a single measurement of S-parameters.

The primary objective of this paper is to demonstrate that even in a non-standard waveguide, most of the published data extraction procedures can be accomplished by a single two port measurement of scattering (S)-parameters. Also, an alternative procedure of determining the real and imaginary parts of the complex permittivity $\varepsilon_r = \varepsilon'_r - j\varepsilon''_r$ will be described which minimizes the difference between the measured S-parameters and the computed S-parameters.

The secondary objective of the paper is to describe the Q-factor measurement technique for obtaining the average values of ε'_r and ε''_r . The same measured S-parameters are used to derive the virtual short circuit (VSC) and the virtual open circuit (VOC) values of the input reflection coefficient. From the values of unloaded Q , it is namely possible to determine ε''_r , and from the resonant frequency it is possible to determine ε'_r . This method requires that the sample length is a multiple of one-quarter wavelength at some frequency within the measured range of frequencies.

The reason for using a non-standard waveguide of the square cross section 20×20 mm is that one will be able to measure two orthogonal polarizations of the dominant mode, necessary for a later investigation of anisotropic materials.

2. TRL MEASUREMENT PROCEDURE

The waveguide fixture consists of three sections which are connected to each other with waveguide flanges. A through section of approximately quarter wavelength is used for Thru-Reflect-Line (TRL) calibration [5]. Figure 1 shows a schematic diagram of the fixture.

For a non-standard waveguide, the well-matched coaxial-to-waveguide transitions are not available. But, even for custom-made transitions, it is possible to use the TRL calibration in the waveguide, so that all the transition imperfections are calibrated out. All the measured S-parameters are later shifted to the sample surface through lengths ℓ_1 and ℓ_2 .

The Nicholson-Ross-Weir (NRW) procedure [6,7] is a popular technique to extract complex permittivity using the measured S-parameters S_{11m} , S_{21m} , S_{12m} , S_{22m} , where the subscript m is used for measured parameters and subscript c will later be used for calculated parameters. This procedure yields an explicit expression for permittivity as a function of the measured S-parameters. We have compared the NRW procedure with the results obtained by minimization of the difference between the computed and measured

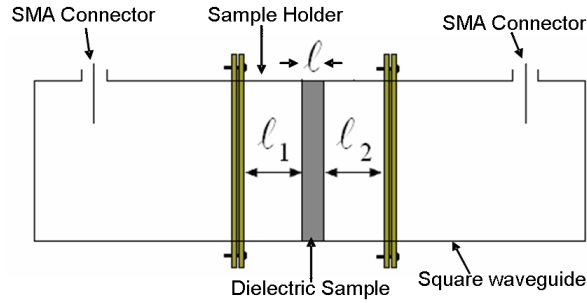


Figure 1. Waveguide fixture with a dielectric sample of thickness ℓ placed at the center of sample holder.

S-parameters, and obtained a reasonable agreement ($\sim 0.8\%$). This minimization technique calculates S-parameters for an assumed value of permittivity ϵ_r as follows.

$$S_{11} = \frac{\left(\frac{1}{R} - R\right) (e^{-\gamma\ell} - e^{\gamma\ell})}{D} \quad (1)$$

$$S_{21} = \frac{4}{D} \quad (2)$$

where the denominator is given by:

$$D = \left(R + \frac{1}{R} + 2\right) e^{\gamma\ell} - \left(R + \frac{1}{R} - 2\right) e^{-\gamma\ell} \quad (3)$$

Quantity R is the ratio of the characteristic impedance of the waveguide filled with the sample material and that of the empty waveguide:

$$R = \frac{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}{\sqrt{\epsilon_r' - j\epsilon_r'' - \left(\frac{f_c}{f}\right)^2}} \quad (4)$$

where f_c is the cut-off frequency of the empty waveguide and f is the frequency of operation. The propagation constant in the waveguide, filled with dielectric material, is a complex quantity

$$\gamma = j\frac{2\pi f}{c} \sqrt{\epsilon_r' - j\epsilon_r'' - \left(\frac{f_c}{f}\right)^2} \quad (5)$$

For an idealized model, $S_{22} = S_{11}$ and $S_{12} = S_{21}$. However, due to the mechanical imperfections of the waveguide and due to the limited accuracy of the Network Analyzer, the measured S-parameters are not fully symmetric.

The squared difference Δ^2 between the measured and calculated values of S-parameters is minimized by varying the values of ε'_r and ε''_r :

$$\begin{aligned} \Delta^2 = & |S_{11m} - S_{11c}|^2 + |S_{21m} - S_{21c}|^2 \\ & + |S_{12m} - S_{21c}|^2 + |S_{22m} - S_{11c}|^2 \end{aligned} \quad (6)$$

“fminsearch” command in MATLAB [8] is used for minimization based on the simplex search method [9]. This optimization procedure results in frequency dependent values of ε'_r and ε''_r . On a typical personal computer, the optimization of 201 measured points takes about six seconds.

3. VIRTUAL REFLECTION COEFFICIENTS

It is possible to obtain reflection coefficient Γ_{in} for a short-circuited sample, without the need of a physical short-circuit. Virtual Short Circuit (VSC) value can namely be expressed in terms of all the four S-parameters. Figure 2 shows the schematic diagram to calculate the VSC reflection coefficient.

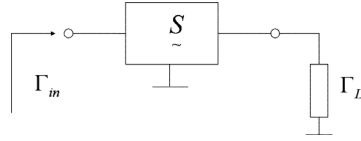


Figure 2. Input reflection coefficient of a two-port.

The input reflection coefficient of a two-port network in Figure 2 is:

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \quad (7)$$

The virtual input reflection of a short-circuited sample is obtained by substituting the load reflection coefficient $\Gamma_L = -1$. Likewise, the Virtual Open Circuit (VOC) reflection coefficient is obtained by substituting $\Gamma_L = +1$. These VSC and VOC values will be used for post-processing in the Q-factor method. Also, a displaced VSC and VOC data will be used in the SCL method [10] as explained in Section 5.

4. Q-FACTOR METHOD

Resonant cavity methods are known to yield the most accurate determination of the permittivity. Both a short-circuited sample as well as an open-circuited sample for which the lengths are multiples of $\lambda_g/4$ constitute such resonant structures. From the value of the resonant frequency and the value of the unloaded Q factor one can deduce the values of ϵ'_r and ϵ''_r . Instead of actually short-circuiting or open-circuiting the sample, the VSC and VOC data are used.

The input impedance of a short-circuited transmission line, one quarter wavelength long, displays a behavior of a parallel resonant circuit, exhibiting very high values of impedance when normalized to characteristic impedance of the waveguide R_{c1} . To obtain a Q-circle suitable for data fitting, the input reflection coefficient Γ_2 should be renormalized to a much higher value of the characteristic impedance R_{c2} . The relationship between the input reflection coefficient Γ_1 , relative to R_{c1} , and Γ_2 , relative to R_{c2} is given by:

$$\Gamma_2 = \frac{\Gamma_1 + A}{\Gamma_1 A + 1} \quad (8)$$

where

$$A = \frac{1 - \frac{R_{c2}}{R_{c1}}}{1 + \frac{R_{c2}}{R_{c1}}} \quad (9)$$

Figure 3 shows the virtual short circuit input reflection coefficient after renormalization to $R_{c2}/R_{c1} = 100$. This value was selected so that the diameter of the Q-circle comes out to be smaller than unity. A fractional linear data fitting method [11] is used to determine the corresponding unloaded Q-factor Q_0 and the resonant frequency f_r . This procedure accepts the input reflection coefficients in an arbitrary order, and does not require any previous data conditioning. As a function of frequency, the input reflection coefficient is fitted to the fractional linear expression:

$$\Gamma_i = \frac{a_1 t + a_2}{a_3 t + 1} \quad (10)$$

In the above, t is a normalized frequency variable given as

$$t = 2 \frac{\omega - \omega_r}{\omega_r} \quad (11)$$

The complex coefficients a_1 , a_2 , a_3 are evaluated by solving a system of overdetermined linear equations. These coefficients are then

used to compute the unloaded Q factor and the resonant frequency of the resonator. The details of the procedure can be found in [10, 11]. Figure 3 shows the results obtained by QZERO software (Student version of software QZERO for Windows is distributed freely to students and faculty at educational institutions by sending a request to eedarko@olemiss.edu. The full version is available from Vector Forum, P. O. B. 757, University, MS 38677, USA.), for a Plexiglas sample of thickness $\ell = 5.95$ mm, for an impedance ratio of $R_{c2}/R_{c1} = 100$. The unloaded Q factor in this case is $Q_0 = 123.7 \pm 2.2$, and the resonant frequency is 8.9999 GHz. For the same sample, the VOC input reflection coefficient is shown in Figure 4, which is plotted by QZERO after processing the input coefficient by the impedance ratio of 1/500. Here, one obtains $Q_0 = 123.9 \pm 2.5$, and the resonant frequency 9.0010 GHz.

From the fact that the resonant frequency corresponds to a 1/4 of a waveguide wavelength, one can compute the real part of the dielectric constant:

$$\epsilon'_r = \left(\frac{c}{4\ell f_r} \right)^2 + \left(\frac{f_c}{f_r} \right)^2 \quad (12)$$

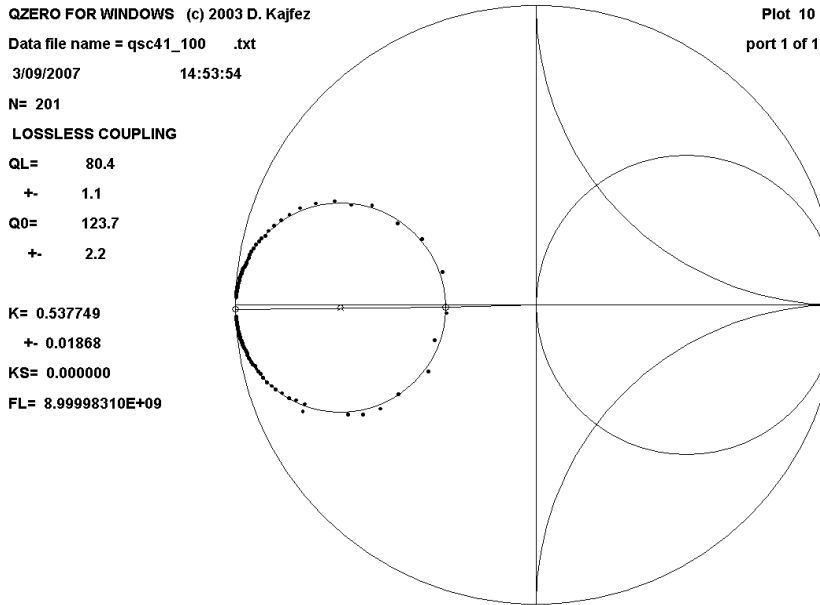


Figure 3. The input reflection coefficient of a short-circuited sample renormalized to characteristic impedance ratio $R_{c2}/R_{c1} = 100$.

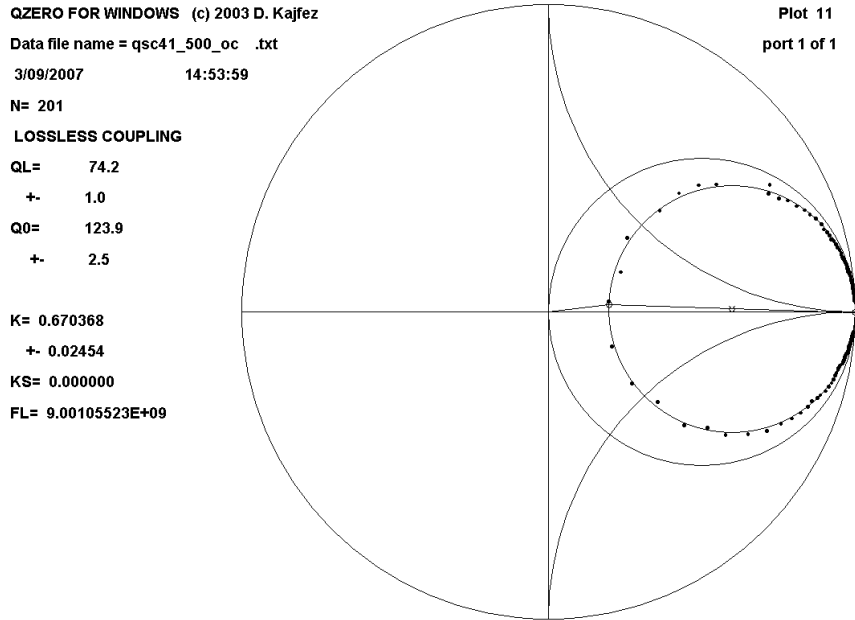


Figure 4. Determination of the Q-factor from the virtual open circuit data, normalized to characteristic impedance ratio $R_{c2}/R_{c1} = 1/500$.

The imaginary part is determined with the help of Q_0 as:

$$\varepsilon_r'' = \frac{\varepsilon_r'}{Q_0} \quad (13)$$

The above two values can be considered as the averaged values valid for the range of frequencies within which the measured data fit the Q-circle. It can be seen that at least 40 frequency points contribute to the best-fit circles in Figures 3 and 4, so that even with noisy data, one obtains a good estimate of Q_0 . Therefore, the VSCQ and VOCQ methods should be well suited for measuring the low-loss materials.

Simple estimates of the uncertainties of the above two quantities can be determined as follows. The dominant uncertainty in (12) comes from the inaccuracy of the sample length measurement, while the uncertainty of the cutoff frequency and the resonant frequency is an order of magnitude smaller. Therefore, the standard deviation of ε_r' is simply

$$\sigma(\varepsilon_r') = 2 \left(\frac{c}{4\ell f_r} \right)^2 \frac{\sigma(\ell)}{\ell} \quad (14)$$

Similarly, from (13), the standard deviation of ε_r'' is

$$\sigma(\varepsilon_r'') = \frac{1}{Q_0} \sqrt{\sigma^2(\varepsilon_r') + \left(\varepsilon_r' \frac{\sigma(Q_0)}{Q_0}\right)^2} \quad (15)$$

5. EXPERIMENTAL RESULTS

Measurements were conducted using the square waveguide fixture shown in Figure 5 over the frequency range from 8 to 10 GHz, with the Network Analyzer model HP8510C. The measured two-port S parameters have been referenced to the sample surfaces and then processed in several ways.

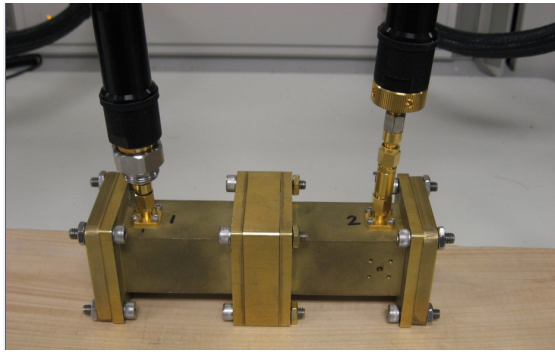


Figure 5. Waveguide fixture with cables leading to the Network Analyzer.

To find the accurate value of the propagation constant in the waveguide, which is needed for shifting the reference planes, the entire length of the empty sample holder was processed as being the sample to be measured. By using the Q-factor method, the waveguide losses were found to correspond to $Q_0 = 1351$. The minimization procedure yielded the real part of permittivity to be 1.0025, which may be caused by the humidity of air in the laboratory. These two values were afterwards used for computing the S-parameters at the sample surface planes.

It has been observed that placing the sample exactly at the center of the sample holder is very critical. For a passive and reciprocal sample material, the scattering coefficients S_{11} and S_{22} should be identical to each other. However, the observed values differ from each other, as shown in Figure 6. We have concluded that the difference was mainly caused by the inequality of lengths ℓ_1 and ℓ_2 in Figure 1. Therefore, we

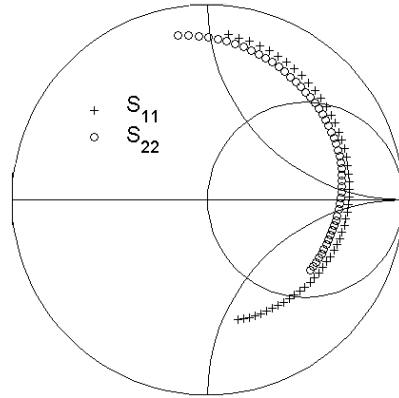


Figure 6. The measured values S_{11} and S_{22} (each fourth value plotted).

measured the phase difference between S_{11} and S_{22} and corrected the values of lengths ℓ_1 and ℓ_2 , so that the phases coincide at the center frequency. The corresponding correction of position came out to be 0.5 mm, which is entirely plausible for manually placing the sample into the waveguide. After this correction, the permittivities resulted in more consistent values.

Figure 7 shows the results for the real part of the relative dielectric constant as a function of frequency. Minimization of difference Δ^2 from (6) is denoted Method #1. The two Q factor methods are denoted Method #3 and Method #4. They are shown as single points which fall on top of each other. The processing of data by Nicholson-Ross-Weir formulas given in [12] is denoted as Method #2. This line passes through the Q-factor points. The virtual short-circuit-line (VSCL) method [12] is denoted as Method #5. This method calls for the measured data of a displaced short-circuited sample. Instead of actually moving the sample away from the short circuit plate, we computed the VSCL data displaced by zero.

Another novel method of processing the measured data (Method # 6) is the virtual-open-circuit-line method (VOCL). In that method, the back face of the sample is displaced by distance ΔL from an ideal open circuit ($\Gamma_L = +1$). For VSCL and VOCL methods, the data are processed by using the following equations [12]:

$$S_{11c} = \frac{-2R\delta + [R^2(\delta + 1) + (\delta - 1)] \tanh \gamma L}{2R + [R^2(\delta + 1) - (\delta - 1)] \tanh \gamma L} \quad (16)$$

$$\delta = \pm e^{-2\gamma_0 \Delta L} \quad (17)$$

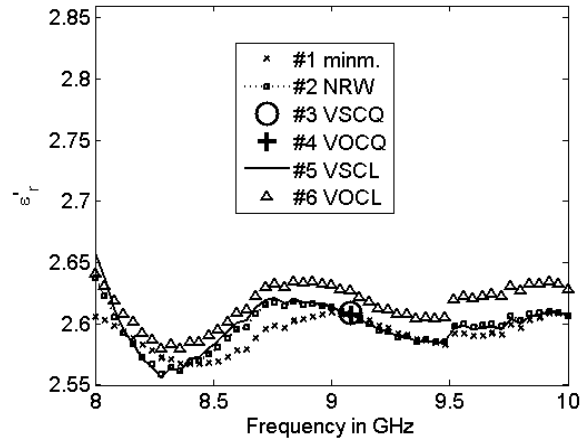


Figure 7. Real part of permittivity measured for Plexiglas sample.

The positive sign is used for VSCL case, and the negative sign is used for VOCL case. For the sample at hand, the displacement is $\Delta L = 0$.

The imaginary part of permittivity is shown in Figure 8. The variation of results with frequency is considerably larger than for the real part of permittivity.

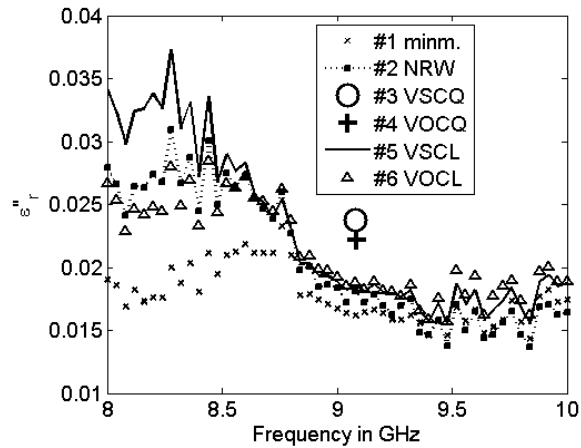


Figure 8. Imaginary part of permittivity measured for Plexiglas sample.

The frequency-dependent data of Methods #1, #2, #5, and #6 were statistically processed to evaluate their means and their standard deviations. Together with the mean and standard deviations for Methods #3 and #4 they are summarized in Table 1. It can be seen that the mean values for ε'_r of all five methods differ from each other for less than 1 percent, and the mean values for ε''_r for less than 25 percent.

Table 1. Measurements for ε'_r and ε''_r , for 5.95 mm Plexiglas sample.

Method #	#1	#2	#3	#4	#5	#6
	Minim	NRW	VSCQ	VOCQ	VSCL	VOCL
ε'_r	2.5918	2.5970	2.6086	2.6069	2.5983	2.6155
$\sigma(\varepsilon'_r)$	0.0129	0.0173	0.0648	0.0647	0.0193	0.0171
ε''_r	0.0177	0.0206	0.0238	0.0222	0.0228	0.0213
$\sigma(\varepsilon''_r)$	0.0022	0.0052	0.0015	0.0012	0.0065	0.0038

The influence of the air gap between the sample and the top and bottom walls of the waveguide has been also investigated. Corrugated samples such as described in [13] have been fabricated, with an air gap half the sample width and 0.5 mm and 1 mm in depth. For the 0.5 mm half gap, the averaged ε'_r of all five methods is 3% lower, and the averaged ε''_r is 10% higher than the values shown above. For the 1 mm half gap, ε'_r is 4% lower, and ε''_r remains to be 10% higher than the values shown above. Thus, for samples with low relative dielectric constant like Plexiglas, a moderate air gap is not a critical factor.

The waveguide fixture of square cross section has been also used for measurement of the carbon nanotube composites [14]. The anisotropy of that material was checked by rotating sample from 0°–360° with 90° step. For that measurement, additional matched waveguide loads had to be inserted at each side of the waveguide fixture, to prevent interior resonances of the system. With a simple coaxial-to-waveguide transition it was possible to maintain –40 dB of decoupling between the two orthogonal polarizations. The minimization procedure was working well also for the high loss materials with $\varepsilon'_r \leq \varepsilon''_r$. For more details, see [14].

6. CONCLUSION

Two modifications suitable for measurement of low-loss dielectric materials are described namely (1) the minimization of the difference between the measured and computed S-parameters and (2) the Q-factor method based on the linear fractional data fitting. The two-port measurements are performed on a dielectric sample in a non-standard waveguide, based on the TRL calibration. Instead of measuring separately the short-circuited sample and/or the sample displaced from the short circuit, the virtual data for the short-circuit, the open-circuit and/or the displaced short-circuit can be computed from the measured two-port S parameters. The comparison with the other published procedures demonstrates the validity of using the virtual data.

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